

They are obtained from

$$r_{kk'} = \int_0^\infty dr r^2 \phi_k(r) r \phi_{k'}(r)$$

The eigenvalues of $(r)_{kk'}$ are $\{x_\alpha\}$

The eigenvectors are

$$T_{k\alpha} = \phi_k(x_\alpha) \sqrt{w_\alpha} \longrightarrow T_{0\alpha} = \sqrt{w_\alpha}$$

The matrix T is unitary

$$\sum_\alpha T_{k\alpha} T_{k'\alpha} = \delta_{kk'}$$

From the definition

$$V_{kk'}^{FBR} = \sum_{\alpha=0}^{N-1} w_\alpha \phi_k(x_\alpha) V(x_\alpha) \phi_{k'}(x_\alpha)$$

We obtain

$$V_{\alpha\beta}^{DVR} = (T^\dagger V^{FBR} T)_{\alpha\beta} = V(x_\alpha) \delta_{\alpha\beta}$$