RPA quasi-elastic responses in infinite and finite nuclear systems

Eduardo Bauer*

Departamento de Física, Facultad de Ciencias Exactas,
Universidad Nacional de La Plata,
La Plata, 1900, Argentina

and

Giampaolo Co'

Dipartimento di Fisica, Università di Lecce
and Istituto Nazionale di Fisica Nucleare, sezione di Lecce,
I-73100 Lecce, Italy

Quasi-elastic responses in nuclear matter and in ^{12}C and ^{40}Ca nuclei are calculated in ring approximation to investigate the finite size effects on the electromagnetic quasi-elastic responses. A method to simulate these effects in infinite systems calculations is proposed. The sensitivity of the results to the various terms of the residual interaction is studied. The results of nuclear matter RPA calculations are compared with those obtained in ring approximation to evidence the importance of the exchange terms.

PACS number: 21.60.Jz, 25.30.Fj, 27.20.+n, 27.40.+z

I. INTRODUCTION

The quasi-elastic excitation of the nucleus is particularly interesting for nuclear structure studies because of the interplay between single particle and many-body effects. The excitation energies characterizing the quasi-elastic peak are well above the nucleon emission threshold, therefore, one or more nucleons are ejected from the nucleus. Before being ejected, any nucleon can interact with the others nucleons composing the nucleus. Considering this effect in a finite nucleus formalism is a difficult task because the nuclear final states are described in terms of their total angular momentum and the presence of the continuum implies the sum on a large amount of possible configurations.

In a nuclear matter formalism one takes advantage of the translational invariance to simplify the description of the final state of the hadronic system. The use of the infinite system formalism is rather appropriate for the quasi-elastic electron excitation, since excitation energies and transferred momenta are such that the excitation process is well localized within the nucleus and collective surface excitations are negligible. In any case additional approximation (say, a variable Fermi momentum or the local density approximation) have been considered in the literature to simulate finite size effects.

We can classify the necessary ingredients to describe the process in three main issues: the single particle basis, the inclusion of initial and/or final state interactions and the operator describing the action of the external probe. Regarding the first point we have already said that some work treats the nucleus as a finite system [1]- [11], while other one use a nuclear matter approach [12]- [25].

Referring to the second point, one of the simplest approaches to include initial and final state interactions is the Random Phase Approximation (RPA), with or without exchange terms (the last approach named ring approximation). In the RPA [1]- [5], [12]- [14] one particle-one hole ($1p1h$) excitations are summed up to infinite order. One step further in complexity is the so-called Second RPA (SRPA) [6], [15] which, in addition to the $1p1h$ excitations, considers also those generated by $2p2h$. The Extended RPA (ERPA) [7], [16], [17]; contains ground state correlations beyond RPA. On a philosophy similar to that of the SRPA is based the Green function approach of Ref. [8], where the relationship between forward virtual Compton scattering and inclusive electron scattering is used to construct a one-body approximation to quasi-elastic electron scattering. The role of the short-range correlations has been investigated in the framework of the correlated basis function theory [10], [18].

Concerning the third point, the external operator is usually represented by electromagnetic one-body operators, but in the transverse channel, it is important to include two-body terms as the Meson Exchange Currents (MEC) [9], [11], [23], [25], and the excitation of the virtual or real $\Delta(1232)$ resonance [19]- [22].

In the present paper we investigate the sources of some inconsistencies between finite and infinite nucleus calculations. For example it seems that for a given residual interactions the effects of the ring approximation are larger in nuclear matter than in finite nuclei. To simulate the finite size effects in the quasi-elastic peak, we have done infinite systems calculations with a diffused Fermi surface. Then, we have studied the effects on the responses of the various characteristics of the interaction, such as the different channels, the range and the density dependence. The infinite systems results have been compared with the calculations done for the ^{12}C and ^{40}Ca nuclei, two doubly magic nuclei with the same number of protons and neutrons. The agreement we have finally obtained between the two kinds of calculations is satisfactory, even for the nucleus ^{12}C, which is supposed to be relatively light to be well represented as an infinite system of nucleons.

Encouraged by this result, we have evaluated the response functions within the nuclear matter RPA framework of Ref. [13] to investigate the effects of the exchange terms neglected in the ring approximation.

The paper is organized as follows. In Sec. II we briefly present the effective theories used to perform the calculations. In Sec. III we compare the results obtained in finite nuclei and nuclear matter both for the free and the ring responses, and we present the nuclear matter RPA responses. Finally, in Sec. IV we draw our conclusions.

II. FORMALISM

The response function for inclusive quasi-elastic electron scattering is given by,

$$R(\boldsymbol{q},\omega) = -\frac{1}{\pi}\ Im < 0|\mathcal{O}^\dagger(\boldsymbol{q})G(\omega)\mathcal{O}(\boldsymbol{q})|0> \tag{1}$$

where ω represents the excitation energy and $\boldsymbol{q}$ the three-momentum transferred by the electron. The nuclear ground state is denoted as $|0>$, while $\mathcal{O}(\boldsymbol{q})$ is the excitation operator and $G(\omega)$ the polarization propagator,

$$G(\omega) = \frac{1}{\omega - H + i\eta}\ - \frac{1}{\omega + H + i\eta} \tag{2}$$

where H is the nuclear Hamiltonian. We introduce the operator P, which projects onto $1p1h$ configurations. The way of building these configurations depends on the particular choice of the single particle basis. In nuclear matter the single particle wave functions are plane waves, while in a finite nucleus they are eigenfunctions of the one-body Schrödinger equation for a mean field potential. In our case we use a real, spherical, Woods-Saxon potential.

The response function generated by $1p1h$ excitations can be expressed as:

$$R_{PP}(\boldsymbol{q},\omega) = -\frac{1}{\pi}\ Im < 0|\mathcal{O}^\dagger(\boldsymbol{q})PG(\omega)P\mathcal{O}(\boldsymbol{q})|0> \tag{3}$$

It is not straightforward to calculate R_{PP}, since the nuclear Hamiltonian is in general not diagonal in the $1p1h$ basis. The solution of the problem is the RPA response. In the present work, we shall compare finite nucleus results obtained within the ring approximation of Ref. [5], with the corresponding nuclear matter responses.

