INTRODUCTION TO DIS

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Bibliography

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- V. Barone & P.G. Ratcliffe, *Transverse Spin Physics*, World Scientific, Singapore 2003 (polarized DIS).
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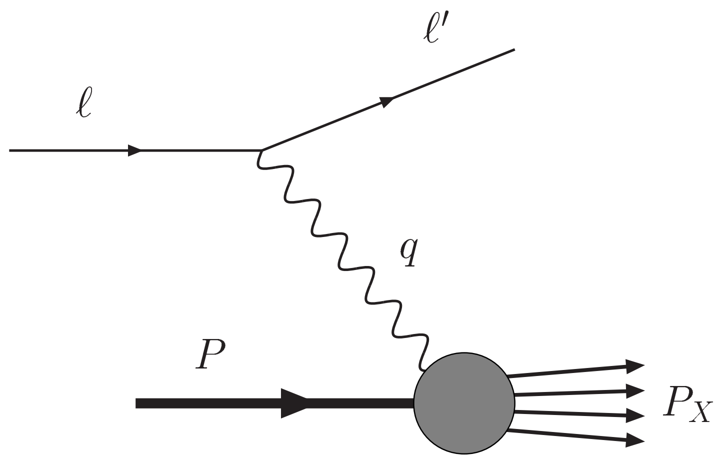
$\boxed{\text{Outline of the lectures}}$

1. General formalism of DIS. Parton model.
2. Structure functions in QCD. Evolution equations.
3. Phenomenology of DIS and global fits. Nuclear effects in DIS.

Deep inelastic scattering

$$l(\ell) + N(P) \rightarrow l'(\ell') + X(P_X)$$

Fully inclusive lepton-nucleon scattering (X is an undetected hadronic system)



(We will consider only unpolarized DIS: all spins are summed or averaged over)

DIS kinematic variables:

$$q^2 \equiv -Q^2 = (\ell - \ell')^2 \qquad \text{momentum transfer}$$

$$W^2 = (P + q)^2 \qquad \text{invariant mass of the } X \text{ system}$$

$$\nu = \frac{P \cdot q}{m_N} = \frac{W^2 + Q^2 - m_N^2}{2m_N} \qquad \text{energy transfer (in Lab)}$$

$$y = \frac{P \cdot q}{P \cdot \ell} = \frac{W^2 + Q^2 - m_N^2}{s - m_N^2} \qquad \text{anelasticity}$$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2m_N \nu} = \frac{Q^2}{Q^2 + W^2 - m_N^2} \qquad \text{Bjorken variable}$$

Since $W^2 \geq m_N^2$, x takes values between 0 and 1

In the target rest frame (TRF):

$$\nu = E - E' \quad \text{and} \quad Q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

where θ is the scattering angle.

The term deep inelastic denotes the kinematic domain where both Q^2 and W^2 are large compared with m_N.

Unpolarized DIS is described by two kinematic variables (besides the beam energy): typical pairs of variables are (E', θ), (ν, Q^2), (x, Q^2), (x, y)

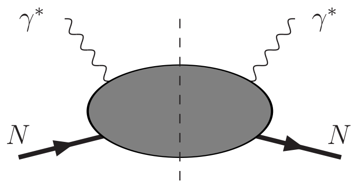
In TRF, the DIS cross section reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2\,m_N\,Q^4}\,\frac{E'}{E}\,L_{\mu\nu}W^{\mu\nu}\,,$$

Leptonic tensor (e.m.) :

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{2}\sum_s\sum_{s'}\left[\bar{u}_{l'}(\ell',s')\gamma_\mu u_l(\ell,s)\right]^*\left[\bar{u}_{l'}(\ell',s')\gamma_\nu u_l(\ell,s)\right] \\ &= \frac{1}{2}\,\mathrm{Tr}\,[\not\ell\,\gamma_\mu\,\not\ell'\,\gamma_\nu] = 2\,(\ell_\mu\ell'_\nu + \ell_\nu\ell'_\mu - g_{\mu\nu}\,\ell\cdot\ell') \end{aligned}$$

Hadronic tensor:



$$\begin{aligned} W^{\mu\nu} &\equiv \frac{1}{2\pi}\sum_X\int\frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2P_X^0}\,(2\pi)^4\,\delta^4(P+q-P_X) \\ &\quad\times\langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle \\ &= \frac{1}{2\pi}\int\mathrm{d}^4z\,\mathrm{e}^{\mathrm{i}q\cdot z}\,\langle P,S|J^\mu(z)J^\nu(0)|P,S\rangle \end{aligned}$$

The hadronic tensor satisfies $W^*_{\mu\nu} = W_{\nu\mu}$, hence $W_{\mu\nu} = W^{(S)}_{\mu\nu} + \mathrm{i}\, W^{(A)}_{\mu\nu}$. Since in the e.m. case $L_{\mu\nu}$ is symmetric, only $W^{(S)}_{\mu\nu}$ contributes to the cross section.

The general decomposition of $W^{(S)}_{\mu\nu}$ is

$$W^{(S)}_{\mu\nu} = A\, g_{\mu\nu} + B\, q_\mu q_\nu + C\,(q_\mu P_\nu + q_\nu P_\mu) + D\, P_\mu P_\nu$$

Imposing current conservation $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$, we get

$$\begin{aligned}
\frac{1}{2m_N}\, W^{(S)}_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(P\cdot q, q^2) \\
&\quad + \frac{1}{m_N^2}\left[\left(P_\mu - \frac{P\cdot q}{q^2}\, q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\, q_\nu\right)\right] W_2(P\cdot q, q^2), \\
&= -g_{\mu\nu} W_1(P\cdot q, q^2) + \frac{P_\mu P_\nu}{m_N^2}\, W_2(P\cdot q, q^2) + \ldots
\end{aligned}$$

(dots denote terms vanishing when contracted with $L^{\mu\nu}$)

In terms of x and y, and of the dimensionless structure functions

$$
\begin{aligned}
F_1(x,Q^2) &\equiv m_N\, W_1(\nu,Q^2) \\
F_2(x,Q^2) &\equiv \nu\, W_2(\nu,Q^2)
\end{aligned}
$$

the DIS cross section becomes

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha_{\mathrm{em}}^2\, s}{Q^4} \left\{ xy^2\, F_1(x,Q^2) + \left(1 - y - \frac{xym_N^2}{s}\right) F_2(x,Q^2) \right\}
$$

In the limit

$$
\nu, Q^2 \to \infty\,, \qquad x = \frac{Q^2}{2m_N\nu} \;\; \text{fixed}
$$

F_1 and F_2 where predicted by Bjorken to be independent of Q^2 (scaling).

Perturbative QCD predicts logarithmic scaling violations.

DIS as virtual photoabsorption

$$\sigma_\lambda^{\gamma^* N} = \frac{4\pi^2 \alpha_{\rm em}}{2\, m_N K}\, \epsilon^{\mu *}(\lambda) W_{\mu\nu}\, \epsilon^\nu(\lambda) \qquad K \text{ photon flux}$$

Polarization vectors of virtual photons:

$$\epsilon^\mu(\pm 1) = \mp \frac{1}{\sqrt{2}}\,(0, 1, \pm \mathrm{i}, 0) \qquad \epsilon^\mu(0) = \frac{1}{\sqrt{Q^2}}\,(\sqrt{\nu^2 + Q^2}, 0, 0, \nu)$$

Cross section for transverse photons: $\sigma_T^{\gamma^* N} \equiv \frac{1}{2}\,(\sigma_{+1}^{\gamma^* N} + \sigma_{-1}^{\gamma^* N})$

Cross section for longitudinal photons: $\sigma_L^{\gamma^* N} \equiv \sigma_0^{\gamma^* N}$

Relations between structure functions and virtual photoabsorption cross sections:

$$W_1 = \frac{K}{4\pi^2 \alpha_{\rm em}} \sigma_T^{\gamma^* N}, \qquad W_2 = \frac{K}{4\pi^2 \alpha_{\rm em}} \frac{Q^2}{Q^2 + \nu^2} (\sigma_T^{\gamma^* N} + \sigma_L^{\gamma^* N})$$

$$\sigma_{\rm tot}^{\gamma^* N} \simeq \frac{4\pi^2 \alpha_{\rm em}}{Q^2} F_2(x, Q^2)$$

Longitudinal to transverse cross section ratio:

$$\begin{aligned} R = \frac{\sigma_L^{\gamma^* N}}{\sigma_T^{\gamma^* N}} &= \frac{W_2}{W_1} \left(1 + \frac{\nu^2}{Q^2}\right) - 1 \\ &= \frac{F_2}{2xF_1} \left(1 + \frac{4m_N^2 x^2}{Q^2}\right) - 1 \simeq \frac{F_2 - 2xF_1}{2xF_1} \end{aligned}$$

Inversely:

$$F_2 = 2xF_1 \frac{1 + R}{1 + 4m_N^2 x^2 / Q^2}$$

In the parton model (and at order α_s^0 in QCD), the longitudinal structure function $F_L = F_2 - 2xF_1$ vanishes and $F_2 = 2xF_1$ (Callan-Gross).

Electroweak DIS

$$\nu + N \to l^- + X, \quad \bar{\nu} + N \to l^+ + X \quad \text{(charged currents, CC)}$$

$$\nu + N \to \nu + X, \quad \bar{\nu} + N \to \bar{\nu} + X \quad \text{(neutral currents, NC)}$$

Virtual boson propagator:

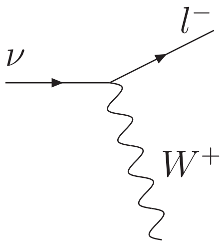
$$\frac{1}{Q^2} \to \frac{1}{Q^2 + M_{W,Z}^2}$$

(suppressed with respect to the e.m. propagator, unless Q^2 is very large)

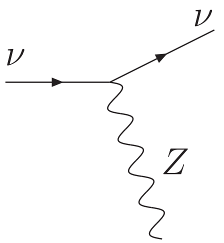
Couplings:

$$\mathrm{CC}: \quad e^2 \to \frac{G\, M_W^2}{\sqrt{2}} = \frac{e^2}{8 \sin^2 \theta_W}, \qquad \mathrm{NC}: \quad e^2 \to \frac{G\, M_W^2}{2\sqrt{2}} = \frac{e^2}{16 \sin^2 \theta_W \cos^2 \theta_W}$$

Remember that $\dfrac{G}{2\pi} = \dfrac{\alpha_{\mathrm{em}}}{2\sqrt{2}\, \sin^2 \theta_W\, M_W^2} = \dfrac{\alpha_{\mathrm{em}}}{2\sqrt{2}\, \sin^2 \theta_W \cos^2 \theta_W\, M_Z^2}$



$$-\frac{e}{2\sqrt{2}\sin\theta_W}\,\bar{l}\,\gamma^\alpha(1-\gamma_5)\,\nu\,W^+_\alpha$$



$$\frac{e}{4\sin\theta_W\cos\theta_W}\,\bar{\nu}\,\gamma^\alpha(1-\gamma_5)\,\nu\,Z_\alpha$$

$$L_{\alpha\beta}(\nu,\bar{\nu}) = \ell_\alpha\ell'_\beta + \ell_\beta\ell'_\alpha - g_{\alpha\beta}\,\ell\cdot\ell' \pm \mathrm{i}\,\varepsilon_{\alpha\beta\gamma\delta}\ell^\gamma\ell'^\delta$$

$$\frac{1}{2m_N}\,W_{\alpha\beta} = -g_{\alpha\beta}W_1 + \frac{P_\alpha P_\beta}{m_N^2}\,W_2 - \frac{\mathrm{i}\varepsilon_{\alpha\beta\gamma\delta}P^\gamma q^\delta}{2m_N^2}\,W^3 + \ldots$$

In electroweak DIS there is a third structure function, W_3 (or $F_3 \equiv -\nu W_3$)

Electroweak DIS cross sections:

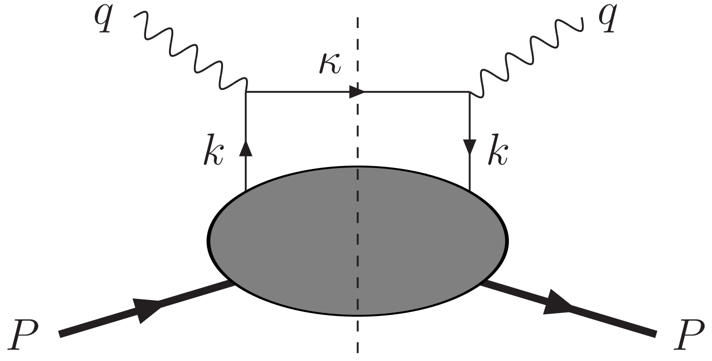
$$\frac{\mathrm{d}^2\sigma^{\nu,\bar\nu}_{\mathrm{CC}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{\pi\alpha^2_{\mathrm{em}}\,s}{4\,\sin^4\theta_W}\,\frac{1}{(Q^2+M_W^2)^2}$$
$$\times\left\{xy^2\,F_1^{W^\pm}(x,Q^2) + \left(1-y-\frac{xym_N^2}{s}\right)F_2^{W^\pm} \pm \left(y-\frac{y^2}{2}\right)xF_3^{W^\pm}(x,Q^2)\right\}$$

$$\frac{\mathrm{d}^2\sigma^{\nu,\bar\nu}_{\mathrm{NC}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{\pi\alpha^2_{\mathrm{em}}\,s}{4\,\sin^4\theta_W\,\cos^4\theta_W}\,\frac{1}{(Q^2+M_Z^2)^2}$$
$$\times\left\{xy^2\,F_1^{Z}(x,Q^2) + \left(1-y-\frac{xym_N^2}{s}\right)F_2^{Z} \pm \left(y-\frac{y^2}{2}\right)xF_3^{Z}(x,Q^2)\right\}$$

At very large Q^2 (i.e., at HERA) there is a non negligible weak (Z^0-exchange) contribution to NC DIS of charged leptons, $\ell^\pm\,N \to \ell^\pm\,X$. In this case, besides $F_1^\gamma, F_2^\gamma, F_3^\gamma$ and F_1^Z, F_2^Z, F_3^Z, there are three interference structure functions $F_1^{\gamma Z}, F_2^{\gamma Z}, F_3^{\gamma Z}$

The quark-parton model

Basic assumption: DIS is an incoherent sum of elastic scatterings of the virtual photon on the nucleon constituents, treated as free particles



Handbag diagram

The Bjorken variable as a momentum fraction

In terms of two Sudakov vectors p^μ and n^μ, such that $p^2 = n^2 = 0$, $p \cdot n = 1$ and $p^- = n^+ = 0$, the relevant momenta in DIS can be parametrized as:

$$P^\mu = p^\mu + \frac{m_N^2}{2}\, n^\mu \simeq p^\mu \quad \text{(proton)}$$

$$q^\mu \simeq (P \cdot q)\, n^\mu - x\, p^\mu \quad \text{(virtual photon)}$$

$$k^\mu = \xi\, p^\mu + \frac{k^2 + \boldsymbol{k}_\perp^2}{2\xi}\, n^\mu + k_\perp^\mu \quad \text{(quark)}$$

In the parton model one assumes that diagrams are dominated by small values of k^2 and $\boldsymbol{k}_\perp^2$, thus

$$k^\mu \simeq \xi\, P^\mu$$

[Equivalently, in the infinite momentum frame ($P^+ \to \infty$), $n^\mu \sim \mathcal{O}(1/P^+)$ is suppressed by a factor $(P^+)^2$ compared to $p^\mu \sim \mathcal{O}(\mathcal{P}^+)$, and one gets again $k^\mu \simeq \xi P^\mu$]

Now, put the outgoing quark on shell. This implies:

$$\delta((k+q)^2) \simeq \delta(-Q^2 + 2\,\xi\,P\cdot q) = \frac{1}{2\,P\cdot q}\,\delta(\xi - x)$$

that is

$$k^\mu \simeq x\,P^\mu$$

The Bjorken variable x is the fraction of the proton's longitudinal momentum carried by the quark:

$$x = \frac{k^+}{P^+}$$

DIS cross section in the parton model

In the parton model the DIS cross section $\mathrm{d}\sigma$ is the incoherent sum of lepton-quark (antiquark) scattering cross sections $\mathrm{d}\hat{\sigma}$

$$l + q\,(\bar{q}) \rightarrow l + q\,(\bar{q})$$

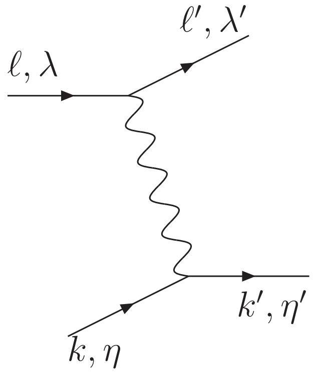
Defining the quark (antiquark) distribution function $q(\xi)$ $(\bar{q}(\xi))$ as the probability of finding a quark q (antiquark $\bar{q}$) carrying a fraction ξ of the proton's longitudinal momentum, $\mathrm{d}\sigma$ is given by

$$\mathrm{d}\sigma = \sum_{q,\bar{q}} \int \mathrm{d}\xi\; q(\xi)\,\mathrm{d}\hat{\sigma}\left(\frac{x}{\xi}\right)$$

In terms of x and y:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y} = \sum_{q,\bar{q}} \int \mathrm{d}\xi\, q(\xi)\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\,\delta(\xi - x) = \sum_{q,\bar{q}} q(x)\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}$$

Elementary cross sections:



$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y} = \frac{1}{16\pi\, \hat{s}^2}\, \frac{1}{4} \sum_{\lambda\lambda'\eta\eta'} \mathcal{M}_{\lambda\eta\lambda'\eta'}\, \mathcal{M}^*_{\lambda\eta\lambda'\eta'}$$

The non-vanishing amplitudes are

$$\mathcal{M}_{++++} = \mathcal{M}_{----} = 2\mathrm{i}\, e^2 e_q\, \frac{1}{y}$$

$$\mathcal{M}_{+-+-} = \mathcal{M}_{-+-+} = 2\mathrm{i}\, e^2 e_q\, \frac{1-y}{y}$$

and therefore

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y} = \frac{2\pi\alpha_{\mathrm{em}}^2\, e_q^2}{\hat{s}\, y^2}\, [1 + (1-y)^2]$$

DIS cross section in the parton model:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,\frac{1}{2}\,\left[1+(1-y)^2\right]\sum_q e_q^2\, x\,[q(x)+\bar{q}(x)]$$

$$= \frac{4\pi\alpha_{\mathrm{em}}^2\, s}{Q^4}\,\{xy^2\,F_1(x)+(1-y)\,F_2(x)\}$$

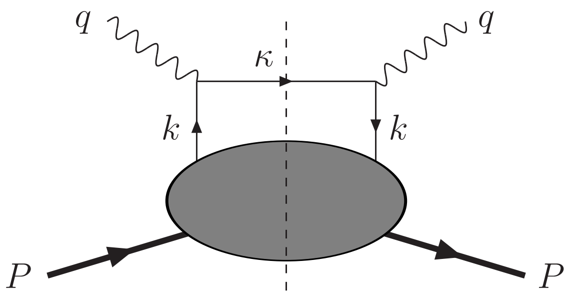
Structure functions in the parton model:

$$F_2(x) = 2xF_1(x) = \sum_q e_q^2\, x\,[q(x)+\bar{q}(x)]$$

The Callan-Gross relation $F_2 = 2xF_1$ implies $R \equiv F_L/F_T = 0$.

Hadronic tensor and field-theoretical definition of quark distributions

The hadronic tensor corresponding to the handbag diagram reads:



$$W^{\mu\nu} = \frac{1}{2\pi} \sum_q e_q^2 \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2P_X^0} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 \kappa}{(2\pi)^4}\, \delta(\kappa^2)$$
$$\times \left[\overline{u}(\kappa)\gamma^\mu \phi(k,P,S)\right]^* \left[\overline{u}(\kappa)\gamma^\nu \phi(k,P,S)\right]$$
$$\times (2\pi)^4\, \delta^4(P-k-P_X)\, (2\pi)^4\, \delta^4(k+q-\kappa)$$

Here $\phi(k,P,S) \equiv \langle X|\psi(0)|P,S\rangle$ are the quark-nucleon vertex functions.

Introducing the quark-quark correlation matrix $\Phi(k, P)$, defined as

$$
\begin{aligned}
\Phi_{ij}(k, P, S) &= \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2P_X^0}\, (2\pi)^4\, \delta^4(P - k - P_X)\, \phi_i(k, P, S)\, \overline{\phi}_j(k, P, S) \\
&= \int \mathrm{d}^4\zeta\ \mathrm{e}^{ik\cdot\zeta}\, \langle P, S|\overline{\psi}_j(0)\psi_i(\zeta)|P, S\rangle,
\end{aligned}
$$

the hadronic tensor can be reexpressed as

$$
\begin{aligned}
W^{\mu\nu} &= \sum_q e_q^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 \kappa}{(2\pi)^4}\, \delta(\kappa^2)\, (2\pi)^4\, \delta^4(k + q - \kappa)\, \mathrm{Tr}\left[\Phi\, \gamma^\mu\, \not{\kappa}\, \gamma^\nu\right] \\
&= \sum_q e_q^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta((k+q)^2)\, \mathrm{Tr}\left[\Phi\, \gamma^\mu\, (\not{k} + \not{q})\, \gamma^\nu\right]
\end{aligned}
$$

Using the identity

$$\gamma^{\mu}\gamma^{\rho}\gamma^{\nu} = g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\rho\sigma} - i\varepsilon^{\mu\rho\nu\sigma}\,\gamma_{\sigma}\gamma^{5}$$

we get

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{S})} &= \frac{1}{2(P\cdot q)}\sum_{q} e_q^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\delta\left(x - \frac{k^+}{P^+}\right) \\ &\quad \times \left[(k_\mu + q_\mu)\,\mathrm{Tr}\,(\Phi\,\gamma_\nu) + (k_\nu + q_\nu)\,\mathrm{Tr}\,(\Phi\,\gamma_\mu) \right. \\ &\quad \left. - \, g_{\mu\nu}(k^\rho + q^\rho)\,\mathrm{Tr}\,(\Phi\,\gamma_\rho)\right] \end{aligned}$$

With $k_\mu + q_\mu \simeq (P\cdot q)\,n_\mu$ this becomes

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{S})} &= \frac{1}{2}\sum_{q} e_q^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\delta\left(x - \frac{k^+}{P^+}\right) \\ &\quad \times \left[n_\mu\,\mathrm{Tr}\,(\Phi\gamma_\nu) + n_\nu\,\mathrm{Tr}\,(\Phi\gamma_\mu) - g_{\mu\nu}n^\rho\,\mathrm{Tr}\,(\Phi\gamma_\rho)\right] \end{aligned}$$

$W_{\mu\nu}^{(S)}$ contains terms of the type:

$$\begin{aligned} \langle \gamma^\mu \rangle &\equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta\left(x - \frac{k^+}{P^+}\right) \mathrm{Tr}\,(\Phi\, \gamma^\mu) \\ &= P^+ \int \frac{\mathrm{d}\zeta^-}{2\pi}\, \mathrm{e}^{\mathrm{i}x P^+ \zeta^-}\, \langle P, S | \overline{\psi}(0)\, \gamma^\mu\, \psi(0, \zeta^-, 0_\perp) | P, S \rangle \end{aligned}$$

Considering only $\mathcal{O}(P^+)$ contributions in the IFM (that is., leading twist contributions), the only available vector to parametrize $\langle \gamma^\mu \rangle$ is P^μ. Thus

$$\langle \gamma^\mu \rangle = 2\, q(x)\, P^\mu$$

$q(x)$ is the quark number density (or, simply, quark distribution):

$$q(x) = \int \frac{\mathrm{d}\zeta^-}{4\pi}\, \mathrm{e}^{\mathrm{i}x P^+ \zeta^-}\, \langle P, S | \overline{\psi}(0) \gamma^+ \psi(0, \zeta^-, 0_\perp) | P, S \rangle$$

$W_{\mu\nu}^{(\mathrm{S})}$ takes the form

$$W_{\mu\nu}^{(\mathrm{S})} = \sum_q e_q^2 \, (n_\mu p_\nu + n_\nu p_\mu - g_{\mu\nu}) \, q(x)$$

The structure functions F_1 and F_2 can be obtained from $W^{\mu\nu}$ by means of two projectors:

$$F_1 = \mathcal{P}_1^{\mu\nu} W_{\mu\nu} = \frac{1}{4} \left(\frac{4x^2}{Q^2} P^\mu P^\nu - g^{\mu\nu} \right) W_{\mu\nu} \ ,$$

$$F_2 = \mathcal{P}_2^{\mu\nu} W_{\mu\nu} = \frac{x}{2} \left(\frac{12x^2}{Q^2} P^\mu P^\nu - g^{\mu\nu} \right) W_{\mu\nu}$$

Since $(P^\mu P^\nu / Q^2) \, W_{\mu\nu} \sim \mathcal{O}(m_N^2 / Q^2)$, one finds that F_2 is proportional to F_1, and

$$F_2(x) = 2x F_1(x) = -\frac{x}{2} \, g^{\mu\nu} W_{\mu\nu} = \sum_q e_q^2 \, x \, q(x) \ ,$$

Introducing the antiquark distributions

$$\bar{q}(x) = \int \frac{\mathrm{d}\zeta^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\zeta^-}\, \langle P,S|\mathrm{Tr}\,[\gamma^+\psi(0)\overline{\psi}(0,\zeta^-,0_\perp)]|P,S\rangle$$

the full expression of F_1 and F_2 in the parton model becomes

$$F_2(x) = 2xF_1(x) = \sum_q e_q^2\, x\, [q(x) + \bar{q}(x)]$$

Flavor structure of nucleons

Quark distributions: $u(x), d(x), s(x), \ldots$

Antiquark distributions: $\bar{u}(x), \bar{d}(x), \bar{s}(x), \ldots$

Quark distributions incorporate a valence component and a sea component:

$$q(x) = q_v(x) + q_s(x)$$

Valence quarks give the nucleon its general (e.m., SU(2), ...) properties.

