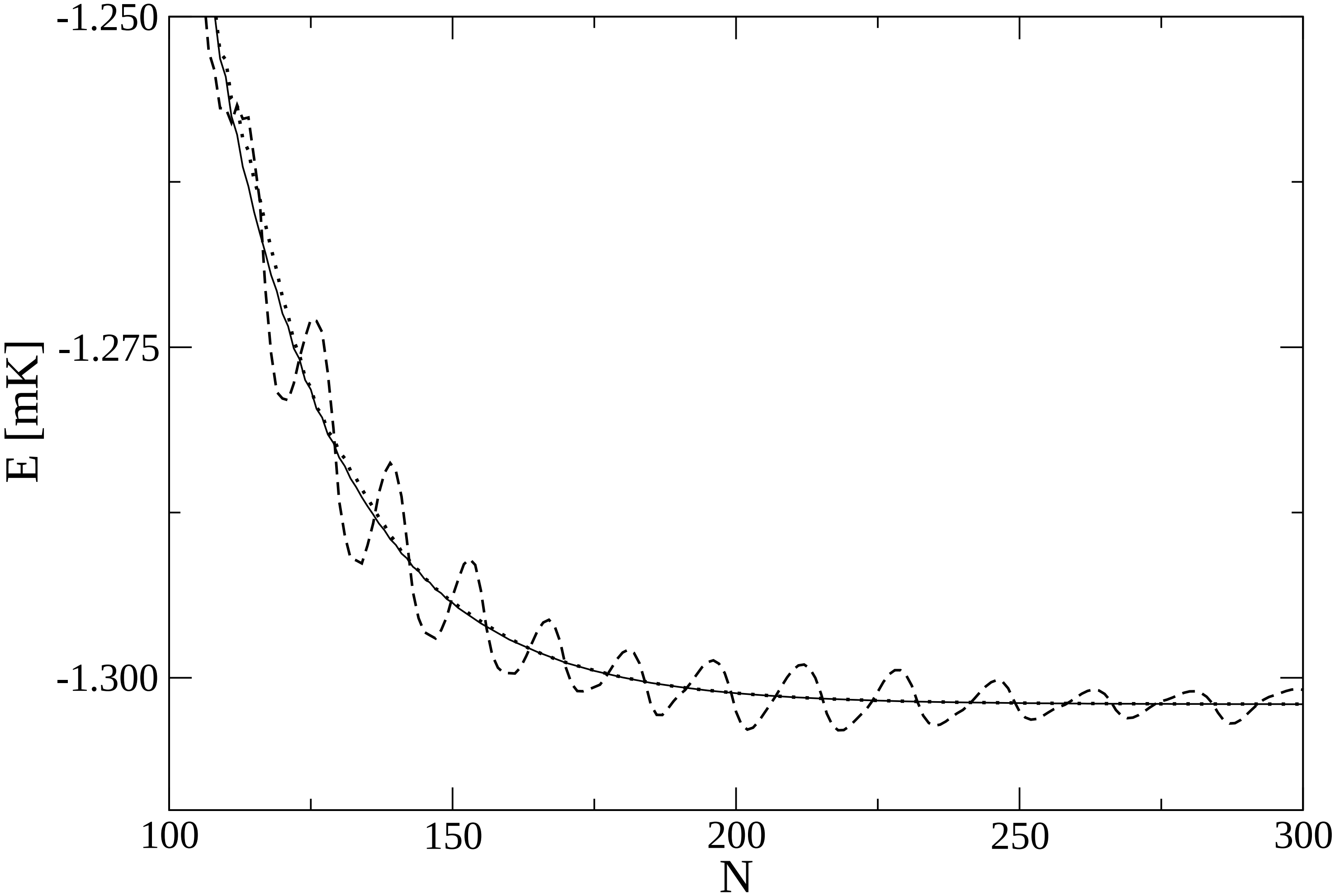
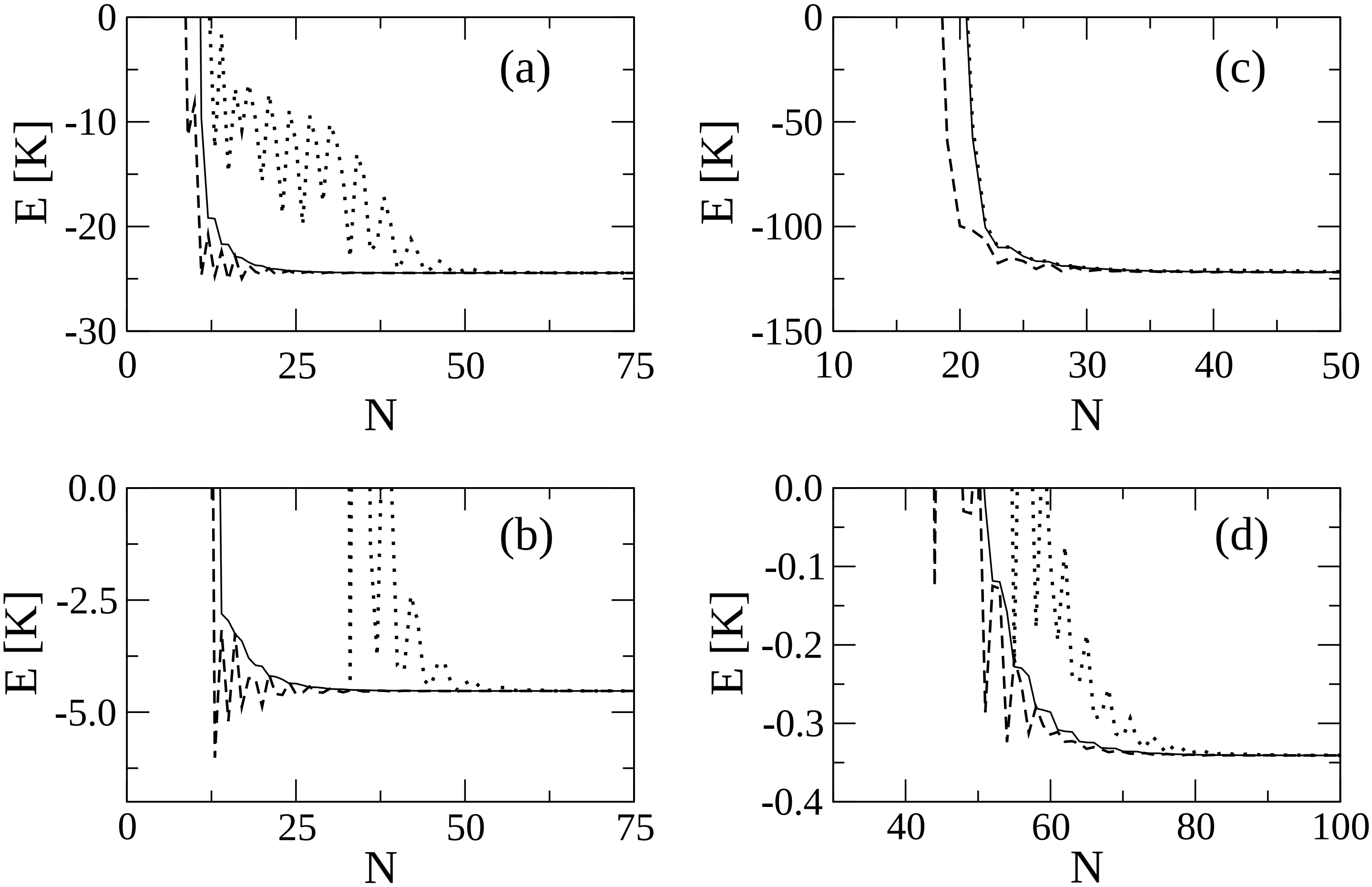
Table 1: The He_2 binding energy, potential energy, calculated with the VBR, DVR and $<$ DVR $>$ methods, as a function of the size N of the basis set employed. Energies are in mK

