Contents

Nuclear excited states within the Random Phase Approximation theory

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Abstract. Results of low-lying excitation spectra and electromagnetic responses of doubly closed shell nuclei within Random Phase Approximation theory using four phenomenological effective interactions are presented. The results obtained with phenomenological interactions are compared with those obtained by self-consistent calculations done with the finite range D1 Gogny interaction. In all the cases investigated, we found that this interaction inverts the energies of all the unnatural parity excitations forming isospin doublets. While the low-lying unnatural parity excitations are not very sensitive to the truncation of the single particle configuration space, this is a relevant problem for the giant resonance excitations. We present a formalism which enables us to solve the RPA equations without truncation of the configuration space but still considering finite range interactions with tensor terms. As an example we show an interesting case of specific sensitivity to tensor channels of the interaction in the case of neutrino cross sections, calculated in the energy range of interest for supernova explosion.

1 Introduction

The results presented in this contribution are part of a project aiming to set up a computational scheme, based on the Random Phase Approximation (RPA) theory, able to describe the excitation of nuclei far from the stability line. The first step of this project is to clarify the reliability of the RPA theory against the changes of the phenomenological input, i.e. effective interaction and single particle configuration space. This is the subject of the present report.

Our investigation has been conducted by calculating the excitation of doubly closed shell nuclei, and by comparing our results with experimental data. We first used a phenomenological approach where single particle energies, wave functions and effective nucleon-nucleon interactions were chosen to reproduce some experimental data. By using this phenomenological input we tested the capacity of the RPA to describe other data, mainly electron scattering data [1] of low-lying unnatural parity states. Our study indicates that the description of these states puts severe limits on the values of some specific terms of the interaction, the spin-isospin and tensor dependent ones [2].

This phenomenological approach cannot be extended to exotic nuclei, since it requires the knowledge of experimental data that in this case we aim to predict. The description of these nuclei can be done within self-consistent approaches. In this case the effective nucleon-nucleon (NN) interaction used to build the single particle configuration space by means of a Hartree-Fock (HF) calculation [3, 4] is also used in the RPA calculations. We tested the validity of this approach by recalculating the same quantities calculated with the phenomenological approach. We made our self-consistent calculations with the finite range Gogny D1 interaction [5], widely used in the literature. The results we obtained indicate that the spin-isospin contents of this interaction is unable to provide a reasonable description of the unnatural parity excitations.

The size of the configuration space is another crucial ingredient of the RPA input. In a phenomenological approach, the truncation of the configuration space is effectively considered by changing the values of the interaction parameters. This procedure cannot be used in a self-consistent approach. We verified that the unnatural parity states we calculated are not sensitive to the configuration space truncation. This is not any more true for the description of the giant resonances. In order to avoid the limits of the truncations of the configuration space, we present here a formalism which allows a proper treatment of the continuum in the RPA calculations which use finite range interactions with tensor terms.

2 Results

The inputs required by RPA calculations are a set of single particle energies and wave functions and a residual NN interaction. In the phenomenological approach we used the single particle basis generated by a Woods-Saxon well, whose parameters are taken from the literature [6]. We used a residual interaction expressed as

$$\begin{aligned} V_{\text{eff}}(1,2) = v_1(r_{12}) \, + \, v_1^{\rho}(r_{12}) \, \rho^{\alpha}(r_1, r_2) \\ + \, [v_2(r_{12}) \, + \, v_2^{\rho}(r_{12}) \, \rho^{\alpha}(r_1, r_2)] \, \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}} \\ + \, v_3(r_{12}) \, \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} \, + \, v_4(r_{12}) \, \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} \, \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}} \\ + \, v_5(r_{12}) \, S_{12}(\hat{r}_{12}) \, + \, v_6(r_{12}) \, S_{12}(\hat{r}_{12}) \, \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}} \, , \end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the usual spin and isospin operators and S_{12} is the tensor operator. As suggested by past phenomenological RPA studies [7, 8] we included density dependent terms in the central and isospin channels. We parameterized the interactions according to the following criteria: *(i)* we chose a unique set of parameters for all the nuclei under investigation, with the exception of the density dependent terms which are different for each nucleus; *(ii)* the density dependent terms have been set to reproduce the first 2^+ state in ^{12}C and the first 3^- states in the other nuclei (^{16}O , ^{40}Ca ,^{90}Zr and ^{208}Pb); *(iii)* the remaining contributions in the central and isospin channels were chosen to get a reasonable description of the centroid energy of the isovector giant dipole resonance; *(iv)* the spin, spin-isospin and tensor channels were chosen to reproduce the excitation energies of the 1^+ and 12^- states of ^{208}Pb and at the same time by taking care that the energies of the low-lying 4^- states of ^{16}O and of the 1^+ doublet of ^{12}C were reasonably well reproduced. Following a Landau-Migdal approach, we considered first zero-range interactions without and with a tensor-isospin channel contribution (LM and LMtt in the following). Then we constructed finite-range interactions (FR and FRtt), by keeping the long-range behavior of the Argonne v_{18} potential [9] and substituting its short-range part with a sum of Gaussians. We also used Gaussians to parameterize the density dependent terms of the interaction and, for the FRtt case, we obtained the tensor channel terms by multiplying the corresponding terms of the v_{18} interaction by a correlation function obtained in Correlated Basis Function variational calculations [6].