Sea is composed of $q\bar{q}$ pairs produced by gluon splitting: $q_s(x) = \bar{q}(x)$.

Number sum rules (for proton):

$$\int_0^1 \mathrm{d}x\, u_v(x) = 2, \quad \int_0^1 \mathrm{d}x\, d_v(x) = 1$$

Proton structure function:

$$\begin{aligned} F_2^p(x) &= \sum_q e_q^2 x\,[q(x) + \bar{q}(x)] \\ &= \frac{4}{9} x\,[u(x) + \bar{u}(x)] + \frac{1}{9} x\,[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)] + \ldots \end{aligned}$$

Isospin invariance implies: $u_p(x) = d_n(x),\ d_p(x) = u_n(x),\ s_p(x) = s_n(x)$

Neutron structure function (obtained experimentally from a deuteron target):

$$F_2^n(x) = \frac{4}{9} x\,[d(x) + \bar{d}(x)] + \frac{1}{9} x\,[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)] + \ldots$$

where all distributions refer to the proton.

Assuming

$$\bar{u}(x) = \bar{d}(x)$$

one gets the so-called Gottfried sum rule

$$\int_0^1 \frac{\mathrm{d}x}{x}(F_2^p - F_2^n) = \frac{1}{3}$$

This is not a fundamental relation and is experimentally known to be broken by $\sim 30\%$ (flavor asymmetry of the light sea, $\bar{u} \neq \bar{d}$)

Neutron-to-proton ratio:

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u + \bar{u} + 4(d + \bar{d}) + s + \bar{s}}{4(u + \bar{u}) + d + \bar{d} + s + \bar{s}}$$

Positivity of distributions implies

$$\frac{1}{4} \leq \frac{F_2^n(x)}{F_2^p(x)} \leq 4$$

This is a genuine parton model prediction in good agreement with data.

Leading–order expressions of charged-current structure functions

	$F_{2,l}$	$F_{2,c}$
$\ell^{\pm}p$	$x\{\frac{4}{9}[u(x)+\bar{u}(x)]+\frac{1}{9}[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$	$\frac{4}{9}\left(\frac{\alpha_s}{2\pi}\right)C_{2,g}^{c,(0)}\otimes xg$
$\ell^{\pm}n$	$x\{\frac{4}{9}[d(x)+\bar{d}(x)]+\frac{1}{9}[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$	$\frac{4}{9}\left(\frac{\alpha_s}{2\pi}\right)C_{2,g}^{c,(0)}\otimes xg$
νp	$2x[\bar{u}(x)+\lvert V_{ud}\rvert^2 d(x)+\lvert V_{us}\rvert^2 s(x)]$	$2\xi[\lvert V_{cd}\rvert^2 d(\xi)+\lvert V_{cs}\rvert^2 s(\xi)]$
νn	$2x[\bar{d}(x)+\lvert V_{ud}\rvert^2 u(x)+\lvert V_{us}\rvert^2 s(x)]$	$2\xi[\lvert V_{cd}\rvert^2 u(\xi)+\lvert V_{cs}\rvert^2 s(\xi)]$
$\bar{\nu}p$	$2x[u(x)+\lvert V_{ud}\rvert^2 \bar{d}(x)+\lvert V_{us}\rvert^2 \bar{s}(x)]$	$2\xi[\lvert V_{cd}\rvert^2 \bar{d}(\xi)+\lvert V_{cs}\rvert^2 \bar{s}(\xi)]$
$\bar{\nu}n$	$2x[d(x)+\lvert V_{ud}\rvert^2 \bar{u}(x)+\lvert V_{us}\rvert^2 \bar{s}(x)]$	$2\xi[\lvert V_{cd}\rvert^2 \bar{u}(\xi)+\lvert V_{cs}\rvert^2 \bar{s}(\xi)]$

[$\xi = x\,(1+m_c^2/Q^2)$ is the slow rescaling variable]

Note that:

- $F_2^{\nu N} \simeq 2x\,(u+\bar{u}+d+\bar{d}+2s)$
- ν probes strangeness, $\bar{\nu}$ probes antistrangeness

	$xF_{3,l}$	$xF_{3,c}$
νp	$2x[\lvert V_{ud}\rvert^2 d(x) + \lvert V_{us}\rvert^2 s(x) - \bar{u}(x)]$	$2\xi[\lvert V_{cd}\rvert^2 d(\xi) + \lvert V_{cs}\rvert^2 s(\xi)]$
νn	$2x[\lvert V_{ud}\rvert^2 u(x) + \lvert V_{us}\rvert^2 s(x) - \bar{d}(x)]$	$2\xi[\lvert V_{cd}\rvert^2 u(\xi) + \lvert V_{cs}\rvert^2 s(\xi)]$
$\bar{\nu} p$	$2x[u(x) - \lvert V_{ud}\rvert^2 \bar{d}(x) - \lvert V_{us}\rvert^2 \bar{s}(x)]$	$-2\xi[\lvert V_{cd}\rvert^2 \bar{d}(\xi) + \lvert V_{cs}\rvert^2 \bar{s}(\xi)]$
$\bar{\nu} n$	$2x[d(x) - \lvert V_{ud}\rvert^2 \bar{u}(x) - \lvert V_{us}\rvert^2 \bar{s}(x)]$	$-2\xi[\lvert V_{cd}\rvert^2 \bar{u}(\xi) + \lvert V_{cs}\rvert^2 \bar{s}(\xi)]$

$\nu - \bar{\nu}$ combinations with isoscalar targets:

- $xF_3^{\nu N} - xF_3^{\bar{\nu} p} \simeq 2\xi\,(s + \bar{s})$
- $xF_3^{\nu p} + xF_3^{\bar{\nu} p} \simeq 2x\,(u_v + d_v) + 2\xi\,(s - \bar{s})$

Something is missing: gluons

The fraction of the proton's momentum carried by a quark q is $\int_0^1 \mathrm{d}x\, x\, q(x)$. Thus, quarks and antiquarks (up to charm) carry a total momentum fraction:

$$\mathcal{F} = \int_0^1 \mathrm{d}x\, x\, [u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x)]$$

It turns out (one can measure $F_2^{\nu p} + F_2^{\nu n}$, which is proportional to $\mathcal{F}$) that at ordinary Q^2 scales $\mathcal{F} \approx 0.4 - 0.5$: there are missing constituents, not seen directly by the γ^*. These electrically neutral partons are the gluons, the quanta of the color field of QCD.

The momentum sum rule therefore reads

$$\int_0^1 \mathrm{d}x\, x \sum_i [q_i(x) + \bar{q}_i(x)] + \int_0^1 \mathrm{d}x\, x\, g(x) = 1$$

Regge theory predictions for low-x DIS

Regge theory applies at large W^2 (c.m. energy of the $\gamma^* N$ system), i.e. at low x.

Virtual photoabsorption cross section:

$$\sigma^{\gamma^* p} \sim \mathcal{A}_{I\!P}\,(W^2)^{\alpha_{I\!P}(0)-1} + \mathcal{A}_{I\!R}\,(W^2)^{\alpha_{I\!R}(0)-1}$$

Regge trajectories: $I\!P$ = Pomeron, $I\!R$ = Reggeon

In terms of the structure function F_2:

$$F_2 \sim \mathcal{A}_{I\!P}\,x^{1-\alpha_{I\!P}(0)} + \mathcal{A}_{I\!R}\,x^{1-\alpha_{I\!R}(0)}$$

With typical intercepts $\alpha_{I\!P}(0) = 1$ and $\alpha_{I\!R}(0) = 1/2$, one has

$$F_2(x) \sim \mathcal{A}_{I\!P}\,x^0 + \mathcal{A}_{I\!R}\,x^{\frac{1}{2}}$$

Using $F_2(x) \sim x\,q(x)$, the quark distributions $q(x)$ behave as

$$q(x) \sim \mathcal{A}_{I\!P}\,x^{-1} + \mathcal{A}_{I\!R}\,x^{-\frac{1}{2}}$$

Since the pomeron has isospin $I = 0$, the valence $V(x) = \sum_q [q(x) - \bar{q}(x)]$, contains only the $I\!\!R$ contribution,

$$V(x) \sim x^{-1/2}$$

and the proton-neutron structure function difference is expected to vanish for $x \longrightarrow 0$ as

$$F_2^p(x) - F_2^n(x) \sim x^{1/2}$$

The sea $S(x)$ and the gluon density $g(x)$, which drives the structure functions at low x, are predicted to diverge

$$S(x), g(x) \sim x^{-1}$$

Notice that the Regge exponents are Q^2-independent. Empirically (and in QCD) the low-x rise of structure functions depends on Q^2.

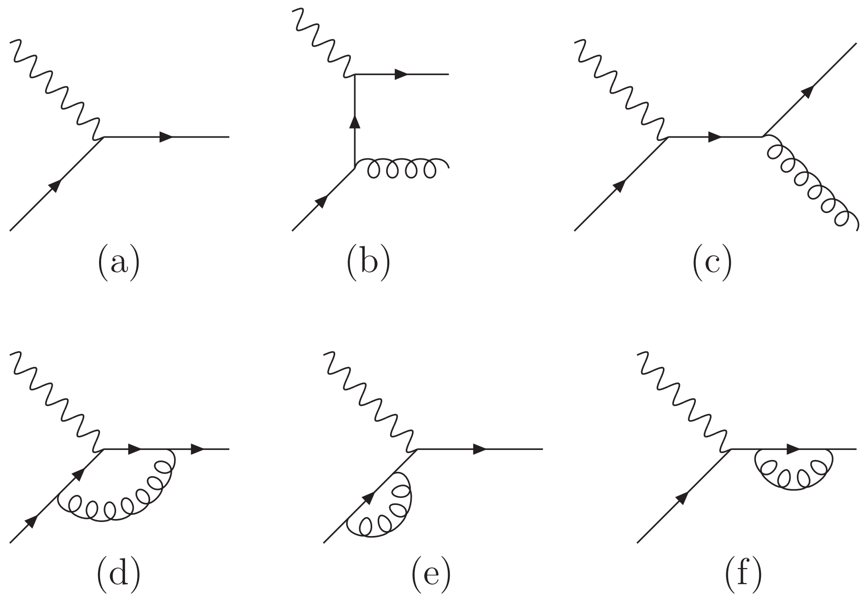
Structure functions in QCD

Parton model is the zero-th order approximation to the truth. In the real world, in fact, the constituents of hadrons are interacting objects, described by quantum chromodynamics (QCD), the theory of strong interactions.