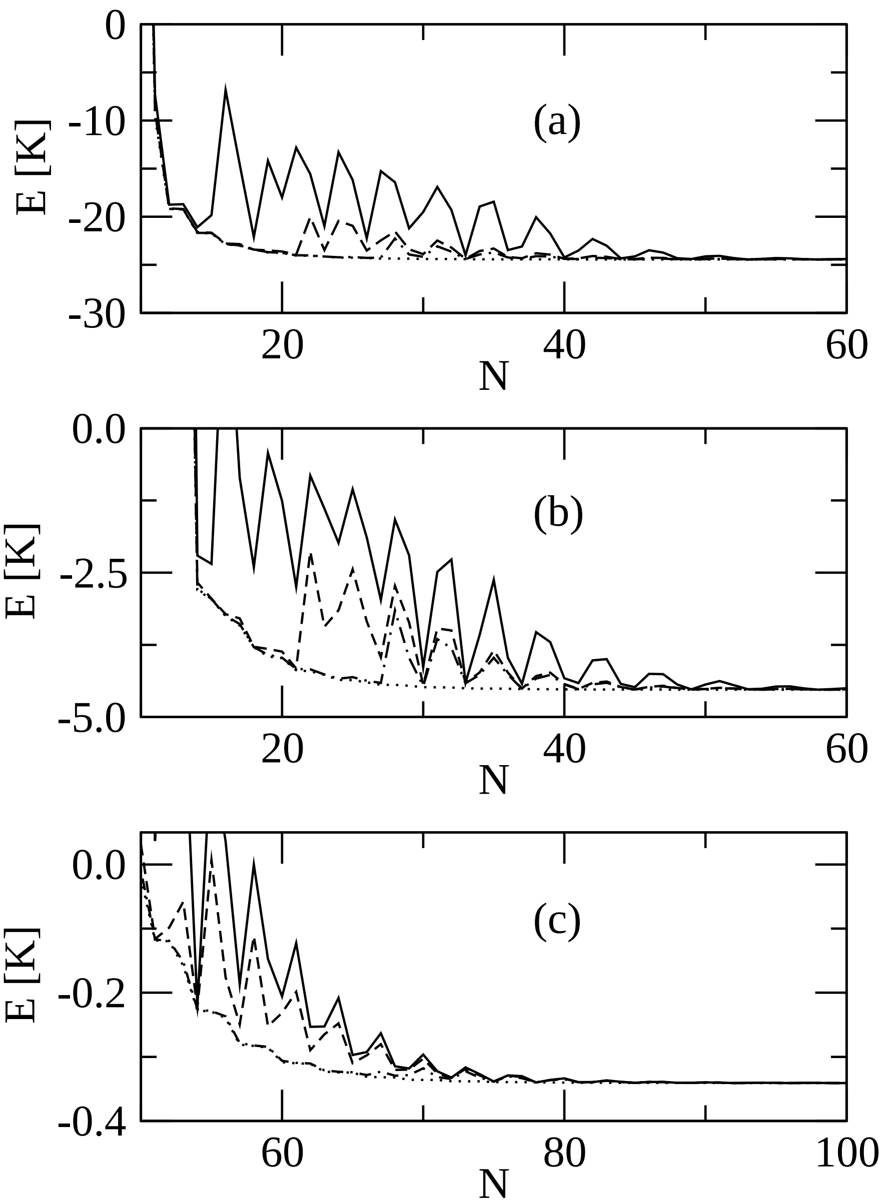
	VBR		DVR		$< DVR >$	
N	E	V	E	V	E	V
50	-0.0588	-141.786	-0.0742	-140.622	0.1002	-140.448
100	-1.2261	-108.303	-1.2337	-108.480	-1.2177	-108.464
150	-1.2943	-102.049	-1.2938	-102.034	-1.2942	-102.035
200	-1.3012	-100.904	-1.3026	-100.958	-1.3012	-100.957
250	-1.3019	-100.718	-1.3025	-100.742	-1.3019	-100.742
300	-1.3020	-100.690	-1.3009	-100.648	-1.3020	-100.649
350	-1.3020	-100.686	-1.3027	-100.714	-1.3020	-100.713
400	-1.3020	-100.686	-1.3014	-100.662	-1.3020	-100.662
450	-1.3020	-100.686	-1.3015	-100.665	-1.3020	-100.665
500	-1.3020	-100.686	-1.3019	-100.682	-1.3020	-100.682



