# Spin density matrix

In a nuclear reaction we have ensembles of $N$ identical particles

The wave function of a particle $i$ with spin $s$ and magnetic component $\alpha$ with respect to a specified axis

$$\psi_i = \sum_\alpha a_\alpha^i |s\alpha\rangle$$

The spin density matrix is defined as an average over the ensemble of the density operator

$$\rho_{\alpha\alpha'} = \frac{1}{N}\sum_{i=1}^{N} \langle s\alpha|\psi_i\rangle\langle\psi_i|s\alpha'\rangle = \frac{1}{N}\sum_{i=1}^{N} a_\alpha^i a_{\alpha'}^{i*}$$

$\rho = \rho^+$ hermitian $\qquad \mathrm{Tr}\,\rho = \sum_\alpha \rho_{\alpha\alpha} = 1$

For every hermitian operator in the spin space

$$\langle A\rangle = \frac{1}{N}\sum_{i=1}^{N} \langle\psi_i|A|\psi_i\rangle = \mathrm{Tr}(\rho A)$$

For $s = \frac{1}{2}$ $\quad\rho\quad$ $2\times 2$ matrix

$\rho = \frac{1}{2}(1 + \vec{P}\cdot\vec{\sigma})$ $\qquad \vec{P}$ vector polarization

$P_a = \dfrac{N_+ - N_-}{N_+ + N}$ $\qquad \langle\vec{\sigma}\rangle = \mathrm{Tr}(\rho\vec{\sigma}) = \vec{P}$