The kinetic energy operator results

$$T = -\frac{\hbar^2}{m}\sum_{i=1}^{A}\nabla_i^2 = T_{CM} + \left(-\frac{\hbar^2}{m}\right)\sum_{i=1}^{N}\nabla_{x_i}^2$$

$$\sum_{i=1}^{N}\nabla_{x_i}^2 = \underbrace{\frac{\partial^2}{\partial\rho^2} + \frac{3N-1}{\rho}\frac{\partial}{\partial\rho}}_{\text{hyperradial}} + \frac{L^2(\Omega)}{\rho^2}$$

hyperangular

$$\left[L^2(\Omega) + K(K+3N-2)\right] Y_{[K]}(\Omega) = 0$$

$A = 3$

$$Y_{[K]}(\Omega) = \left[Y_{l_1}(\hat{x}_1)\,Y_{l_2}(\hat{x}_2)\right]_{LM}\, {}^{(2)}P_m^{l_1 l_2}(\phi)\, N_m^{l_1 l_2}$$

$${}^{(2)}P_m^{l_1 l_2}(\phi) = (\sin\phi)^{l_1}(\cos\phi)^{l_2}\, P_m^{l_1+1/2,\, l_2+1/2}(\cos 2\phi)$$

$K = l_1 + l_2 + 2m$

$A = 4$

$$Y_{[K]}(\Omega) = \left[Y_{l_1}(\hat{x}_1)\,Y_{l_2}(\hat{x}_2)\,Y_{l_3}(\hat{x}_3)\right]_{LM}\, {}^{(3)}P_{m_1 m_2}^{l_1 l_2 l_3}(\phi_1\,\phi_2)\, N_{m_1 m_2}^{l_1 l_2 l_3}$$

$${}^{(3)}P_{m_1 m_2}^{l_1 l_2 l_3}(\phi_1\phi_2) = (\sin\phi_1)^{l_1}(\cos\phi_1)^{l_2}(\sin\phi_2)^{l_1+l_2+2m_2}(\cos\phi_2)^{l_3}\times$$

$$\times P_{m_1}^{l_1+1/2,\, l_2+1/2}(\cos 2\phi_1)\; P_{m_2}^{K_2+2,\, l_3+1/2}(\cos 2\phi_2)$$

$K_2 = l_1 + l_2 + 2m_1$

$K = K_2 + l_3 + 2m_2$