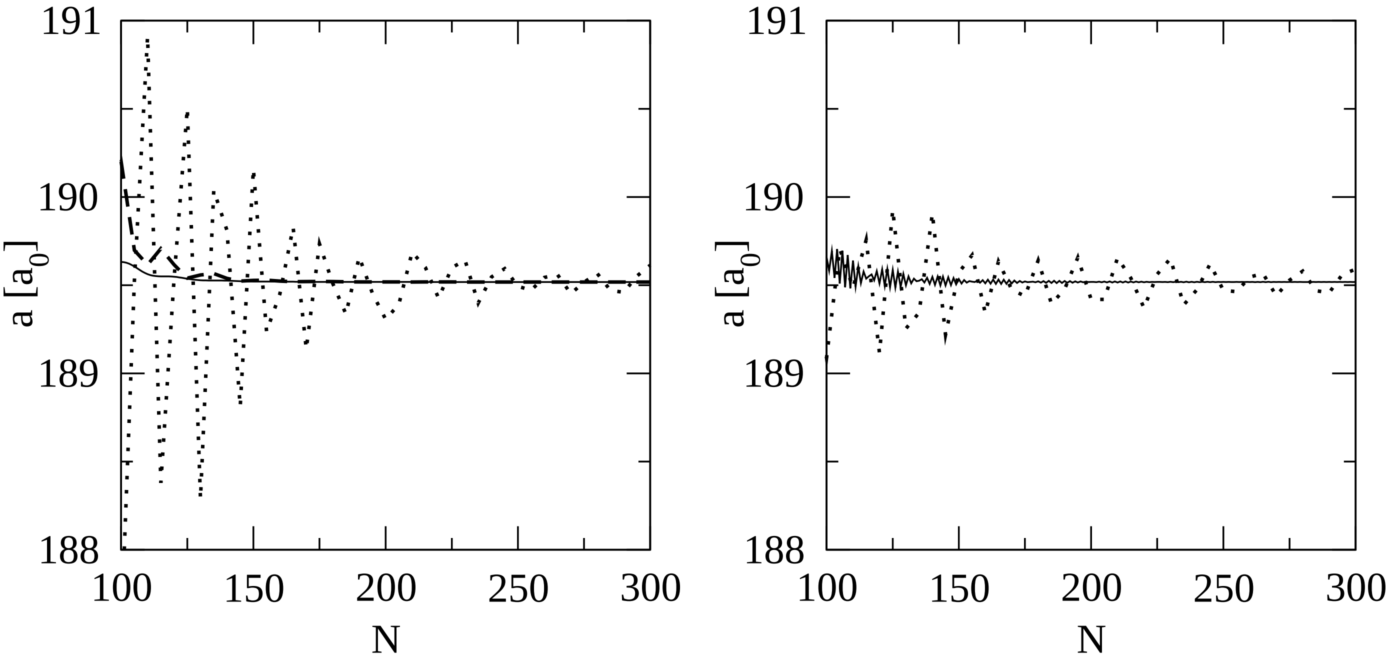
Ground state energy of He_2, calculated using the VBR (solid line), DVR(dashed line) and $<\mathrm{DVR}>$ (dotted line) methods, as a function of the dimension N of the basis.