In perturbative QCD, structure functions can be written in a factorized form (for the moment, we consider only their quark and antiquark component):

$$F_2(x, Q^2) = \sum_{q,\bar{q}} e_q^2 \, x \int_x^1 \frac{\mathrm{d}\xi}{\xi} \, q_0(\xi) \, \hat{F}_2^q \left(\frac{x}{\xi}, Q^2, \alpha_s \right)$$

Here $q_0(\xi)$ are the bare parton distributions (universal non-perturbative quantities), whereas $\hat{F}_2^q$ are the "quark structure functions", i.e. the $\gamma^* q$ virtual photoabsorption cross sections (process-dependent perturbative quantities, called Wilson coefficients)



a) $\mathcal{O}(\alpha_s^0)$ contribution:

$$\hat{F}_2^q(z) = \delta(1-z) \;\Rightarrow\; F_2 = \sum_q e_q^2 x \left[q(x) + \bar{q}(x)\right]$$

b-f) $\mathcal{O}(\alpha_s^1)$ contributions (b-c: real gluon emission; d-f: virtual gluon radiation):

$$\hat{F}_2^q(z, Q^2) = \frac{\alpha_s}{2\pi}\left[P(z)\,\ln\frac{Q^2}{\kappa_0^2} + h(z)\right]$$

Up to $\mathcal{O}(\alpha_s)$, the nucleon structure functions is

$$F_2(x,Q^2) \quad = \quad \sum_{q,\bar{q}} e_q^2\, x \left\{ q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, q_0(\xi) \right.$$
$$\left. \times \left[P\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\kappa_0^2} + h\left(\frac{x}{\xi}\right) \right] + \ldots \right\}$$

Introduce a factorization scale μ:

$$\ln \frac{Q^2}{\kappa_0^2} = \ln \frac{Q^2}{\mu^2} + \ln \frac{\mu^2}{\kappa_0^2} \ .$$

and split (arbitrarily) the finite function $h(z)$ in two parts (this defines the factorization scheme):

$$h(z) = \widetilde{h}(z) + h'(z)$$

The singularity $\ln\left(\mu^2/\kappa_0^2\right)$ and the term $h'(z)$ are reabsorbed in a redefinition of the quark distributions, which become scale dependent:

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, q_0(\xi) \left[P\left(\frac{x}{\xi}\right) \ln \frac{\mu^2}{\kappa_0^2} + h'\left(\frac{x}{\xi}\right) \right] + \ldots$$

The factorization formula becomes

$$F_2(x, Q^2) = \sum_{q,\bar{q}} e_q^2\, x \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, q(\xi, \mu^2)\, \hat{F}_2^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s\right) \;,$$

where the (regularized and subtracted) partonic structure function $\hat{F}_2^q$ is

$$\hat{F}_2^q\left(z, \frac{Q^2}{\mu^2}, \alpha_s\right) = \delta(1-z) + \frac{\alpha_s}{2\pi}\left[P(z)\, \ln\frac{Q^2}{\mu^2} + \widetilde{h}(z)\right] + \ldots$$

Since $F_2(x,Q^2)$ is a physical observable, it cannot depend on the unphysical scale μ^2. This leads to the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equation governing the scale dependence of the quark distributions:

$$\frac{\partial\, q(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y}\, P\left(\frac{x}{y}\right)\, q(y,\mu^2)$$

$P(z)$ is the splitting function, i.e. the probability for a quark to emit another quark with momentum fraction z. Splitting functions can be expanded as

$$P(z) = \sum_{n=0}^{\infty} \alpha_s^n\, P^{(n)}(z)$$

DGLAP equation effectively resums powers of $\alpha_s \ln(Q^2/\mu^2)$

At zero-th order in α_s, quark distributions are scale-independent and $\hat{F}_2^q = \delta(1-z)$. Thus

$$F_2(x) = \sum_{q,\bar{q}} e_q^2 \, x \int_x^1 \frac{\mathrm{d}\xi}{\xi} \, q(\xi) \, \delta\left(1 - \frac{x}{\xi}\right) = \sum_q e_q^2 \, x \, [q(x) + \bar{q}(x)]$$

At first order in α_s, the splitting function is $P(z) = P^{(0)}(z)$ and $\hat{F}_2^q = \delta(1-z) + (\alpha_s/2\pi) \, P^{(0)}(z) \, \ln(Q^2/\mu^2)$. Thus

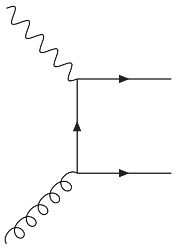
$$\begin{aligned} F_2(x, Q^2) &= \sum_{q,\bar{q}} e_q^2 \, x \int_x^1 \frac{\mathrm{d}\xi}{\xi} \, q(\xi, \mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} \, P^{(0)}\left(\frac{x}{\xi}\right) \, \ln \frac{Q^2}{\mu^2} \right] \\ &= \sum_q e_q^2 \, x \, [q(x, Q^2) + \bar{q}(x, Q^2)] \end{aligned}$$

with

$$\frac{\partial \, q(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} \, P^{(0)}\left(\frac{x}{y}\right) \, q(y, Q^2)$$

Introducing gluons

Photon-gluon fusion



Gluons contribute to DIS via the photon-gluon fusion diagram, which appears at order α_s^1.

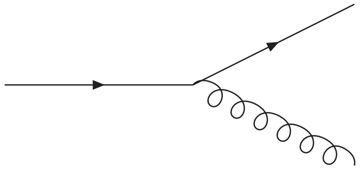
The complete factorization formula for the structure function is

$$F_2(x,Q^2) \;=\; \sum_{q,\bar{q}} e_q^2\, x \int_x^1 \frac{\mathrm{d}\xi}{\xi} \left[q(\xi,\mu^2)\, \hat{F}_2^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s\right) \right. \\ \left. + \, g(\xi,\mu^2)\, \hat{F}_2^g\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s\right)\right]$$

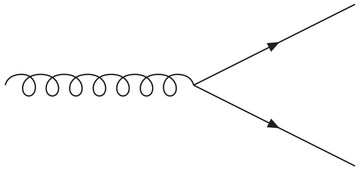
Taking gluons into account, the scale-dependent quark distributions become

$$
\begin{aligned}
q(x,\mu^2) \quad = \quad & q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, q_0(\xi) \left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{\mu^2}{\kappa_0^2} + h'_q\left(\frac{x}{\xi}\right) \right] \\
& + \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, g_0(\xi) \left[P_{qg}\left(\frac{x}{\xi}\right) \ln\frac{\mu^2}{\kappa_0^2} + h'_g\left(\frac{x}{\xi}\right) \right] + \ldots
\end{aligned}
$$

The two splitting functions correspond to



P_{qq}



P_{qg}

DGLAP equations in detail

Non-singlet: $q_-(x,Q^2) = q(x,Q^2) - \bar{q}(x,Q^2)$

Singlet: $\Sigma(x,Q^2) = \sum_q q_+(x,Q^2) = \sum_q [q(x,Q^2) + \bar{q}(x,Q^2)]$

$$\frac{\partial q_-(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}\left(\frac{x}{y}\right) q_-(y,t) \qquad [t \equiv \ln(Q^2/\mu^2)]$$

$$\frac{\partial}{\partial t}\left\{\begin{array}{c} \Sigma(x,t) \\ g(x,t) \end{array}\right\} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} \left\{\begin{array}{cc} P_{qq}\left(\frac{x}{y}\right) & 2n_f P_{qg}\left(\frac{x}{y}\right) \\ P_{gq}\left(\frac{x}{y}\right) & P_{gg}\left(\frac{x}{y}\right) \end{array}\right\} \left\{\begin{array}{c} \Sigma(y,t) \\ g(y,t) \end{array}\right\}$$

Non-singlet distributions (valence) do not mix with gluons (no gluon splitting, no pair annihilation) and their evolution is much simpler.

The momentum fraction of valence (sea, glue) decreases (increases) with Q^2. Sea and gluon distributions grow with Q^2 at low x.

Splitting functions at leading order

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x) \right] \qquad (q \to qg)$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2 \right] \qquad (g \to q\bar{q})$$

$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x} \right] \qquad (q\bar{q} \to g)$$

$$P_{gg}^{(0)}(x) = 2\,C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \frac{11\,C_A - 2\,n_f}{6}\,\delta(1-x) \qquad (g \to gg)$$

where $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$, and the $+$ distributions are such that

$$\int_0^1 \mathrm{d}x\, \frac{f(x)}{(1-x)_+} = \int_0^1 \mathrm{d}x\, \frac{f(x) - f(1)}{1-x}$$

Moments of parton distributions

The moments of pdf's are the Mellin transforms

$$q(n,t) = \int_0^1 \mathrm{d}x\, x^{n-1}\, q(x,t)$$

The inverse transforms are

$$q(x,t) = \frac{1}{2\pi i} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}n\, x^{-n}\, q(n,t)$$

The moments of the splitting functions are the so-called anomalous dimensions

$$\int_0^1 \mathrm{d}x\, x^{n-1}\, P(x,\alpha_s) = \gamma(n,\alpha_s)$$

Since the Mellin transform of a convolution of two functions is the product of the Mellin transforms of the two functions, the DGLAP evolution equations become algebraic equations in the n-space.

$$\frac{\partial\, q_-(n,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi}\,\gamma_{qq}\, q_-(n,t)$$

$$\frac{\partial}{\partial t}\left\{\begin{array}{c}\Sigma(n,t)\\ g(n,t)\end{array}\right\} = \frac{\alpha_S(t)}{2\pi}\left(\begin{array}{cc}\gamma_{qq} & 2n_f\gamma_{qg}\\ \gamma_{gq} & \gamma_{gg}\end{array}\right)\left\{\begin{array}{c}\Sigma(n,t)\\ g(n,t)\end{array}\right\}$$

At leading order the anomalous dimensions are

$$\begin{aligned}
\gamma^0_{qq}(n) &= C_F\left[-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{k=2}^{n}\frac{1}{k}\right]\\
\gamma^0_{qg}(n) &= \frac{1}{2}\left[\frac{2+n+n^2}{n(n+1)(n+2)}\right]\\
\gamma^0_{gq}(n) &= C_F\left[\frac{2+n+n^2}{n(n^2-1)}\right]\\
\gamma^0_{gg}(n) &= 2\,C_A\left[-\frac{1}{12}+\frac{1}{n(n-1)}+\frac{1}{(n+1)(n+2)}-\sum_{k=2}^{n}\frac{1}{k}\right]-\frac{n_f}{3}
\end{aligned}$$

LO evolution of non-singlet moments

Using

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \, \ln(Q^2/\Lambda^2)} \qquad [\beta_0 = 11 - 2\, n_f/3]$$

the evolution equation for $q_-(n,t)$ becomes

$$\frac{\mathrm{d}q_-}{q_-} = \frac{2\gamma^0_{qq}}{\beta_0} \, \frac{\mathrm{d}t}{t}$$

and its solution is

$$q_-(n,Q^2) = q_-(n,Q_0^2) \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]^{2\gamma^0_{qq}(n)/\beta_0}$$

Note that:

i) $\gamma^{(0)}_{qq}(1) = 0$: the number of valence quarks does not change;

ii) $\gamma^{(0)}_{qq}(2) = -4/9$: the momentum fraction of valence decreases.

Parton distributions in QCD

QCD adds two new ingredients to

$$q(x) = \int \frac{\mathrm{d}\zeta^-}{4\pi} \, \mathrm{e}^{\mathrm{i}xP^+\zeta^-} \, \langle P,S|\overline{\psi}(0)\gamma^+\psi(0,\zeta^-,0_\perp)|P,S\rangle$$

1) Scale dependence (arising from field renormalization):

$$q(x) \to q(x,\mu^2)$$

2) Wilson line between the quark fields (to ensure gauge invariance):

$$\mathcal{U}(\zeta,0) = \mathcal{P}\exp\left[\mathrm{i}g\int_0^\zeta \mathrm{d}x_\mu \, A_a^\mu(x)\, t^a\right],$$

In the axial gauge $A_a^+ = 0$, by choosing a suitable path, $\mathcal{U}$ reduces to identity (caveat: this is not true for $\boldsymbol{k}_T$-dependent distributions).

QCD vs. quark models

Distribution functions contain matrix elements of quark operators

$$\langle P, S | \overline{\psi}(0) \gamma^{+} \psi(\zeta) | P, S \rangle$$

In quark models these are numbers, with no scale dependence. QCD, on the other hand, predicts a scale dependence.

How to reconcile quark models with QCD?

Parisi & Petronzio, Glück & Reya, Jaffe & Ross:
Matrix elements computed in quark models describe the nucleon at some fixed, low scale Q_0^2 (typically ~ 0.5 GeV^2).

In other terms, quark models provide the input for the QCD evolution.

It turns out that the model scale cannot be taken as a valence scale: in order to reproduce the experimental data, the nucleon at the scale Q_0 must contain not only valence, but also some sea and glue.