In most calculations done in infinite systems the Fermi surface which separates, in momentum or energy space, the hole from the particle states, is a sharp step function. In the next section, we shall show that this approximation reproduces rather well the position and the shape of the free responses calculated in finite nuclei. However, the ring response are appreciably different when evaluated in nuclear matter and in finite nuclei. The original motivation of the present work was to explain this discrepancy. The first attempt to improve our nuclear matter model, is to replace the step function representing the momentum distribution of particles and holes, by a more realistic one. For the holes with momentum $\boldsymbol{h}$ we make the substitution,

$$\theta(k_F - |\boldsymbol{h}|)\ \rightarrow\ n(h) \tag{4}$$

where k_F is the Fermi momentum, $h \equiv |\boldsymbol{h}|$, and

$$n(h) = \frac{1}{1 + e^{(h-k_F)/a}} \tag{5}$$

being a a constant to be adjusted. An analogous expression can be obtained for particles.

In the next section we shall compare results obtained with this smoothed Fermi surface in ring approximation with those obtained in finite systems calculations. With the same smoothed momentum distribution we have done RPA calculations following the computational scheme developed in Ref. [13] which we briefly describe. In nuclear matter, direct RPA terms can be summed up to infinite order (ring series), but, in general, it is not possible to find a closed form to sum all the exchange terms. Normally these last terms are perturbatively considered and, for numerical reasons, this is done up to the second order. On the other hand, it possible to make the full summation of the exchange terms when a contact interaction is used. Full sums are also possible for separable interactions. To exploit this feature we rewrite the residual interaction V, as:

$$V = V_{contact} + \tilde{V} \tag{6}$$

The $V_{contact}$ term is a contact interaction conveniently chosen to make $\tilde{V}$ small. For $V_{contact}$ both direct and exchange terms are summed up to infinite order while $\tilde{V}$ is perturbatively considered up to the second order which we found to account reasonably well for the whole sum. Interference terms between $V_{contact}$ and $\tilde{V}$ are included up to infinite order in $V_{contact}$ and up to second order in $\tilde{V}$. A more detailed description of the method can be found in Ref. [13]

The finite nucleus calculations have been done within the Fourier-Bessel computational scheme adopted in Ref. [5]. The single particle basis is generated by using a Woods-Saxon mean field whose parameters have been fixed to reproduce experimental rms radii and single particle energies of the bound states close to the Fermi surface. By neglecting the exchange diagrams, the RPA equations are rewritten in terms of local density functions which are expanded on a Fourier-Bessel basis. In this manner the problem to be solved, for every value of the excitation energy, is the diagonalization of a matrix whose dimensions are four times the number of the Fourier-Bessel expansion coefficients. More details about the method can be found in Ref. [24].

III. RESULTS

In this section we compare finite nucleus results with those of nuclear matter. This is done for two nuclei: ^{12}C and ^{40}Ca. The ^{12}C nucleus is perhaps too light to be appropriately described in terms of nuclear matter. On the other hand we wanted to test our model also in extreme situations and, last but not least, the finite nucleus calculations are much less involved than in the ^{40}Ca case.

The transferred momenta analyzed are $q = 400$ MeV/c and $q = 500$ MeV/c. These values are sufficiently large to eliminate the presence of collective surface vibrations, and at the same time, sufficiently small to require a limited number of partial waves. In the finite nucleus calculations we sum multipole excitations up to angular momentum J=12 [9].

The first step of our calculations consists in fixing the values of k_F and a in eq. (5) to reproduce the finite nucleus free responses. For ^{12}C we have obtained the values k_F=0.85 fm^{-1} and a=0.20 fm^{-1} , and for ^{40}Ca, k_F=1.0 fm^{-1} and a=0.17 fm^{-1}. The comparison with the finite nuclei responses is shown in Figs. 1 and 2, where we have also added the results obtained with a step function Fermi surface represented by the dashed lines. In this last case we have used the procedure of Ref. [11] to fix the value of the Fermi momentum obtaining the values k_F=1.09 fm^{-1} for ^{12}C and k_F=1.19 fm^{-1} for ^{40}Ca. These values are noticeably different from those fixed by the smoothed momentum distribution. As expected, k_F increases with increasing mass number. We succeeded in obtaining a satisfactory agreement with the finite nucleus responses, especially for ^{40}Ca, in the case of a diffused Fermi surface. As expected the high energy tail of the finite nucleus responses can be reproduced only by the calculations with a diffused Fermi surface.

The comparison between the various responses calculated in ring approximation is shown in Figs. 3 and 4 for the ^{12}C and ^{40}Ca nuclei respectively. In these figures the meaning of the symbols is analogous to that of the previous figures. The interaction used in these calculations is the finite range polarization potential utilized also in Ref. [5]. The nuclear matter calculations have been done with the values of k_F and a previously fixed.

Two observations should be done about these results. A first one is about the fact that, in general, the full curves reproduce better the finite nucleus results than the dashed ones. In a second place we observe that the longitudinal responses are better reproduced than the transverse ones.

The first observation induces to conclude that the sharp Fermi surface, even with an effective value of the Fermi momentum, is unable to reproduce the finite nucleus results. This feature does not depend from the residual interaction, as we show in Fig. 5 where the various ^{12}C responses have been calculated, always in ring approximation, with a zero range Migdal interaction. We have used the following values of the parameters of this force: $f_0 = 386$, $f'_0 = 289.5$, $g_0 = 106.2$ and $g'_0 = 135.1$, expressed in MeV fm^3 units. These values have been chosen to magnify some of the effects we want to discuss. Specifically, the big value of f_0, ten times larger than the normally used one [26], enhances the difference between the nuclear matter calculations in the longitudinal response. Here it becomes more evident the poor quality of the sharp Fermi surface calculations. Our diffused Fermi surface calculation is able to reproduce rather well the finite nucleus results even in this rather extreme conditions.

Concerning the observation that, the longitudinal responses are always better reproduced than the transverse ones, we have verified that this fact is due to the differences between infinite and finite systems. The isospin channel of the force does not contribute in ring approximation calculations of nuclear matter transverse responses, while it does in finite nuclei calculations. The effects of this difference become evident by comparing the transverse responses of Fig. 5 with Fig. 6. In this last figure the ^{12}C transverse responses have been calculated with the contact interaction above described but without the isospin channel of the force, i.e. by setting $f'_0 = 0$. The agreement between the finite nucleus responses and those obtained with the diffused Fermi surface is comparable with that obtained in the longitudinal case.