We performed systematic calculations of the low energy spectra of ^{12}C , ^{16}O , ^{40}Ca , ^{48}Ca , ^{90}Zr and ^{208}Pb . For each state we also calculated electromagnetic response functions, and we compared them with the available experimental data. We obtained a general good agreement with the excitation spectrum, and, except for few cases we shall consider explicitly below, scarse sensitivity of the results to the details of the interactions.

One of the interesting cases we want to discuss is that of the transverse response functions of the 12^- states of ^{208}Pb that are shown in the right panels of Figure 1. The data of the higher state at 7.08 MeV (lower right panel) are reasonably well reproduced, and all the NN interactions we have used produce very similar results. The situation is completely different for the lower energy state, that at 6.43 MeV, whose response is shown in the upper right panel. In this case we observe a very strong dependence on the residual interaction, both when finite-range and when tensor channel contributions are included. We obtained a much better description of the electromagnetic response of this state by changing some parameters of the residual interaction, but this is worsening the global description of the various magnetic states we have considered. We show in the left panels of the same figure the transverse responses of two 10^- states. We observe that in both cases the presence of the tensor term of the interaction allows a good description of the peaks at about 2.5 fm^{-1}.

We paid particular attention to the study of the isospin doublets. As an example we show in Figure 2 the transverse responses of the 1^+ excitation in ^{12}C . In the upper panel there is the isoscalar excitation, and in the lower panel the isovector one. The energies indicated in the figure are the experimental ones. In the case of the isoscalar excitation we obtain an excellent agreement with the data, but the isovector case indicates an overestimation of the data before the first minimum and a totally wrong behaviour at high q values. Our calculations produce excitation energies which may differ from 13% up to 20% from the experimental values, see Table 1, but in any case the correct order of the excitations is reproduced: the isoscalar excitation has always a lower energy with respect to the isovector one.

The results we presented are a sample of the type of results we have obtained with the phenomenological approach [1,2]. The general agreement with experimental data of the low lying magnetic states of double magic nuclei can be interpreted as if we were able to include in the effective NN interaction some general features necessary to describe well this type of excitations.

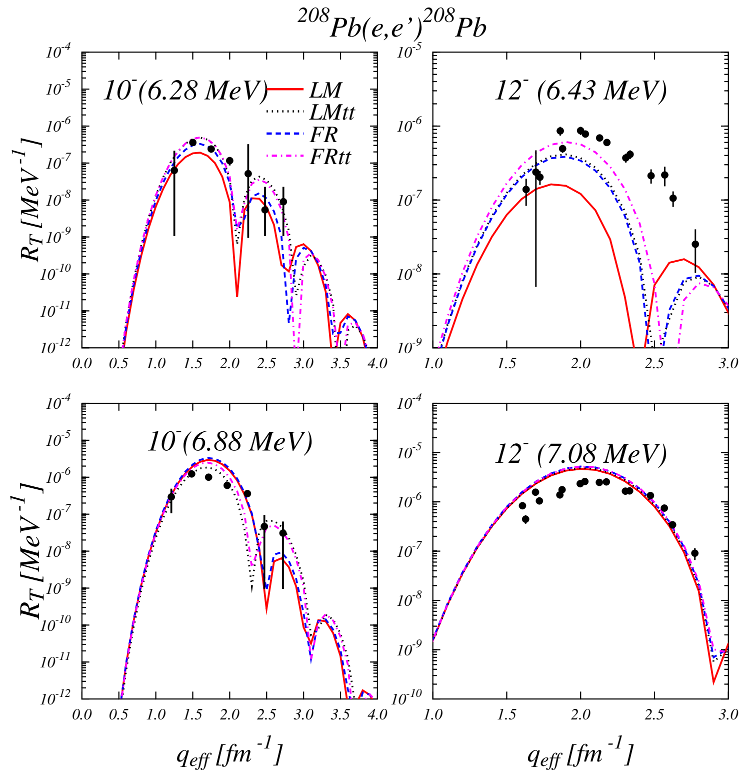

Figure 1. Right panels: electron scattering transverse responses for the first 12^- states of ^{208}Pb , versus the effective momentum transfer. Different residual interactions are used as indicated. Left panels: the same for the first 10^- states. Experimental data from [10].