DIS phenomenology

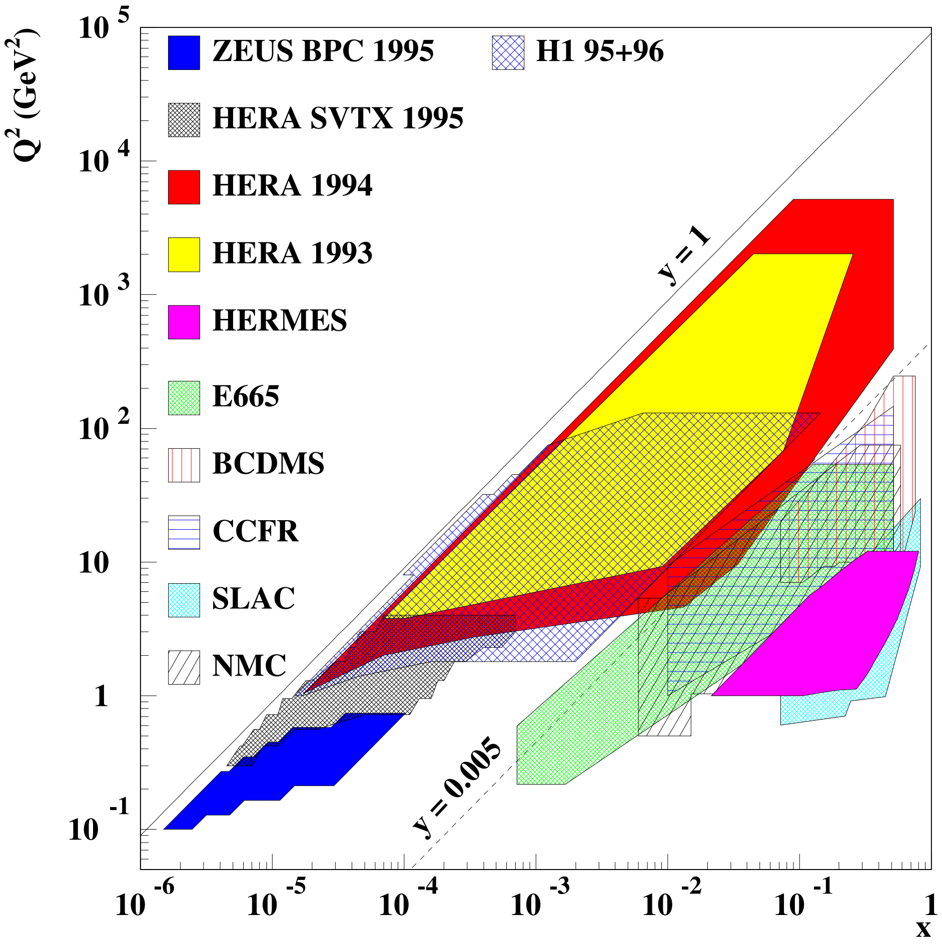
- The knowledge of the quark-gluon structure of nucleons is essentially based on DIS (four decades of measurements of increasing precision, starting from the celebrated SLAC experiment (1968) that identified partons with quarks).
- DIS provides the most successful test of perturbative QCD: the evolution of structure functions is correctly predicted over four orders of magnitude in Q^2.
- Due to the universality of pdf's and to QCD factorization theorems, the parton densities extracted from DIS can be used to predict other hard hadronic processes.

Kinematical ranges of DIS experiments

Note the difference between fixed-target experiments (NMC, SLAC, BCDMS) and *ep* collider (HERA)

HERA range:
$0.2 < Q^2 < 10^4$ GeV^2
$10^{-5} < x < 10^{-1}$

Probed length scale:
$d \simeq 10^{-16} - 10^{-14}$ cm



Global fits of DIS

Goal: extract the pdf's from a global analysis of DIS and related data (Drell-Yan processes, W production, etc.).

Strategy: pdf's are parametrized (quite arbitrarily) at a certain scale $Q_0^2 \sim 1 - 4$ GeV^2, then evolved by DGLAP equations to larger Q^2, convoluted with Wilson coefficients to obtain structure functions, and finally compared with data.

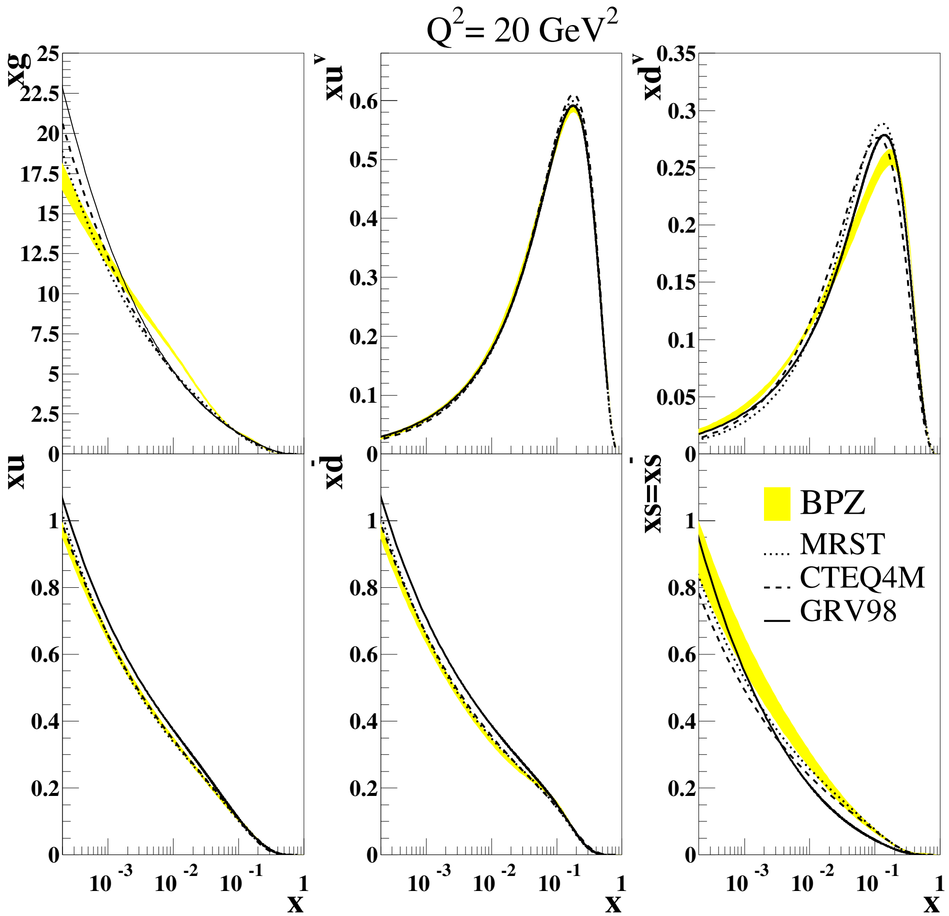
Typical functional form of pdf's:

$$xq(x, Q_0^2) = A\, x^B\, (1-x)^C (1 + D\, x^E)$$

Some of the parameters are fixed by sum rules (number and momentum). Others (if not well constrained by data) can be suggested by Regge theory.

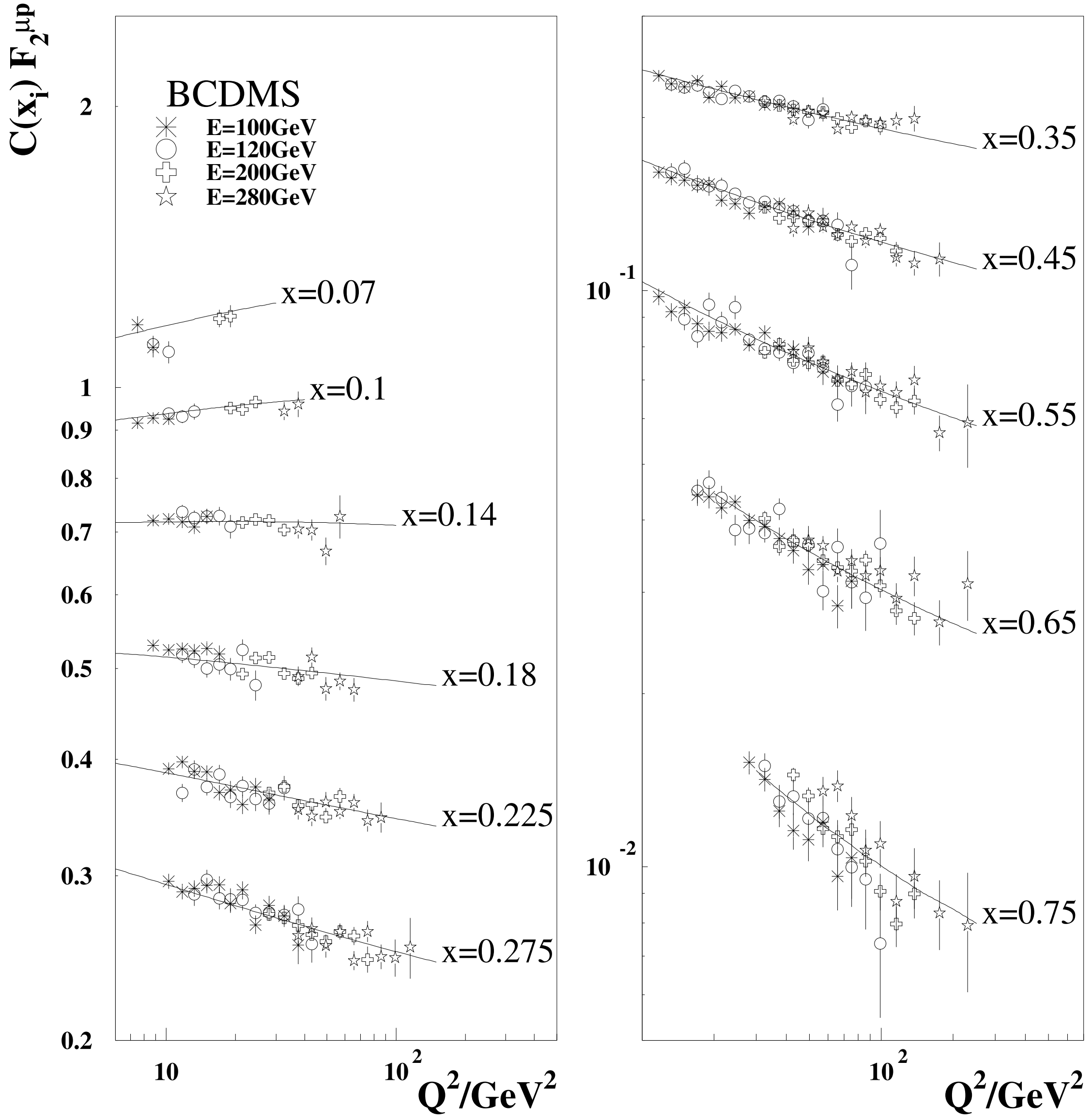
Main global fits: MRST (Martin, Roberts, Stirling, Thorne), CTEQ (Tung et al.), GRV (Glück, Reya, Vogt), BPZ (B., Pascaud, Zomer).

Pdf's from BPZ fit (2000)