We should remark the fact that the polarization potential has a density dependence in the scalar and isospin channels. In the nuclear matter calculations we have used the force parameters defined for the nuclear interior. We have checked the sensitivity of the results by switching off the density dependence in the finite nuclei calculations and comparing with the responses evaluated with the full interactions. The differences found between these two calculations are of the same order of the differences between nuclear matter and finite nuclei longitudinal responses. We conclude that for the calculations of quasi-elastic responses the density dependence of the force is not important.

In Figs. 7 and 8 we compare the RPA nuclear matter responses (full lines) with those evaluated in ring approximation (dashed lines). Both calculations have been done by using the polarization potential and the diffused Fermi surface. This comparison shows the effects of the exchange diagrams evaluated in RPA and neglected in ring approximation. For the particular interaction used these effects are noticeable, especially in the longitudinal responses and for low values of the momentum transfer.

In the same figures we also present the free responses (dotted lines) and the ^{12}C [27] and ^{40}Ca [28] experimental point. Our results show that the major source of disagreement between free responses and data is produced by correlations beyond the RPA. In Ref. [5] the role of the final state interactions and of the effective mass was pointed out. The inclusion of these two effects within a simplified model produces a good agreement with the ^{40}Ca data [9]. The same model is however unable to explain the ^{12}C transverse response data.

IV. CONCLUSIONS

In the present work we have compared nuclear matter and finite nuclei quasi-elastic responses induced by electron scattering. We have proposed a computational scheme which is able to reproduce the main features of the finite nucleus and, at the same time, it has the computational advantages of nuclear matter. This has been achieved by using a diffused momentum distribution of particles and holes in nuclear matter calculations. We have described the momentum distribution with a simple Fermi function depending from two parameters whose values have been adjusted to reproduce the finite nuclei free responses.

With this simple model we have calculated the ^{12}C and ^{40}Ca responses for different values of the momentum transfer in ring approximation. The agreement with the finite nucleus calculations is excellent for the longitudinal responses. We traced the source of the small differences found in the transverse responses to the isospin part of the residual interaction which is not active in nuclear matter calculations.

Using the same residual interaction, the polarization potential, we have done RPA calculations, i. e. we have also considered the exchange diagrams. The aim of this calculation was to test its feasibility. Finite nuclei RPA calculations have been done [1]- [4], but they require a large computational effort, while our method is simpler and more suitable to be used for those extensions beyond RPA which are necessary to describe the experimental data.

With respect to this last point some words of caution are necessary to avoid double counting. A first one is about the residual interaction which in RPA calculations cannot be the polarization potential constructed in Ref. [29] to consider in average manner the exchange diagrams. A second warning is about the diffuseness of the Fermi surface which is partially produced by correlations effects beyond RPA.

Our approach should be compared with the most commonly used method to account for finite nucleus effects in nuclear matter calculations: the local density approximation (LDA). Unfortunately this comparison is not straightforward. In LDA the responses are calculated for several values of the Fermi momentum and then they are appropriately averaged to obtain the final result. In the averaging procedure the way how the responses for each k_F are weighted, differs from nucleus to nucleus. However, for each k_F a sharp Fermi surface is employed and this implies that the second term in the right hand side of Eq. (2) does not contribute. This terms, however, gives a contribution when a smooth Fermi surface is considered. This contribution is normally small but it becomes appreciable when $\omega \leq 20$ MeV . This is the formal difference between our method and the LDA. From the pragmatical point of view the poor quality of LDA results in reproducing finite nuclei quasi-elastic responses has already been pointed out in Ref. [11], where the effective momentum approximation was found to be better. Here we have shown the superiority of the diffused Fermi surface scheme even with respect to the effective Fermi momentum approximation. One should also consider that from the computational point of view our scheme is superior to the LDA. In the last case one has to calculate nuclear matter responses for different values of k_F, while in our case a single calculation is necessary. For the fast calculations done in ring approximation, this difficulty is irrelevant, but it becomes an handicap for RPA calculations, or for more elaborated ones like SRPA or ERPA.

ACKNOWLEDGMENTS

We thank A.M. Lallena for useful discussions.

---

* Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas, CONICET.

[1] M. Cavinato, D. Drechsel, E. Fein, M. Marangoni and A.M. Saruis, Nucl. Phys. **A423**, 376 (1984); M. Cavinato, M. Marangoni and A.M. Saruis, Phys. Lett. **B235**, 346 (1990); A.M. Saruis, Phys. Rep. **235**, 57 (1993).

[2] A. Dellafiore, F. Lenz and F.A. Brieva, Phys. Rev. **C31**, 1088 (1985); F.A. Brieva and A. Dellafiore, Phys. Rev. **C36**, 899 (1987).

[3] T. Shigehara, K. Shimizu and A. Arima, Nucl. Phys. **A492**, 388 (1989).

[4] M. Buballa, S. Drożdż, S. Krewald and J. Speth, Ann. Phys. (N.Y.) **208**, 346 (1991); M. Buballa, S. Drożdż, S. Krewald and A. Szczurek, Phys. Rev. C **44**, 810 (1991); S. Jeschonnek, A. Szczurek, G. Co' and S. Krewald, Nucl. Phys. **A570**, 599 (1994).

[5] G. Co', K. F. Quader, D. R. Smith and J. Wambach, Nucl. Phys. **A485** 61 (1988).

[6] S. Drożdż, G. Co', J. Wambach and J. Speth, Phys. Lett. **B185**, 287 (1987); S. Drożdż, S. Nishizaki, J. Speth and J. Wambach, Phys. Rep. **197** 1 (1990).

[7] K. Takayanagi, K. Shimizu and A. Arima 1988 Nucl. Phys. **A477** 205 (1988); K. Takayanagi, Phys. Lett. **B230** 11 (1989); Nucl. Phys. **A510** 162 (1990); *ibid.* **A522** 494 (1991); *ibid.* **A522** 523 (1991); *ibid.* **A556** 14 (1993).

