

Variational estimates using a Discrete Variable Representation

1) Bound states

$$H = -\frac{\hbar^2}{m} \nabla^2 + V(r)$$

$$H \psi_\lambda = E_\lambda \psi_\lambda \quad E_\lambda < 0$$

$$\psi_\lambda = \sum_{k=0}^{N-1} A_k^\lambda \phi_k(r)$$

$$\int dr r^2 \phi_k(r) \phi_{k'}(r) = \delta_{kk'}$$

The eigenvalues of the $N \times N$ Hamiltonian

$$E_{\lambda+1}(N) \geq E_\lambda(N) \geq E_\lambda(N+1)$$

$$\lim_{N \rightarrow \infty} E_\lambda(N) = E_\lambda$$