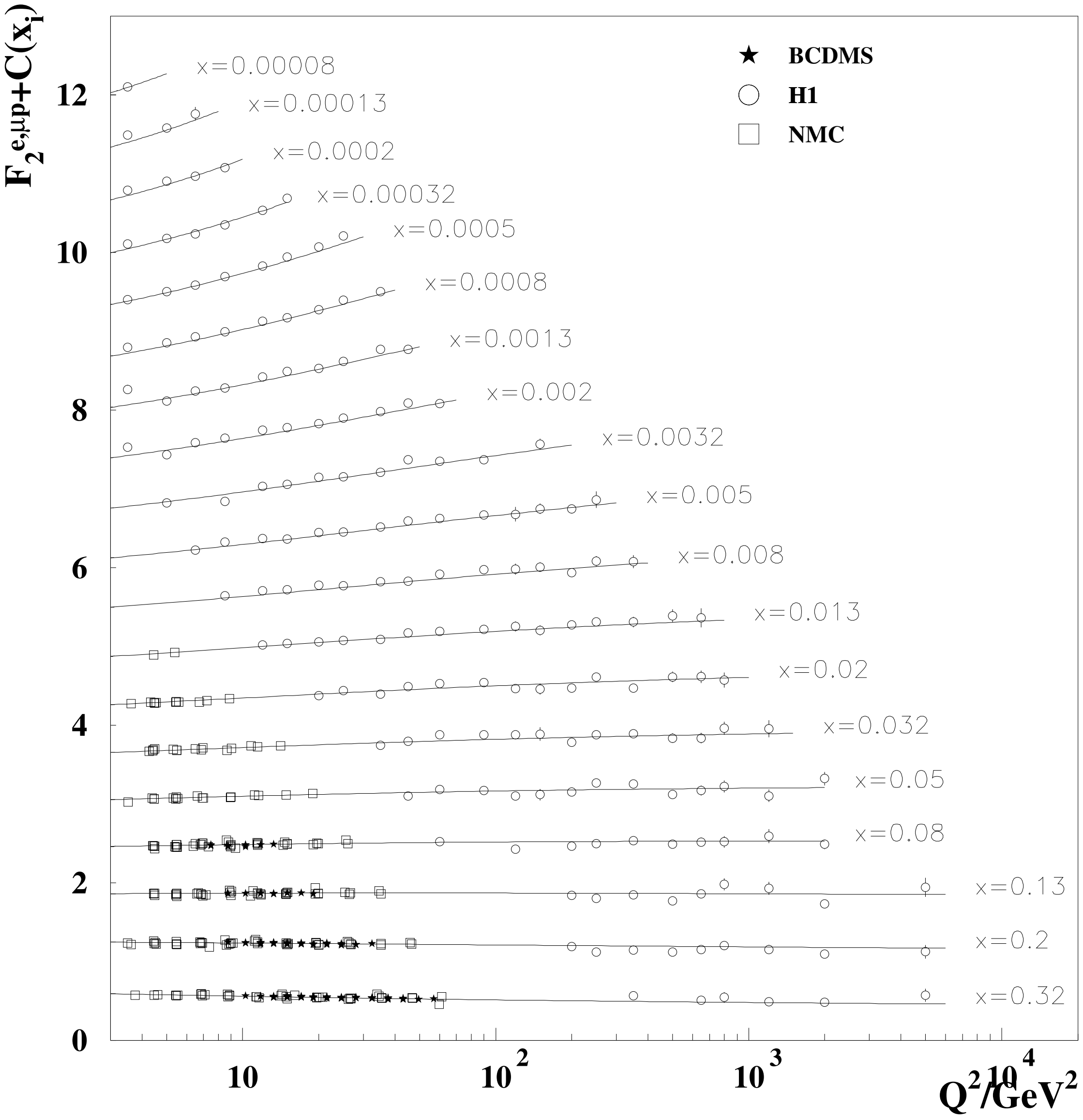


Quality of the fit: $\chi^2 = 2431/(2657 - 44)$

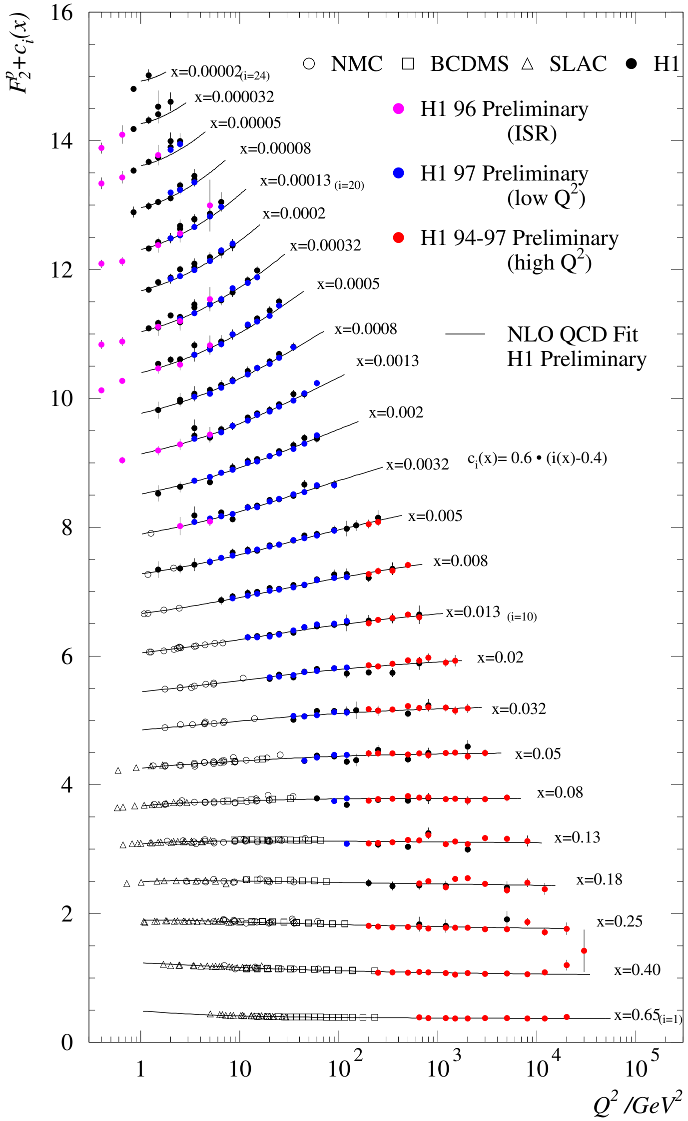
Momentum fractions at $Q^2 = 20$ GeV^2:
glue 47%, valence 34%, sea 19% (strangeness 5%)



BCDMS
E=100GeV
E=120GeV
E=200GeV
E=280GeV
C(x_i) F_2^μp
x=0.07
x=0.1
x=0.14
x=0.18
x=0.225
x=0.275
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
10^-1
10^-2
10
10^2
Q^2/GeV^2

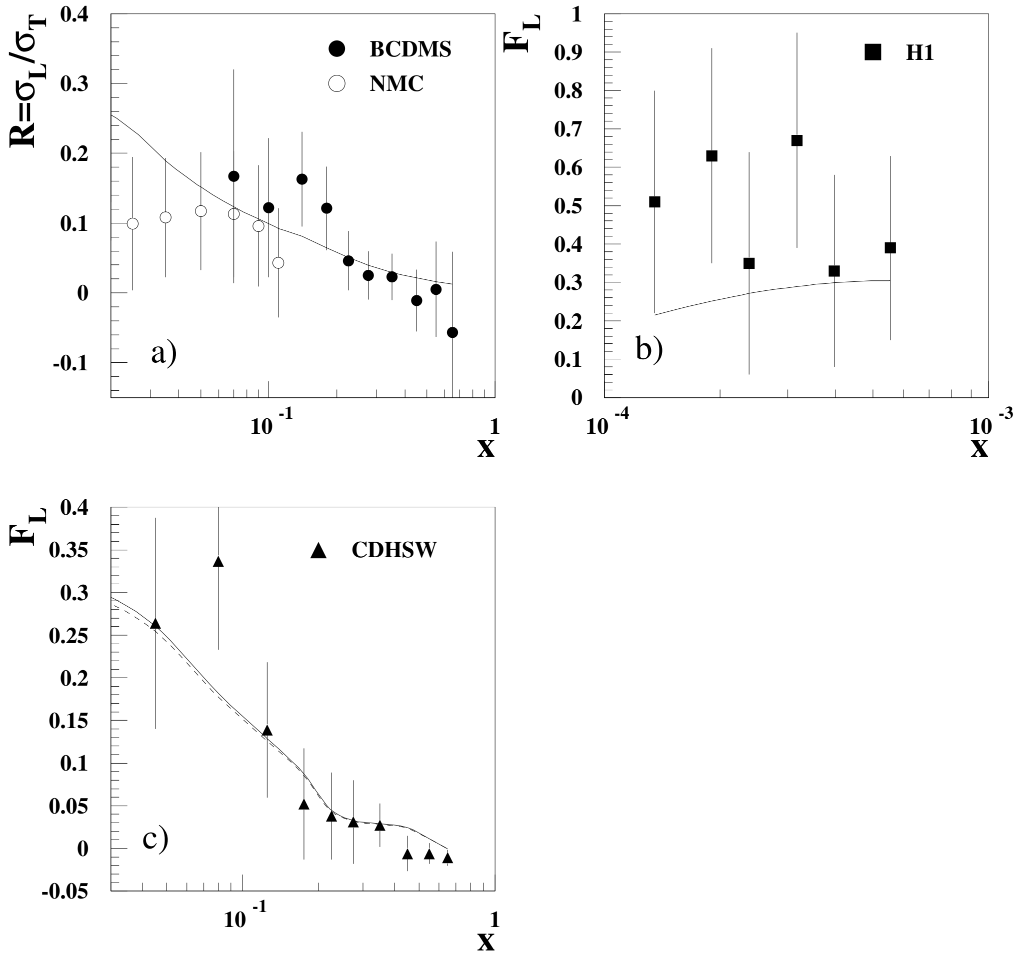


$F_2^{e,\mu p}+C(x_i)$
BCDMS
H1
NMC
x=0.00008
x=0.00013
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.2
x=0.32
0
2
4
6
8
10
12
10
10^2
10^3
10^4
Q^2/GeV^2



$F_2^p+c_i(x)$
Q^2 /GeV^2
NMC
BCDMS
SLAC
H1
H1 96 Preliminary (ISR)
H1 97 Preliminary (low Q^2)
H1 94-97 Preliminary (high Q^2)
NLO QCD Fit H1 Preliminary
$c_i(x)= 0.6 \bullet (i(x)-0.4)$
x=0.00002 (i=24)
x=0.000032
x=0.00005
x=0.00008
x=0.00013 (i=20)
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013 (i=10)
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.40
x=0.65 (i=1)

Longitudinal structure function



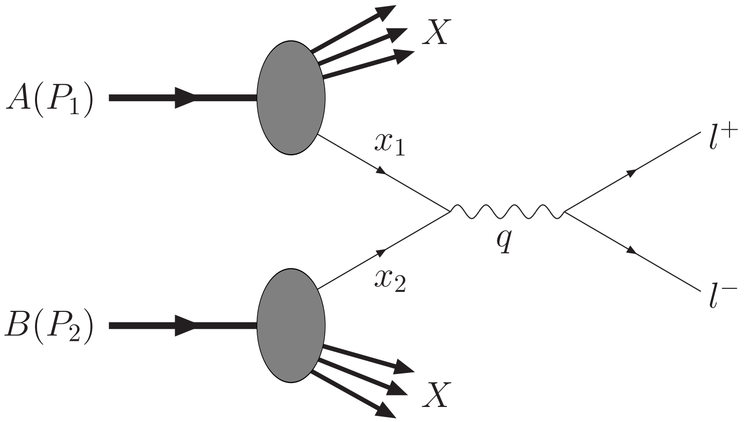
Some open or unsettled questions:

- Flavor asymmetry of the light sea: $\bar{u} \neq \bar{d}$.
- Large-x behavior of valence.
- Strange sea asymmetry: $s \neq \bar{s}$.
- Extraction of standard model parameters $(\alpha_s,\, \theta_W)$ from DIS.
- Small-x rise of structure functions and glue distribution.

The dominant parton distributions (u and d) are known at a few percent level (a huge progress in the past 20 years).