[8] C. R. Chinn, A. Picklesimer and J. W. Van Orden, Phys. Rev. **C40** 1159 (1989); F. Capuzzi, C. Giusti and F. D. Pacati, Nucl. Phys. **A524** 681 (1991); F. Capuzzi, Nucl. Phys. **A554** 362 (1993).

[9] J. E. Amaro, G. Co' and A. M. Lallena, Ann. Phys. (N.Y.) **221**, 306 (1993); Nucl. Phys. **A578**, 365 (1994).

[10] G. Co' and A. M. Lallena, Ann. Phys. (N.Y.) (to be published), arXiv:nucl-th/0007032.

[11] J. E. Amaro, A.M. Lallena and G. Co', Int. J. Mod. Phys. **E3**, 735 (1994).

[12] W. M. Alberico, M. B. Barbaro, A. De Pace, T. W. Donnelly and A. Molinari, Nucl. Phys. **A563**, 605 (1993); M. B. Barbaro, A. De Pace, T. W. Donnelly and A. Molinari, Nucl. Phys. **A596**, 553 (1996); *ibid.* **A598**, 503 (1996); A. De Pace, Nucl. Phys. **A635**, 163 (1998).

[13] E. Bauer, A. Ramos and A. Polls, Phys. Rev. **C54**, 2959 (1996).

[14] E. Bauer and A. Lallena, Phys. Rev. **C59**, 2603 (1999).

[15] E. Bauer, J. Phys. **G26**, 145 (2000).

[16] W. M. Alberico, M. Ericson and A. Molinari, Ann. Phys. (N.Y.) **154**, 356 (1984); W. M. Alberico, A. De Pace, A. Drago and A. Molinari, Rivista del Nuovo Cimento **14**, 1 (1991); W. M. Alberico, T. W. Donnelly and A. Molinari, Nucl. Phys. **A512**, 541 (1990).

[17] E. Bauer, Nucl. Phys. **A589**, 669 (1995); E. Bauer, A. Polls and A. Ramos, Phys. Rev. **C58**, 1052 (1998).

[18] S. Fantoni and V. R. Pandharipande, Nucl. Phys. **A427** 473 (1984); A. Fabrocini and S. Fantoni, Nucl. Phys. **A503** 375 (1989); O. Benhar, A. Fabrocini and S. Fantoni, Nucl. Phys. **A550** 201 (1992).

[19] A. Gil, J. Nieves and E. Oset, Nucl. Phys. **A627** 543 (1997); A. Gil, Ph. D. Thesis, Universitat de València, 1996.

[20] R. Cenni, F. Conte and P. Saracco, Nucl. Phys. **A623**, 391 (1997).

[21] E. Bauer, Nucl. Phys. **A637**, 243 (1998).

[22] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly and A. Molinari, Nucl. Phys. **A657**, 161 (1999).

[23] J. W. Van Orden and T. W. Donnelly, Ann. Phys. (N.Y.) **131** 451 (1981).

[24] R. de Haro, S. Krewald and J. Speth, Nucl. Phys. **A388** 265 (1982); G. Co' and S. Krewald, Nucl. Phys. **A433** 392 (1985).

[25] E. Bauer, Phys. Rev. **C61**, (2000).

[26] G. Rinker and J. Speth, Nucl. Phys. **A306** 360 (1978).

[27] P. Barreau *et al*, Nucl. Phys. **A402** 515 (1983).

[28] C. F. Williamson *et al*, Phys. Rev. **C56**, 3152 (1997).

[29] D. Pines, K.F. Quader and J. Wambach, Nucl. Phys. **A477** 365 (1988).