(a) ground state energy of Ne_2, (b) first excited state energy of Ne_2, (c) ground state energy of Ar_2 and (d) eighth level of Ar_2, calculated using VBR (solid line), DVR (dashed line) and $<\mathrm{DVR}>$ (dotted line), as a function of the dimension N of the basis.



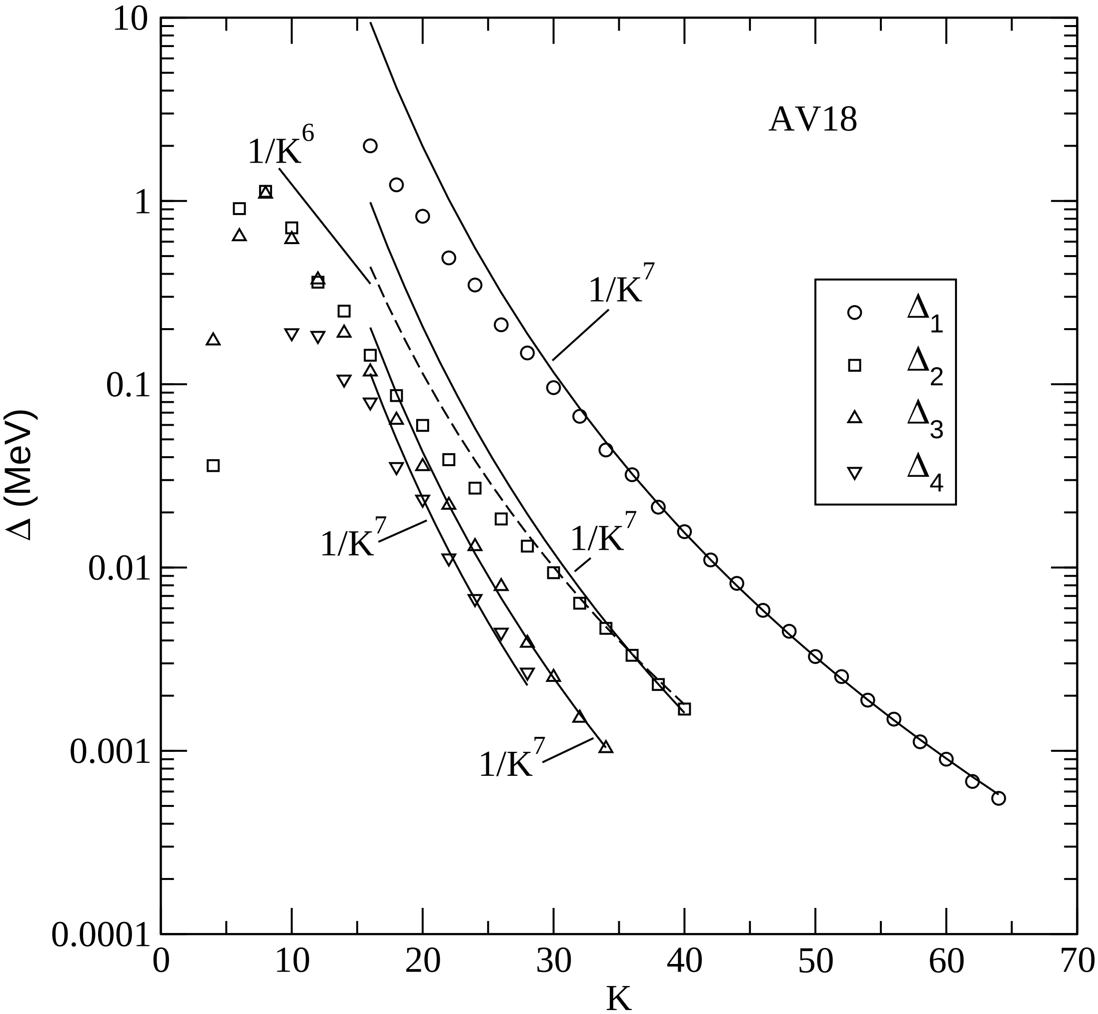
Convergence patterns for the energy of the ground state of Ne_2 (a), the first excited vibrational state of Ne_2 (b), and the eighth level of Ar_2 (c), for different choices of the size n' of the restricted variational problem. For Ne_2 the lines correspond to $n' = 4$ (solid), $n' = 8$ (dashed) and $n' = 12$ (dotted-dashed). For Ar_2 the lines correspond to $n' = 12$ (solid), $n' = 20$ (dashed) and $n' = 30$ (dotted-dashed). The VBR results are shown as a dotted line for reference.



Second order estimate for the He-He scattering length a calculated with the VBR (solid line), $<$ DVR $>$ (dashed line) and DVR$_q$ (dotted line) as a function of N (left panel). First order estimate for a, as obtained with the VBR method (solid line), and DVR (dotted line) as a function of N (right panel).

Table 2: Convergence of α–particle binding energies (MeV) corresponding to the inclusion in the WF of the different classes C1–C6 in which the HH basis has been divided.

K_1	K_2	K_3	K_4	K_5	K_6	MT-V	AV18	AV18+UIX
20						28.928	14.70	14.90
30						29.794	15.99	16.17
40						29.962	16.17	16.34
50						30.008	16.20	16.377
60						30.024	16.213	16.385
70						30.032	16.214	16.386
72						30.033	16.214	16.386
72	8					30.714	18.29	18.99
72	16					31.170	19.75	20.65
72	24					31.240	19.97	20.87
72	32					31.256	20.01	20.912
72	36					31.259	20.022	20.918
72	40					31.261	20.026	20.921
72	40	8				31.300	21.94	24.69
72	40	16				31.336	23.24	27.15
72	40	24				31.340	23.37	27.35
72	40	30				31.341	23.385	27.37
72	40	34				31.341	23.388	27.37
72	40	34	8			31.341	23.54	27.55
72	40	34	16			31.344	24.086	28.31
72	40	34	20			31.346	24.145	28.38
72	40	34	24			31.347	24.163	28.40
72	40	34	28			31.347	24.170	28.41
72	40	34	28	16			24.181	28.43
72	40	34	28	20			24.191	28.439
72	40	34	28	24			24.195	28.444
72	40	34	28	24	4		24.205	28.456
72	40	34	28	24	8		24.209	28.461
72	40	34	28	24	12		24.210	28.462
72	40	34	28	24	16		24.210	28.462
“extrapolated”						31.358	24.23	28.47
“exact”						31.360	24.25	28.50