As we have already pointed out in the introduction, the limit of the phenomenological approach is related to the difficulty of extending it to exotic nuclei. For this reason we have considered a self-consistent RPA approach where the single particle mean field basis is obtained by means of a HF calculation which uses the same effective NN interaction used in the RPA calculation. For these calculations, we have used the Gogny D1 interaction [5, 7, 13] which has finite-range components in the central, isospin, spin and spin-isospin channels and a zero-range density dependent term.

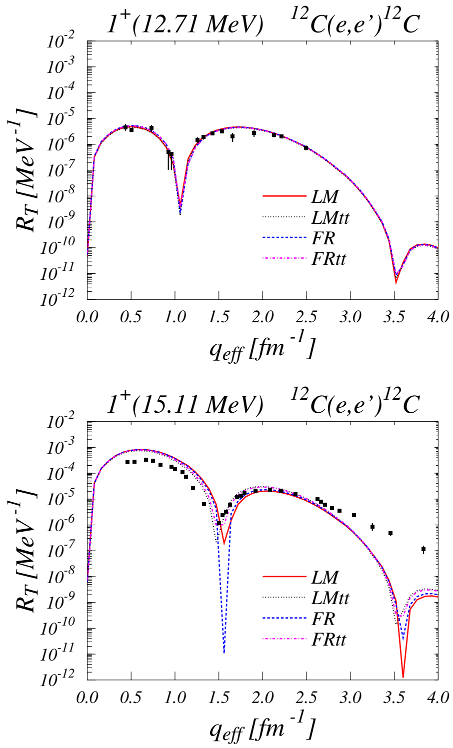

Figure 2. Electromagnetic responses of the 1^+ isospin doublet in ^{12}C. Experimental data from [11].

Table 1. Energies of 1^+ isospin-doublet of ^{12}C (in MeV). Experimental energies from [12]. FR energies from [2]

^{12}C					
excitation	LM	LMtt	FR	FRtt	exp
1^+	14.41	14.41	13.89	13.87	12.71
1^+	18.13	17.97	18.17	18.05	15.11

In the following, we present two different types of results. In the first case the Gogny D1 interaction was used in the RPA calculations, but the single particle wave functions and energies were taken to be the same as in the phenomenological approach. The results of these calculations are represented by the dotted lines in Figures 3 and 4. The second case is fully self-consistent, i.e. the single particle basis was produced by a HF calculation with the same interaction used in RPA. The corresponding results are shown in the figures as dashed lines. The comparison between these two cases allows us to distinguish between the role played by the single particle basis and that played by the residual interaction. In the figures, the full lines

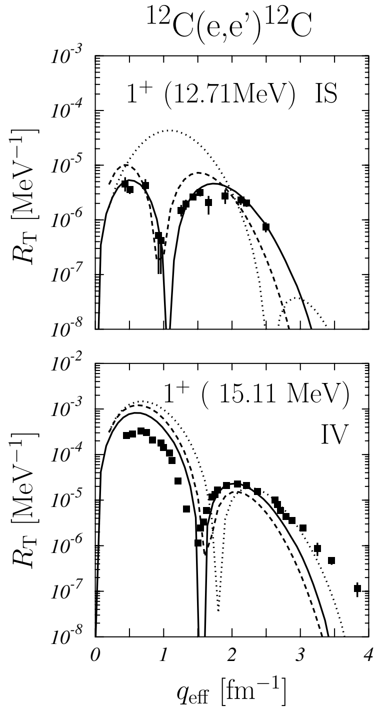

Figure 3. Electromagnetic responses of the 1^+ isospin doublet in ^{12}C. The full lines show the results obtained with the FR interaction [2]. The dotted lines have been obtained with the D1 interaction but using the set of single particle wave functions and energies used in the phenomenological approach. The dashed lines are the results of a self-consistent calculation with the D1 interaction.

show the results obtained in the phenomenological approach by using the FR interaction. This interaction has finite-range but it does not include the tensor terms, therefore it is the most similar to the D1 interaction.

All our calculations were done by discretizing the continuum. We checked that our results are stable with respect to the parameters characterizing the continuum discretization and also with respect to the size of the single particle configuration space used for each nucleus. More details on the role of the continuum discretization in self-consistent RPA calculations can be found in [1].