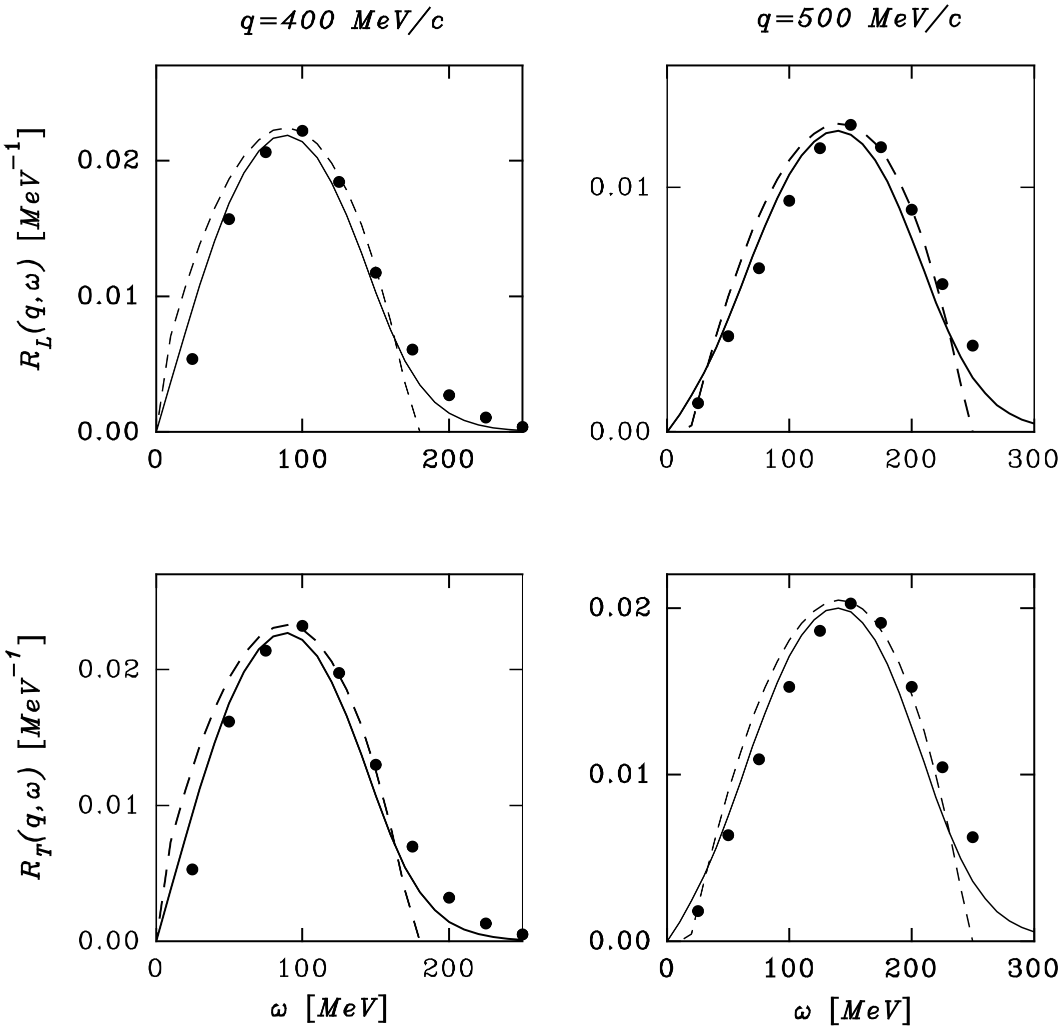

FIG. 1. Nuclear matter Free responses in units of MeV^{-1}, for two values of the momentum transfer compared with the continuum shell model $^{12}\mathrm{C}$ responses (black points). The full lines have been obtained using a diffused Fermi surface, the dashed ones with a step functions Fermi surface. In both cases the values of the parameters have been modified to reproduce the finite nucleus responses.

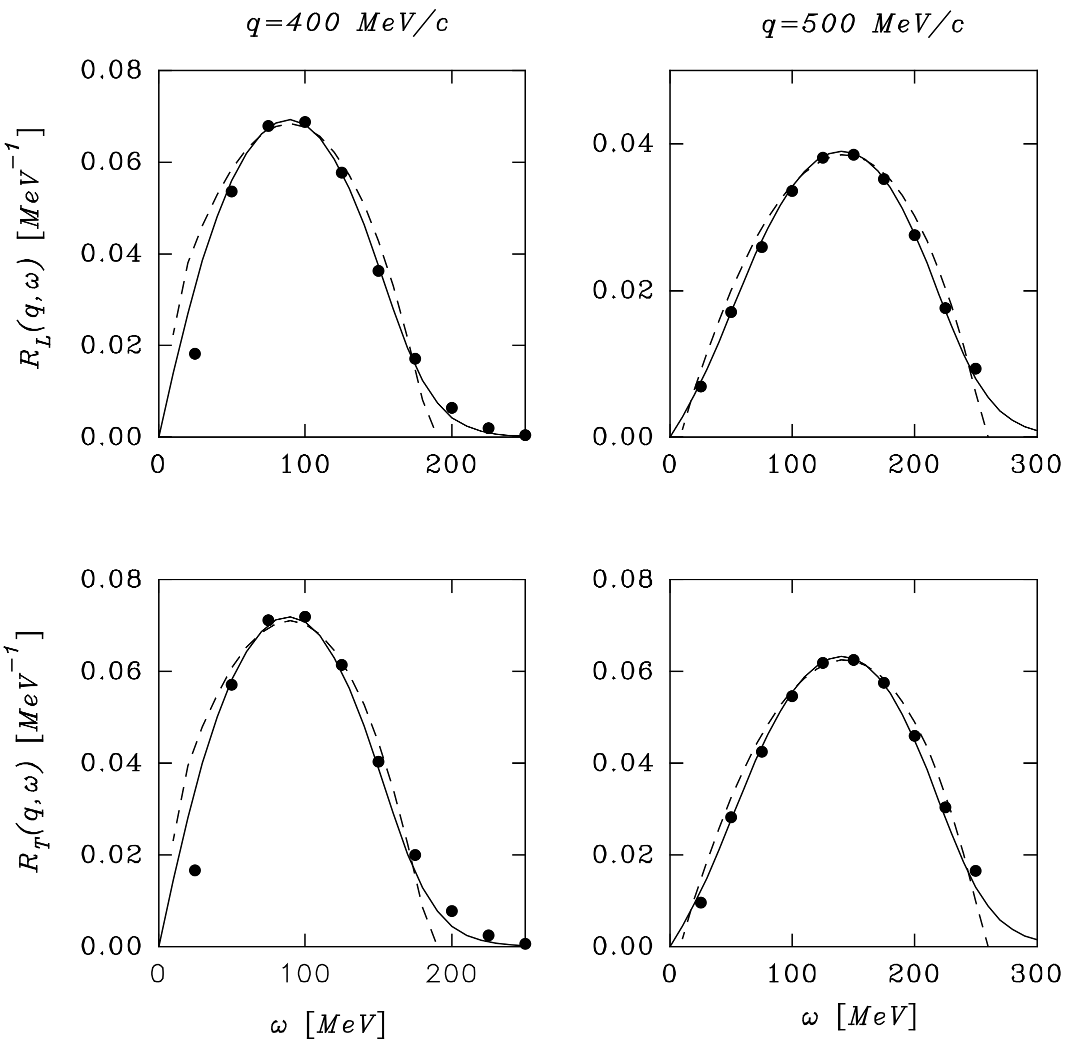

FIG. 2. The same as Fig. 1 but for ^{40}Ca.