Convergence studies: binding energy differences for the classes C1 – C4 as function of the grand angular value K.

$$\Delta_1(K) = B(K,0,0,0,0,0) - B(K-2,0,0,0,0,0)$$
$$\Delta_2(K) = B(K_{1\mathrm{M}},K,0,0,0,0) - B(K_{1\mathrm{M}},K-2,0,0,0,0)$$
$$\Delta_3(K) = B(K_{1\mathrm{M}},K_{2\mathrm{M}},K,0,0,0) - B(K_{1\mathrm{M}},K_{2\mathrm{M}},K-2,0,0,0)$$

$K_{1\mathrm{M}} = 72$
$K_{2\mathrm{M}} = 40$
$K_{3\mathrm{M}} = 34$

Binding Energies of $A = 3$

Hamiltonian	^{3}H	^{3}He
	B(MeV)	B(MeV)
AV18 ($T = 1/2$)	7.618	6.917
AV18 ($T = 1/2, 3/2$)	7.624	6.925
AV18+UR ($T = 1/2$)	8.474	7.742
AV18+UR ($T = 1/2, 3/2$)	8.479	7.750
Expt.	8.482	7.718

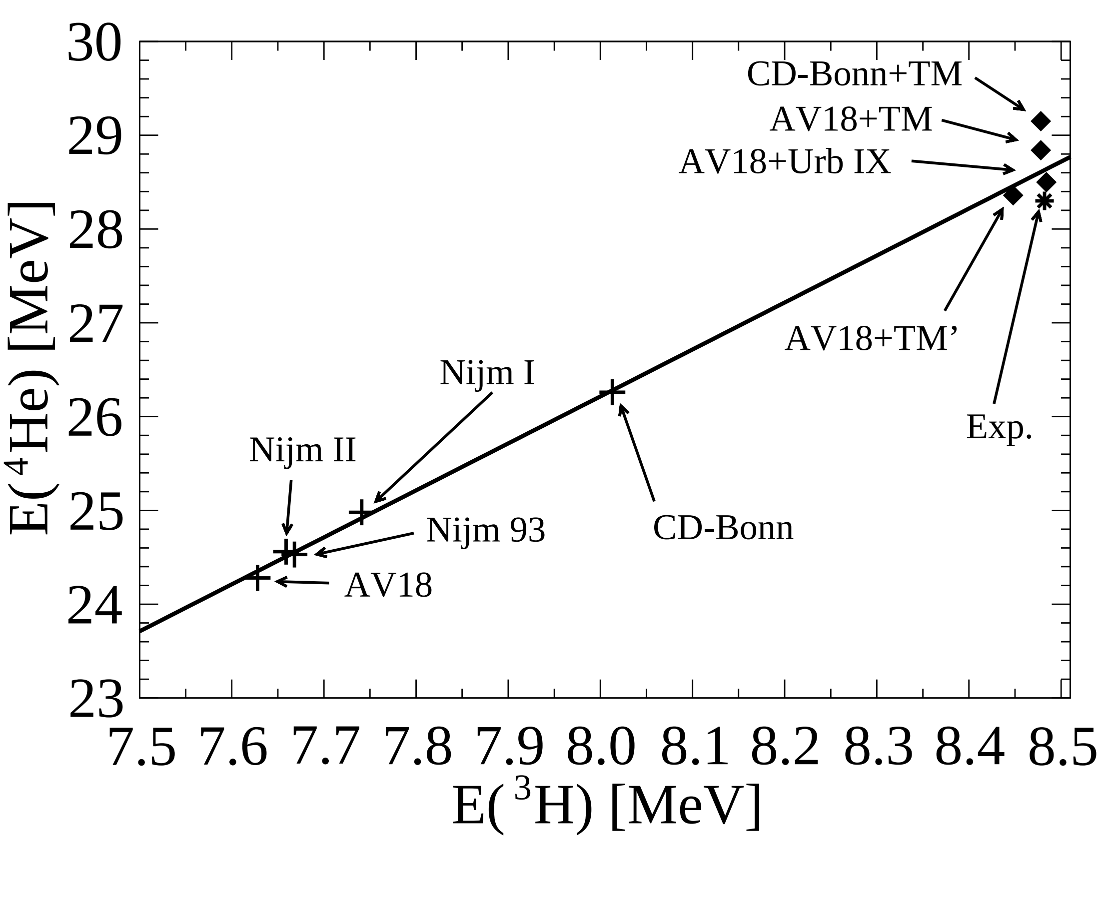
Interaction term	$B(^3\mathrm{H})$-$B(^3\mathrm{He})$
Nuclear CSB	65 keV
Point Coulomb	677 keV
Full Coulomb	648 keV
Magnetic moment	17 keV
Orbit-orbit force	7 keV
n-p mass difference	14 keV
Total (theory)	751 keV
Expt.	764 keV