The electromagnetic responses of the 1^+ isospin doublet in ^{12}C are shown in Fig 3. Therein we compare results of the higher RPA solution with the IS data, and that of the lower RPA solution with the IV data. This inversion of the order in the excitation is obtained in both types of RPA calculations done with the D1 interaction and therefore it does not depend on the single particle basis, but it is related to the characteristics of the interaction itself. We repeated our calculations with another Gogny-like force with different values of the parameters, the D1S interaction [14],

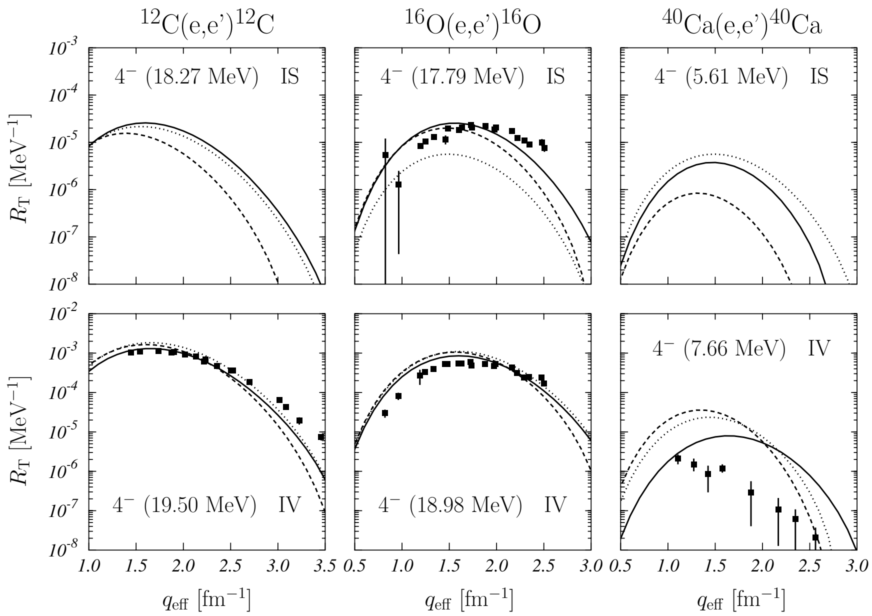

Figure 4. Electromagnetic responses of some 4^- isospin doublets. The meaning of the lines is the same as in Figure 3.

and also in this case we observed sistematically the inversion of the isospin partner states.

We have observed systematically this inversion in all the magnetic states forming isospin partners we studied. Examples are shown in Figures 4 for a set of 4^- states in various nuclei. This result indicates the inadequacy of the D1 interaction in the isospin-dependent channels. A detailed study can be found in [2].

The low-lying states we have considered so far are not very sensitive to the size of the configuration space. We verified that a further increase of the configuration space used in our calculations did not modify sensibly the results. This is not the case for states above the nucleon emission threshold, especially the giant resonances. In phenomenological calculations the truncation of the configuration space can be compensated by changing the parameters of the interaction, but in a more microscopical approaches, such as the self-consistent calculation we did, one has to consider the complete set of single particle wave functions, or, in other words, one has to correctly treat the continuum.

For the Continuum RPA (CRPA) the expression of the operator generating an excited state $|\nu\rangle$ is

$$Q^\dagger_\nu = \sum_{ph} \sum\!\!\!\!\!\!\int_{\epsilon_p > \epsilon_F}^{\infty} \left(X^\nu_{ph}(\epsilon_p) a^\dagger_p a_h - Y^\nu_{ph}(\epsilon_p) a^\dagger_h a_p \right) \quad (2)$$

that considers a sum on the discrete energy states of bount particles and an integration up to infinity on the continuum energy. We reformulated the CRPA equations by changing the variables X and Y in the above equations, which should be in principle known up to infinite values of ϵ, into the unknown channel functions $f_{[p]h}(r)$ and $g_{[p]h}(r)$, defined as

$$f_{[p]h}(r) = \sum\!\!\!\!\!\!\int_{\epsilon_p > \epsilon_F}^{\infty} X_{ph}(\epsilon_p) R_p(r, \epsilon_p) \tag{3}$$

$$g_{[p]h}(r) = \sum\!\!\!\!\!\!\int_{\epsilon_p > \epsilon_F}^{\infty} Y_{ph}(\epsilon_p) R_p(r, \epsilon_p) \tag{4}$$

where $[p]$ indicates all the quantum numbers characterizing the single particle state except the energy ϵ_p and where R_p is the free radial wave function. The new CRPA equations form a set of coupled integro-differential equations. We solved this set of equations by expanding the channel functions on a complete basis of Sturmian functions [19] which already obey the correct continuum boundary conditions: their asymptotic behaviour is like that of the Hankel functions for positive energy and decreases exponentially otherwise. This technology allows us to use finite range interactions and deal with the exchange matrix elements and tensor terms, differently from what has been done in Refs. [16–18].