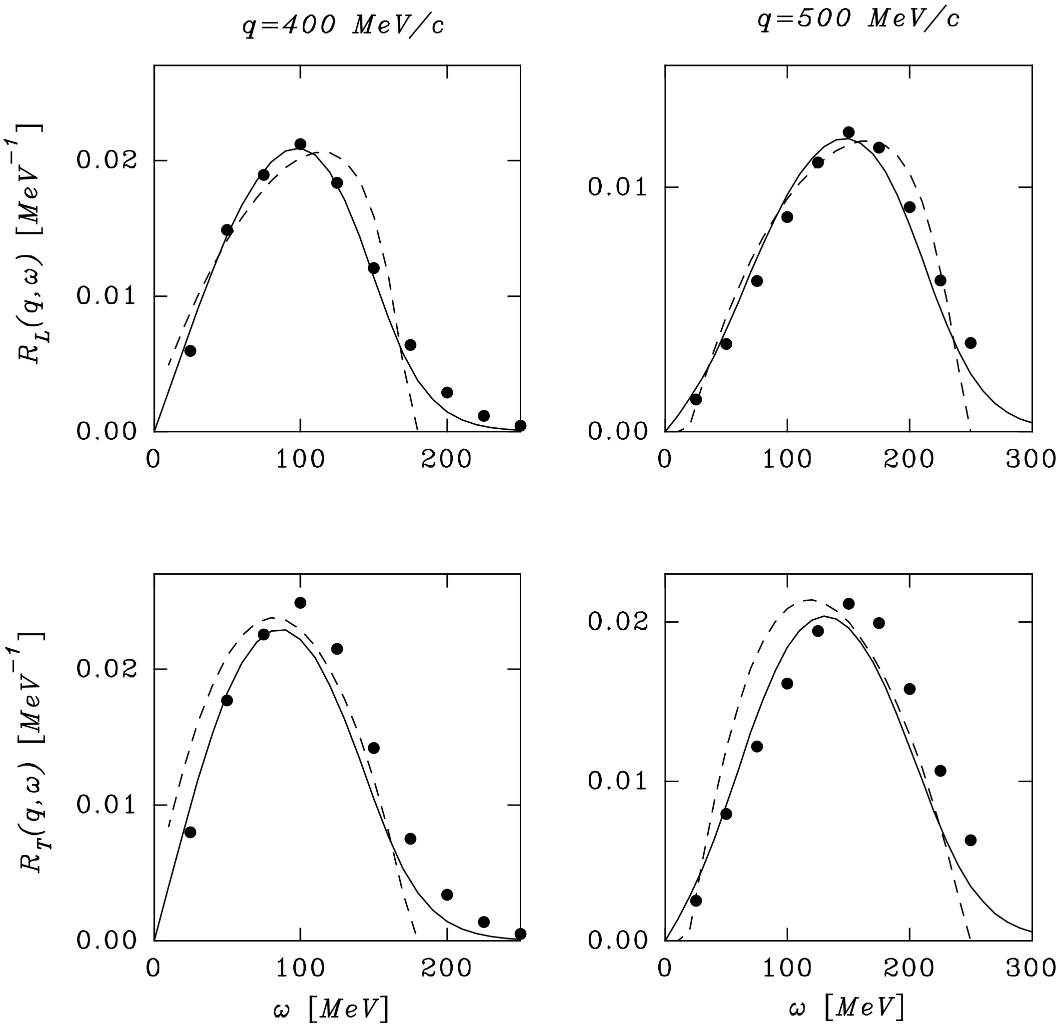

FIG. 3. ^{12}C responses calculated in ring approximation with the polarization potential. The meaning of the lines and of the symbols is the same as in Fig. 1.

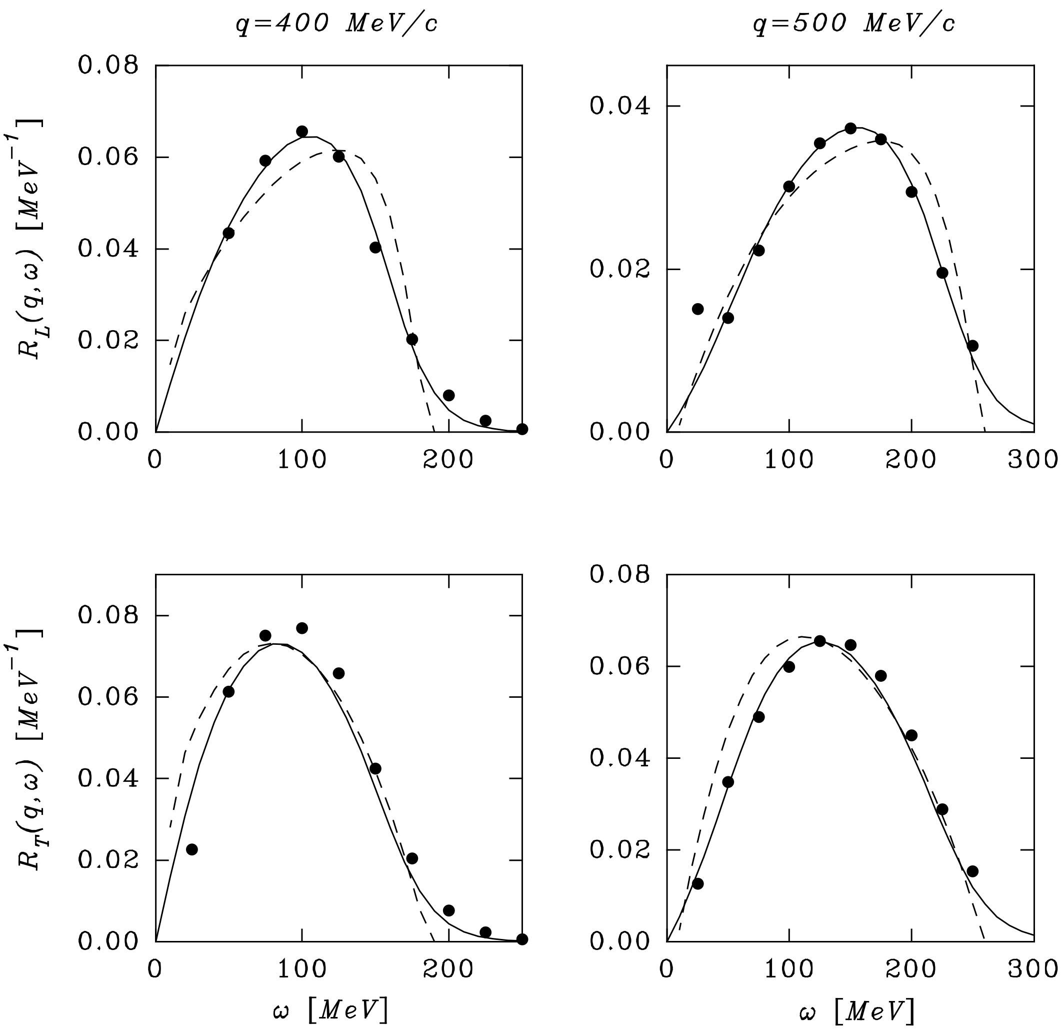

FIG. 4. Same as Fig. 3 for ^{40}Ca.