Another hard process: Drell-Yan production

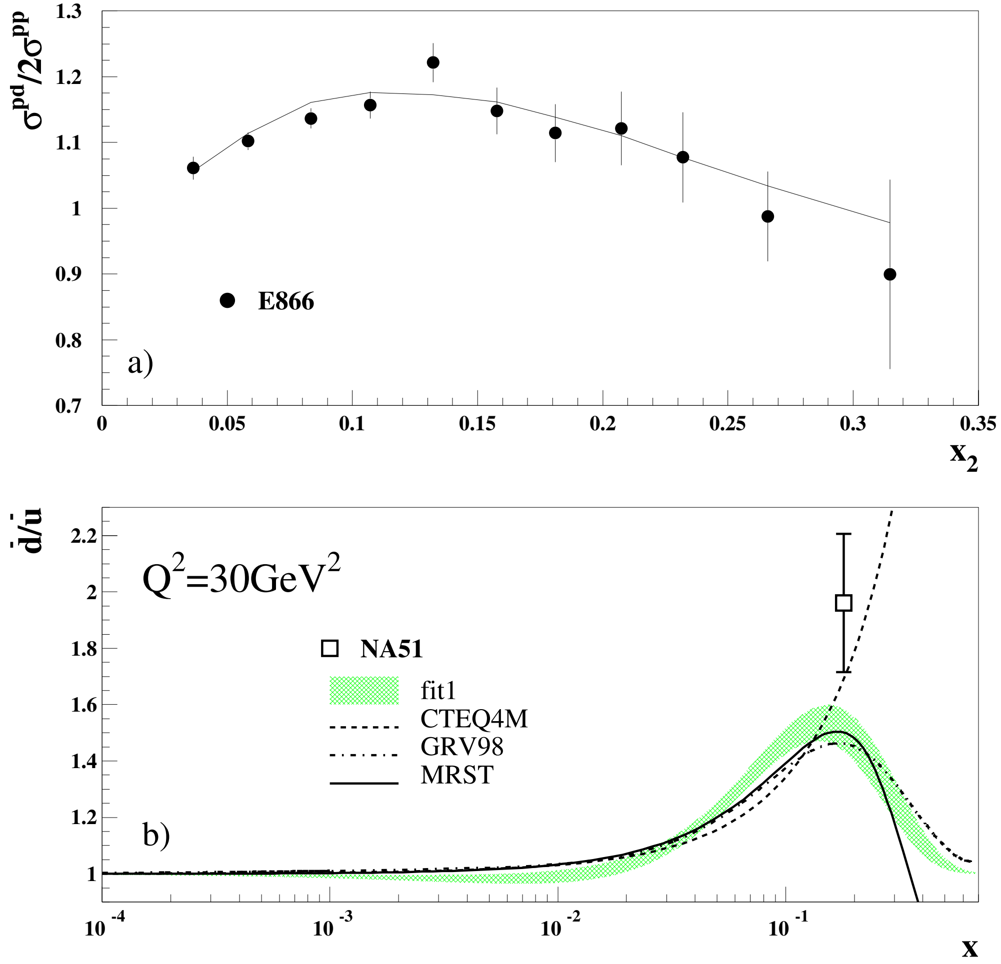
$$A + B \rightarrow l^+ + l^- + X$$



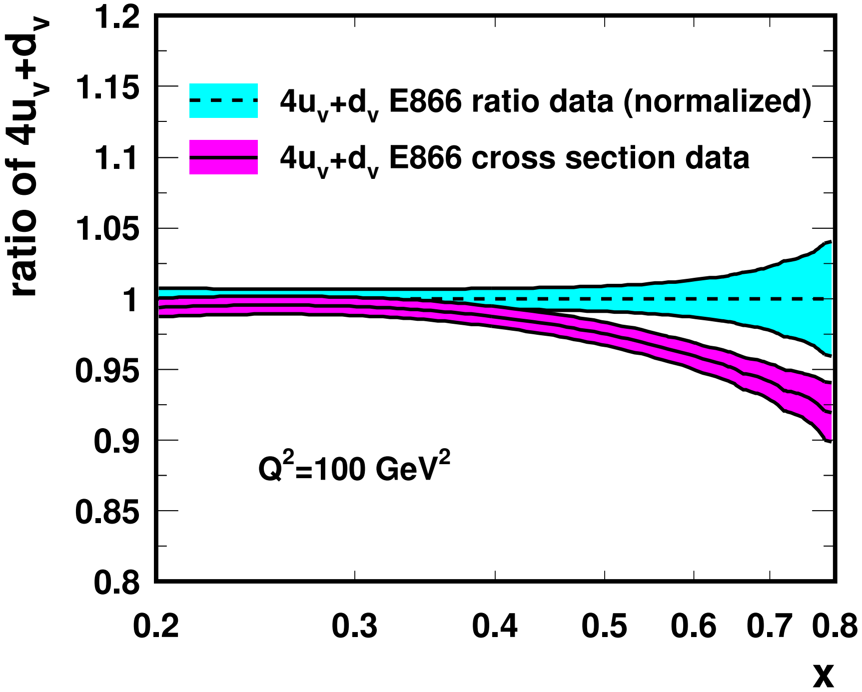
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1 \mathrm{d}x_2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{9Q^2} \sum_i e_i^2 \left[q_i(x_1)\bar{q}_i(x_2) + q_i(x_2)\bar{q}_i(x_1)\right]$$

DY provides crucial information about the flavor asymmetry of the sea and the large-x tails of valence distributions

Flavor asymmetry of the light sea: $\bar{u} \neq \bar{d}$



Large-x behavior of structure function



The large-x tail of valence is determined by DY, but current data on deuteron/proton ratio $\sigma_{DY}^{pD}/\sigma_{DY}^{pp}$ do not constrain it very well.

New preliminary data on separate cross sections, σ_{DY}^{pD} and σ_{DY}^{pp}, seem to imply a 10% depletion of u_v at $x \sim 0.8$.

Strange sea asymmetry

A large $\nu, \bar{\nu}$ DIS statistics allows checking the charge asymmetry of the strange sea

$$s(x) \neq \bar{s}(x)$$

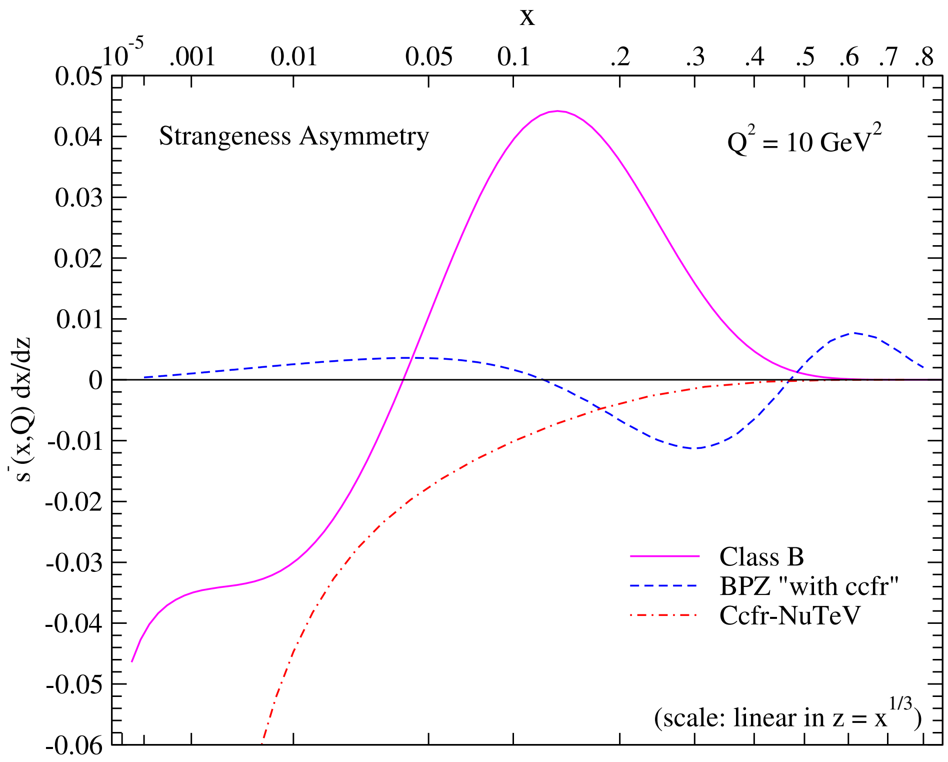
No fundamental principle forbids this asymmetry (of course, we must have $\int \mathrm{d}x(s - \bar{s}) = 0$ because the proton is not strange).

The strange asymmetry cannot be produced (at leading order) by QCD evolution. It must be an intrinsic effect.

According to Brodsky et al., there is a sea of intrinsic long-living $q\bar{q}$ pairs, manifesting as meson-baryon fluctuations.

Consider $p \to \Lambda K^+$. Due to chiral symmetry, pseudoscalar mesons have small masses and therefore the average x of the $\bar{s}$ inside the K is smaller than the average x of the s inside the Λ:

$$\int \mathrm{d}x\, x(s - \bar{s}) > 0$$



BPZ (2000): $\langle x\,(s-\bar{s})\rangle \sim 0.002$

Olness et al. (2004): $-0.001 < \langle x\,(s-\bar{s})\rangle < 0.004$

$\boxed{\text{Extraction of the Weinberg angle from } \nu, \bar{\nu} \text{ DIS}}$

Paschos-Wolfenstein ratio (NC/CC)

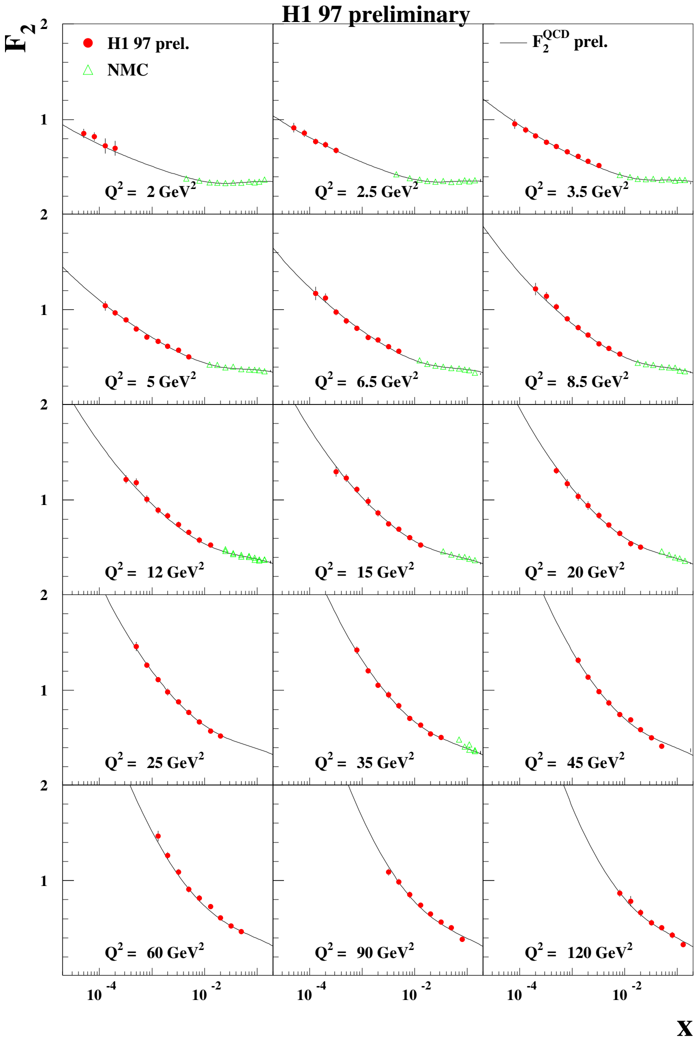
$$R_{\mathrm{PW}} = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{\mathrm{NC}}}{\sigma^{\nu}_{\mathrm{CC}} - \sigma^{\bar{\nu}}_{CC}} = \frac{1}{2} - \sin^2\theta_W$$

Corrections to R_{PW}: 1) order α_s; 2) non-isoscalarity; 3) $s \neq \bar{s}$.

CCFR/NuTeV data give $\sin^2\theta_W = 0.227 \pm 0.016$, about 3 σ larger than world average.

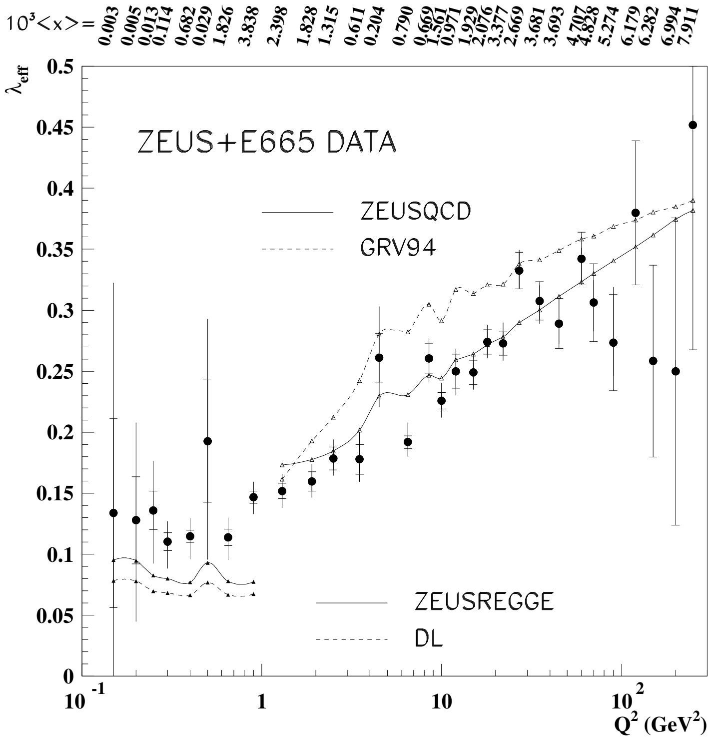
The strange sea asymmetry reduces (and possibly cancel) this discrepancy.