We did CRPA calculations to obtain cross section for electron and neutrino scattering processes paying particular attention to the sensitivity of these quantities to the tensor channel and finite range [1].

Here we present a result related to the neutrino cross section. In Ref. [18] it was found that the most important contributions to the neutrino cross section derive from the 1^- and 2^- multipoles. We found that the natural parity states are not affected by the differences between the interactions. More interesting is the case of the 2^- excitation for charge exchange processes. The calculations were done with the same interactions used in the discrete phenomenological approach.

The total antineutrino and neutrino cross sections on ^{16}O are shown in Figures 5 and 6 as a function of the projectile energy, in both linear and logarithmic scales. The energy values considered were those of interest for supernova neutrinos.

For $(\overline{\nu}, e^+)$ process we obtained the same cross section with interactions without the tensor channel. This term of the interaction produces opposite results if added to the zero-range force, where it reduces the cross section, or to the finite range interaction, where it increases the cross section by almost a factor 2.

The effect of the tensor interaction is rather different for the (ν, e^-) reaction (Figure 6). In this case there is an increase of about a factor 2 at energies above 30 MeV for the zero range interaction, while the results obtained with finite-range interactions are almost insensitive to the presence of tensor term.

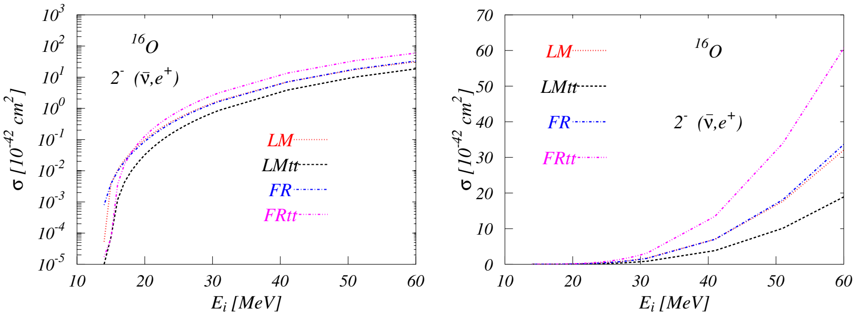

Figure 5. Charge exchange antineutrino cross section as a function of the energy of the projectile in logarithmic and linear scale. Only the contribution of the 2^- excitation has been considered. The CRPA wave functions calculated with the four phenomenological interactions presented in the text have been used.

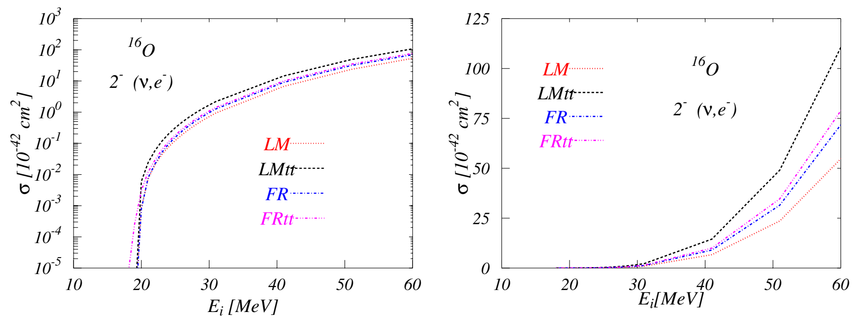

Figure 6. Same as in Figure 5 but for the neutrino scattering.

3 Conclusions

Our results show that the study of unnatural parity low-lying excitations of doubly magic nuclei can provide useful information about the effective NN interaction be used in RPA calculations. Specifically, we found that the Gogny D1 interaction consistently provides the wrong ordering of the isoscalar and isovector partners excitations.

The CRPA formalism based on an expansion of the equations on a Sturmian basis allowed us to consider the complete single particle configuration space and to use finite range interactions with tensor terms. The relevance of the tensor terms has been pointed out by considering the excitation of the 2^- charge exchange excitations. Neutrino, and antineutrino, cross sections in the energy range of interest for supernova explosion are strongly modified by the presence of the tensor term.

Acknowledgments

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