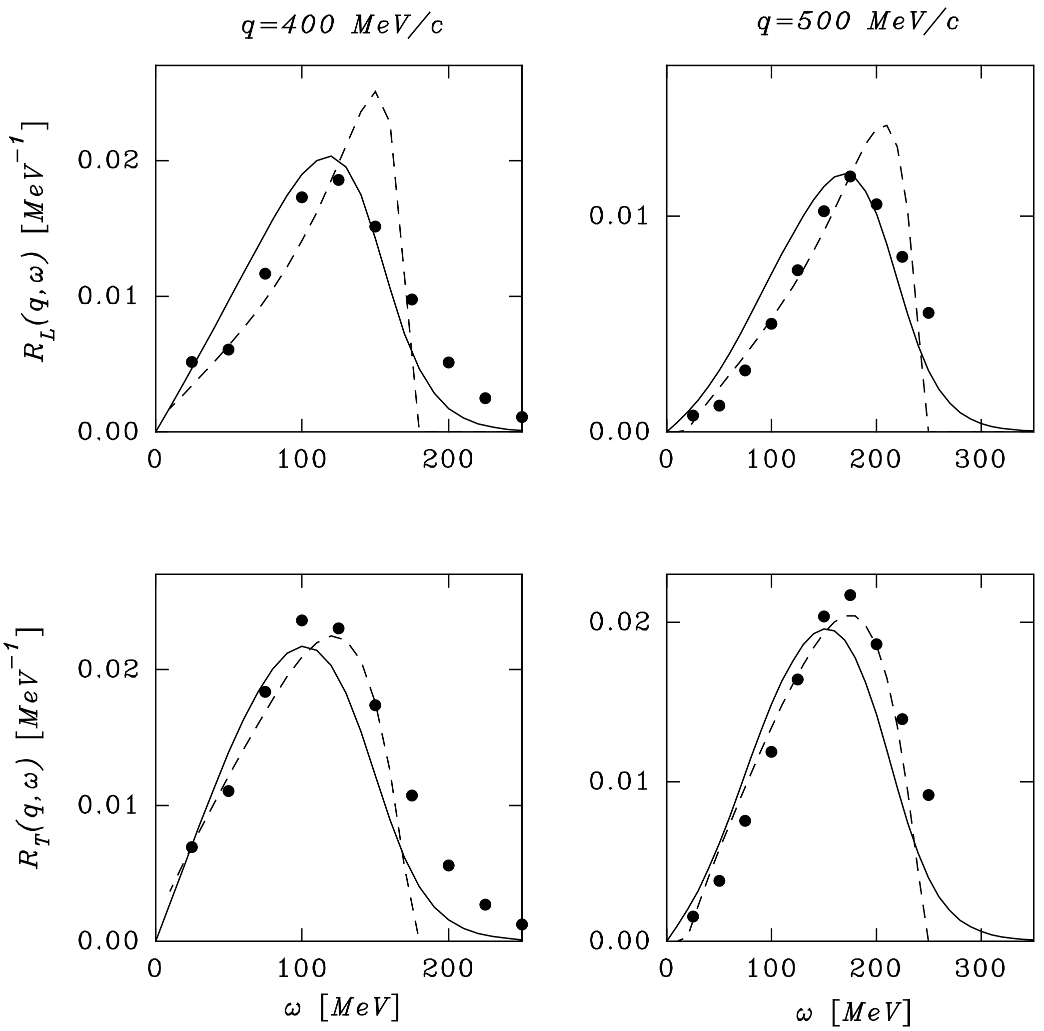

FIG. 5. ^{12}C responses calculated in ring approximation the zero-range Migdal interaction.

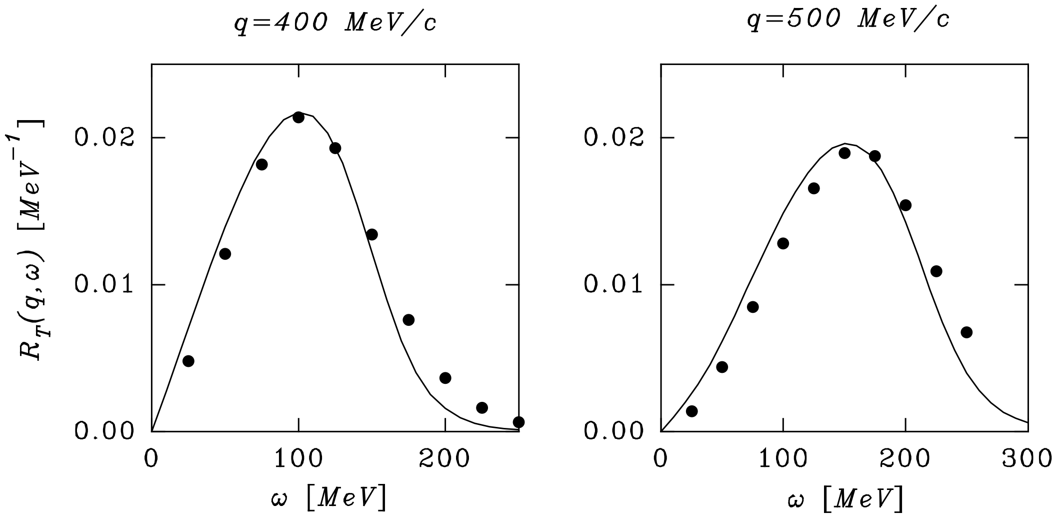

FIG. 6. Transverse ^{12}C responses calculated in ring approximation with a contact interaction. The spin-isospin term was leaft out.