HERA finding: steep rise of F_2 at low x



Fit $F_2(x,Q^2) = A(Q^2)\,x^{-\lambda}$ for $x < 0.1$

$\lambda \simeq 0.1$ for $Q^2 \sim 1\,\mathrm{GeV}^2$, $\lambda \simeq 0.3$ for $Q^2 \sim 100\,\mathrm{GeV}^2$



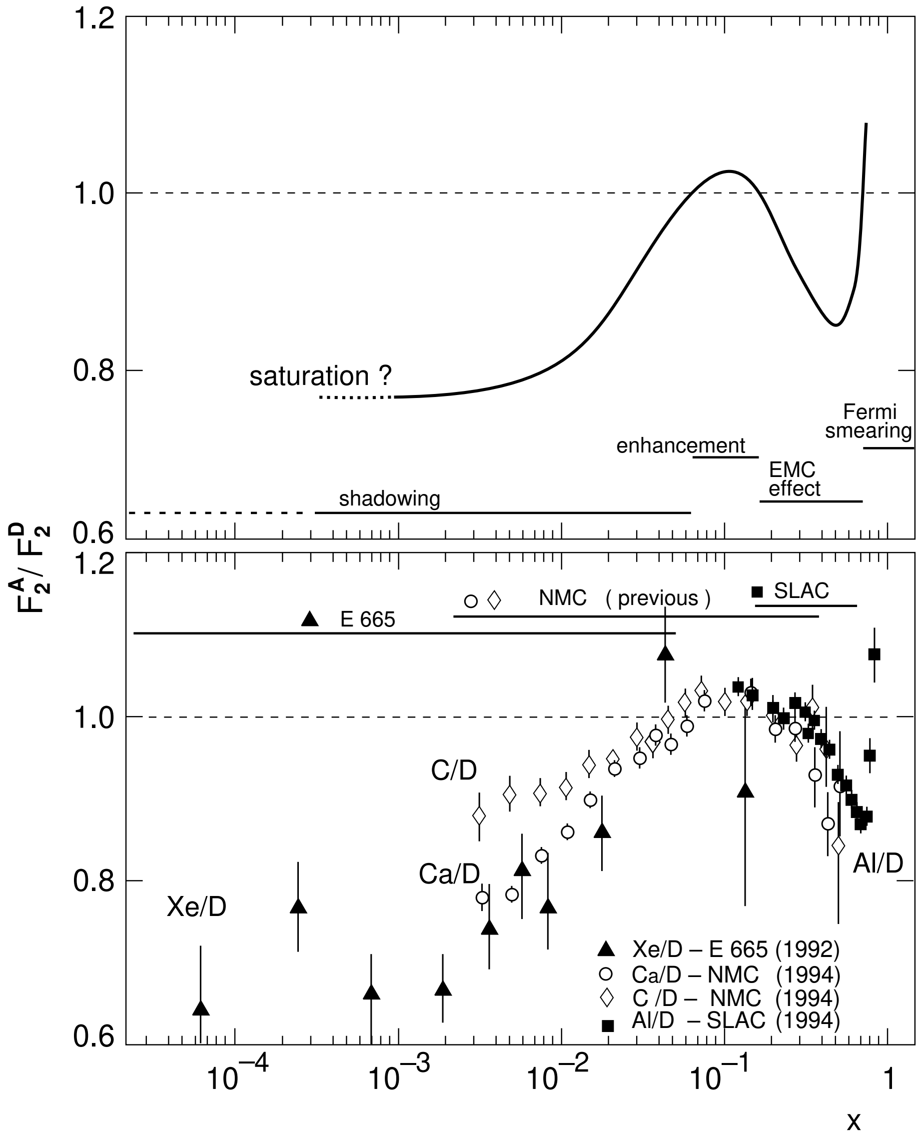
$\boxed{\text{Nuclear effects in DIS}}$

In 1983, the EMC Collaboration showed that the structure functions of iron are sensibly (and non trivially) different from those of free nucleons,

$$\rho_A(x, Q^2) \equiv \frac{F_2^A(x, Q^2)}{F_2^N(x, Q^2)} \neq 1$$

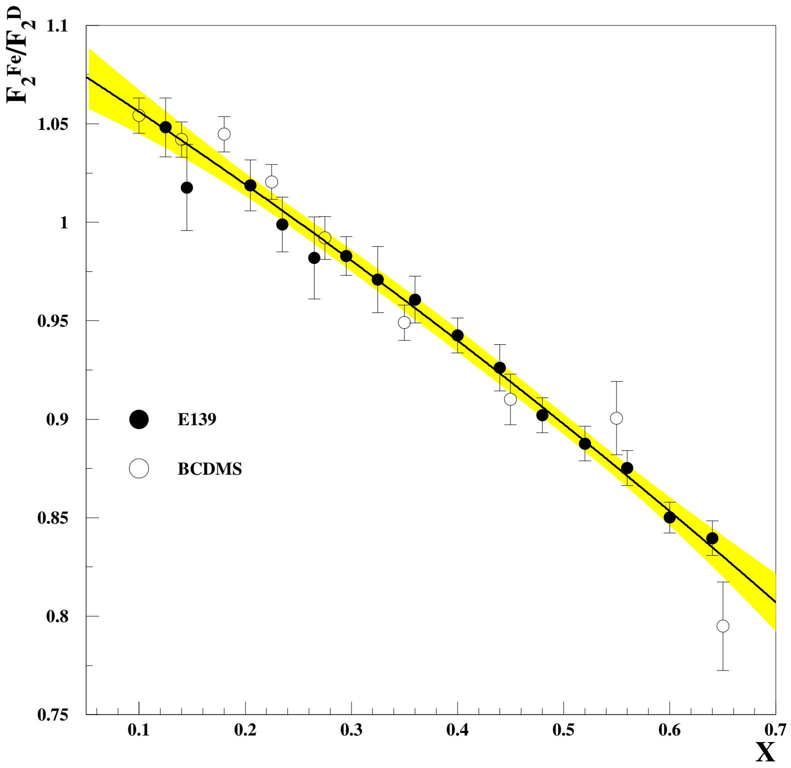
with the following pattern:

- $x < 0.1$: nuclear shadowing, $\rho_A < 1$
- $0.1 < x < 0.7$: EMC effect, ρ_A crosses 1 around $x \approx 0.3$
- $x > 0.7$: Fermi motion, steep rise of ρ_A



1.2
1.0
0.8
0.6
saturation ?
enhancement
Fermi smearing
EMC effect
shadowing
F_2^A / F_2^D
E 665
NMC (previous)
SLAC
C/D
Ca/D
Xe/D
Al/D
Xe/D – E 665 (1992)
Ca/D – NMC (1994)
C /D – NMC (1994)
Al/D – SLAC (1994)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Parametrization of $\rho_{\mathrm{Fe/D}}$ in the intermediate-x region



$$\rho_{\mathrm{Fe/D}}(x) = 1.09 - 0.34\,x - 0.09\,x^2 \qquad \text{(nearly a straight line)}$$

Fermi motion and nuclear binding

Trivial kinematic effect: since $x_A \equiv Q^2/2m_A\nu \leq 1$, the Bjorken variable $x \equiv Q^2/2m_N\nu$ is not constrained to be smaller than 1:

$$0 \leq x \leq \frac{m_A}{m_N}$$

The ratio F_2^A/F_2^N rises steeply as $x \to 1$, because F_2^N is constrained to vanish for $x = 1$.

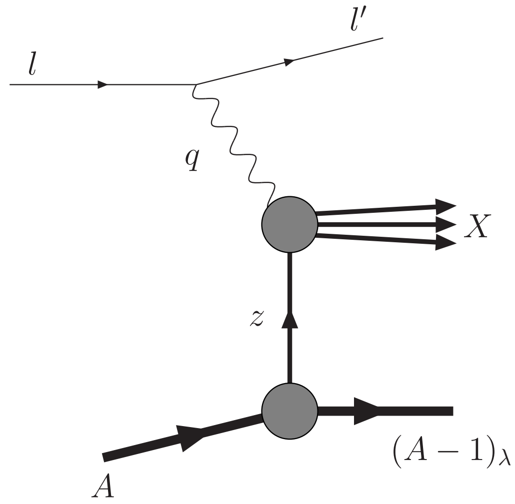
In the region $0.1 < x < 0.7$ nuclear binding is at work.

In the convolution model, DIS on nuclei is pictured as a factorized process

$$F_2^A = f_{N/A} \otimes F_2^N$$

Convolution model

$$F_2^A(x) = \int_x^A \mathrm{d}z\, f(z)\, F_2^N(x/z)$$



Momentum distribution of nucleons:

$$f(z) = \sum_\lambda \int \mathrm{d}^4p\, \frac{p^+}{p^0}\, |\phi_\lambda(\boldsymbol{p})|^2\, \delta(p^0 - m_N - \epsilon_\lambda + T_R)\, \delta(z - A\,p^+/P_A^+)$$

- off-shellness of nucleons
- value of the binding energy $\langle \epsilon_\lambda \rangle$
- relativistic effects
- violation of the energy-momentum sum rule: $\int \mathrm{d}z\, z\, f(z) \neq 1$
- modification of nucleon properties in nuclei

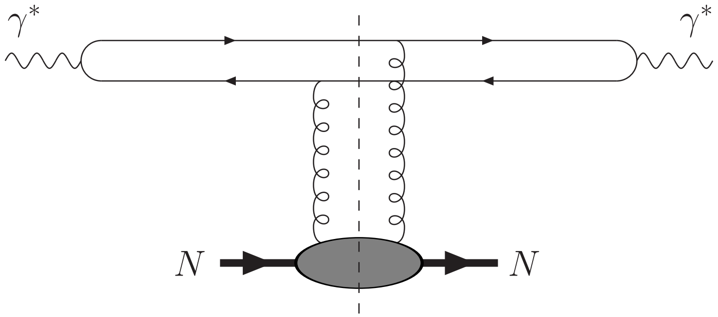
$\boxed{\text{Nuclear shadowing}}$

In the limit of photoproduction ($Q^2 \to 0$), shadowing can be understood by assuming that the photon fluctuates into vector mesons (ρ, ω, ϕ, ...) which interact strongly with the nucleons on the surface of the target nucleus. These nucleons absorb most of the hadronic flux and cast a shadow onto the inner ones.

Naïvely, shadowing is expected to disappear as Q^2 gets larger. The EMC NA28 experiment (1988, 1990) and the NMC experiment (1991) showed that this is not the case: shadowing persists at relatively large Q^2.

The leading-twist character of nuclear shadowing was predicted in 1975 by Nikolaev and Zakharov, and attributed to fusion of partons from different nucleons.

Color-dipole model (Nikolaev & Zakharov, Mueller)



Formation time of the $q\bar{q}$ fluctuation of γ^*:

$$\tau_f \sim \frac{1}{\Delta E} \sim 1/m_N x$$

At low x DIS can be described in terms of the interaction of the $q\bar{q}$ Fock state of the virtual photon (of transverse size ρ) with the target

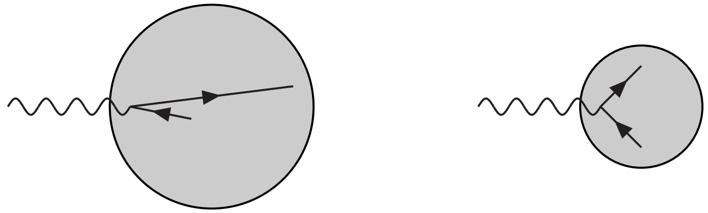
$$\sigma_{L,T}^{\gamma^* N}(x, Q^2) = \langle \sigma(x, \rho) \rangle_{L,T} \equiv \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{\rho}\, |\Psi_{L,T}(z, \rho)|^2 \, \sigma(x, \rho) \, .$$

$\Psi_{L,T}(z, \rho)$ are the wave functions of the $q\bar{q}$ fluctuations of longitudinal and transverse photons (explicitly calculable).

Dipole cross section:

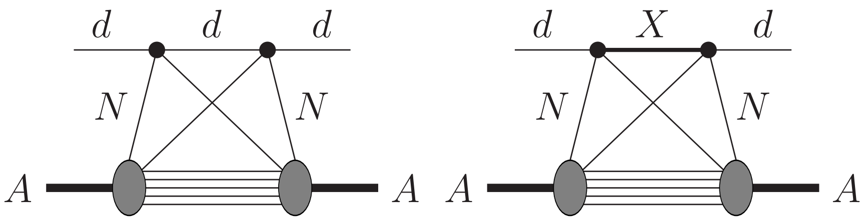
$$\sigma(x,\rho) = \frac{4\pi}{3} \int \frac{\mathrm{d}^2 \boldsymbol{k}_\perp}{\boldsymbol{k}_\perp^4}\, \alpha_s\, \frac{\partial[xg(x,\boldsymbol{k}_\perp^2)]}{\partial \ln \boldsymbol{k}_\perp^2} \left(1 - \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp \cdot \rho}\right)$$

For small ρ, $\sigma(x,\rho) \sim \rho^2 \sim 1/Q^2$ (color transparency). At large ρ, the dipole cross section saturates: $\sigma(x,\rho) \sim 1/\mu^2 \sim R_h^2$.



Small-size symmetric pairs contribute equally to $\sigma_L^{\gamma^* N}$ and $\sigma_T^{\gamma^* N}$.
The contribution of large-size asymmetric pairs (aligned-jet configuration) to $\sigma_L^{\gamma^* N}$ is suppressed by a factor $1/Q^2$.

Glauber theory of multiple rescattering (nucleus conceived as a system of uncorrelated nucleons, with the overall phase shift equal to the sum of phase shifts for each nucleon) + Gribov inelastic corrections:



Shadowing correction (to be subtracted from $\sigma_{L,T}^{\gamma^* N}(x, Q^2)$):

$$\Delta\sigma_{L,T}^{\gamma^* N}(x, Q^2) \sim \frac{\mathrm{d}\sigma_{L,T}^{\mathrm{diffr}}}{\mathrm{d}t} \sim \langle \sigma^2(x,\rho) \rangle_{L,T} \sim \frac{1}{Q^2} \int_{1/Q^2}^{R_h^2} \frac{\mathrm{d}\rho^2}{\rho^4}\, \sigma^2(\rho)$$

Shadowing is dominated by large-size asymmetric pairs (due to saturation of dipole cross sections).

- Nuclear effects can be important even at very high energies.
- Nuclear DIS provides a unique window on quark-gluon dynamics in nuclei.
- Interplay between short-distance and large-distance phenomena.
- Open problems:
  - EMC effect understood only qualitatively.
  - Nuclear effects in ν DIS (especially at low x).