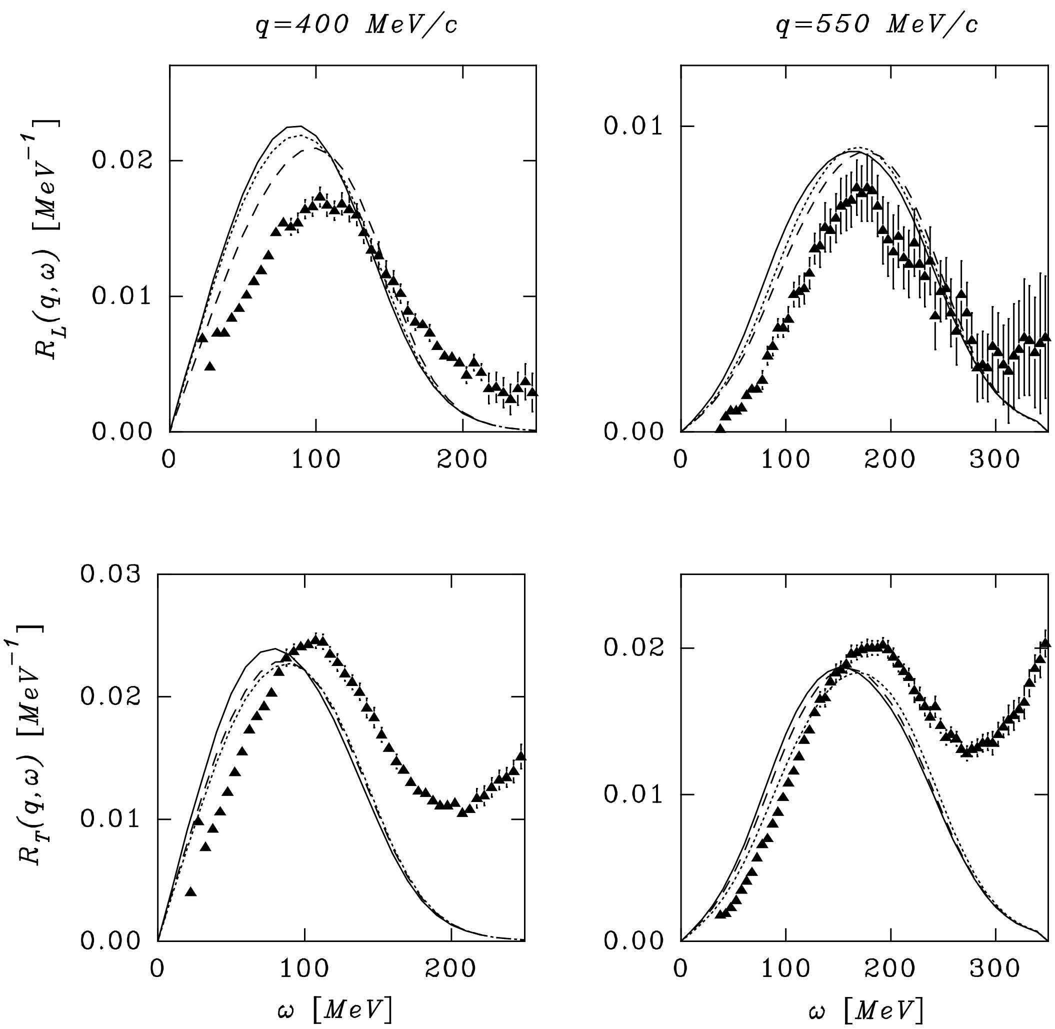

FIG. 7. ^{12}C responses calculated in RPA (solid lines) and ring approximation (dashed lines) with the polarization potential. Dotted line is the free responce. Data were taken from ref. [27]

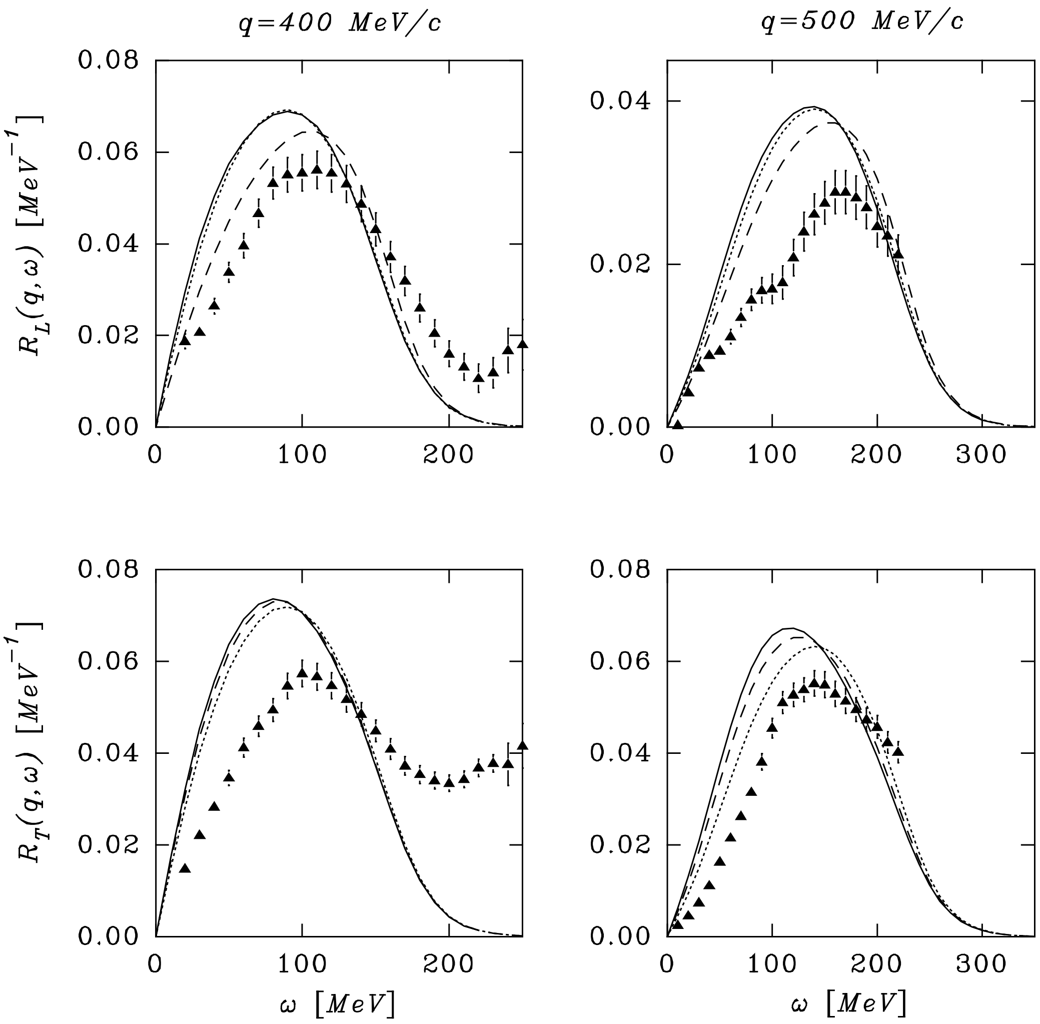

FIG. 8. The same as Fig. 7 but for ^{40}Ca. Data were taken from ref. [28].