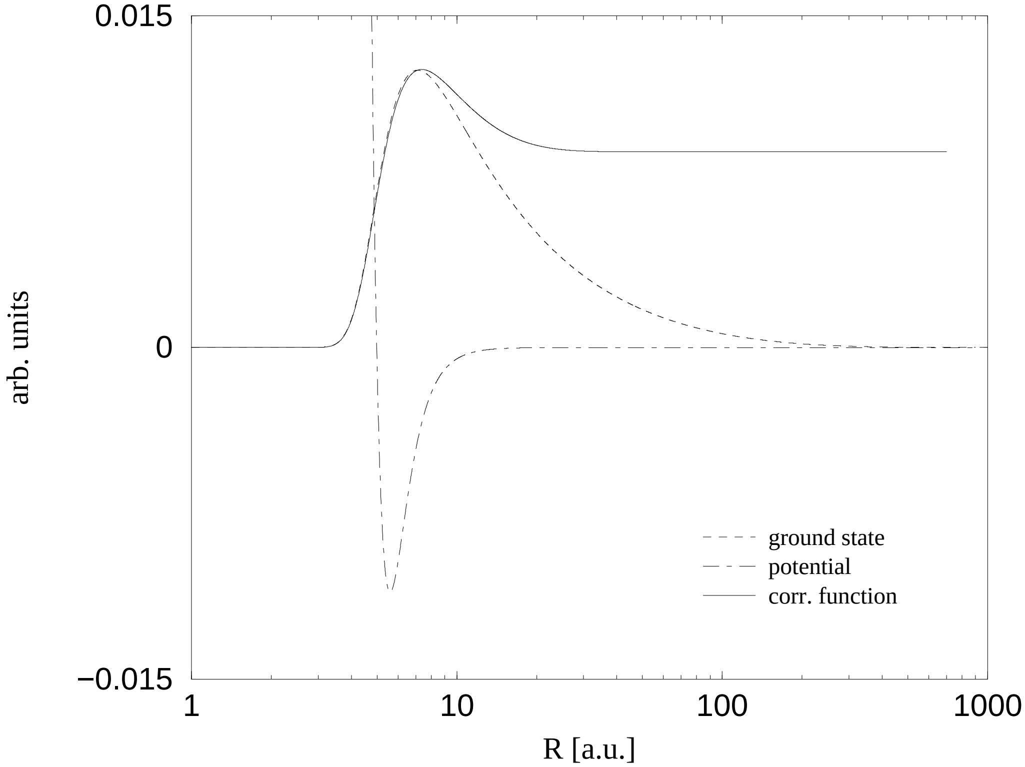
A. Nogga,...,A.K, M.V., L.M.,S.R, Phys.Rev.C 67, 034004 (2003)

The HH Method applied to the A=4 bound state

M. Viviani, A. Kievsky, S. Rosati, PRC71, 024006 (2005)

potential	Method	B(MeV)	T(MeV)	$P_P(\%)$	$P_D(\%)$
AV18	HH	24.22	97.84	0.35	13.74
	FY(MS)	24.25	97.80	0.35	13.78
	FY(CS)	24.22	97.77		
Nijm II	HH	24.43	100.27	0.33	13.37
	FY(CS)	24.56	100.31	0.33	13.37
AV18+UR	HH	28.46	113.30	0.73	16.03
	FY(MS)	28.50	113.21	0.75	16.03
	GFMC	28.34(4)	110.7(7)		
AV18+TM'	HH	28.31	110.27	0.73	15.63
	FY(MS)	28.36	110.14	0.75	15.67
Exp.		28.30			

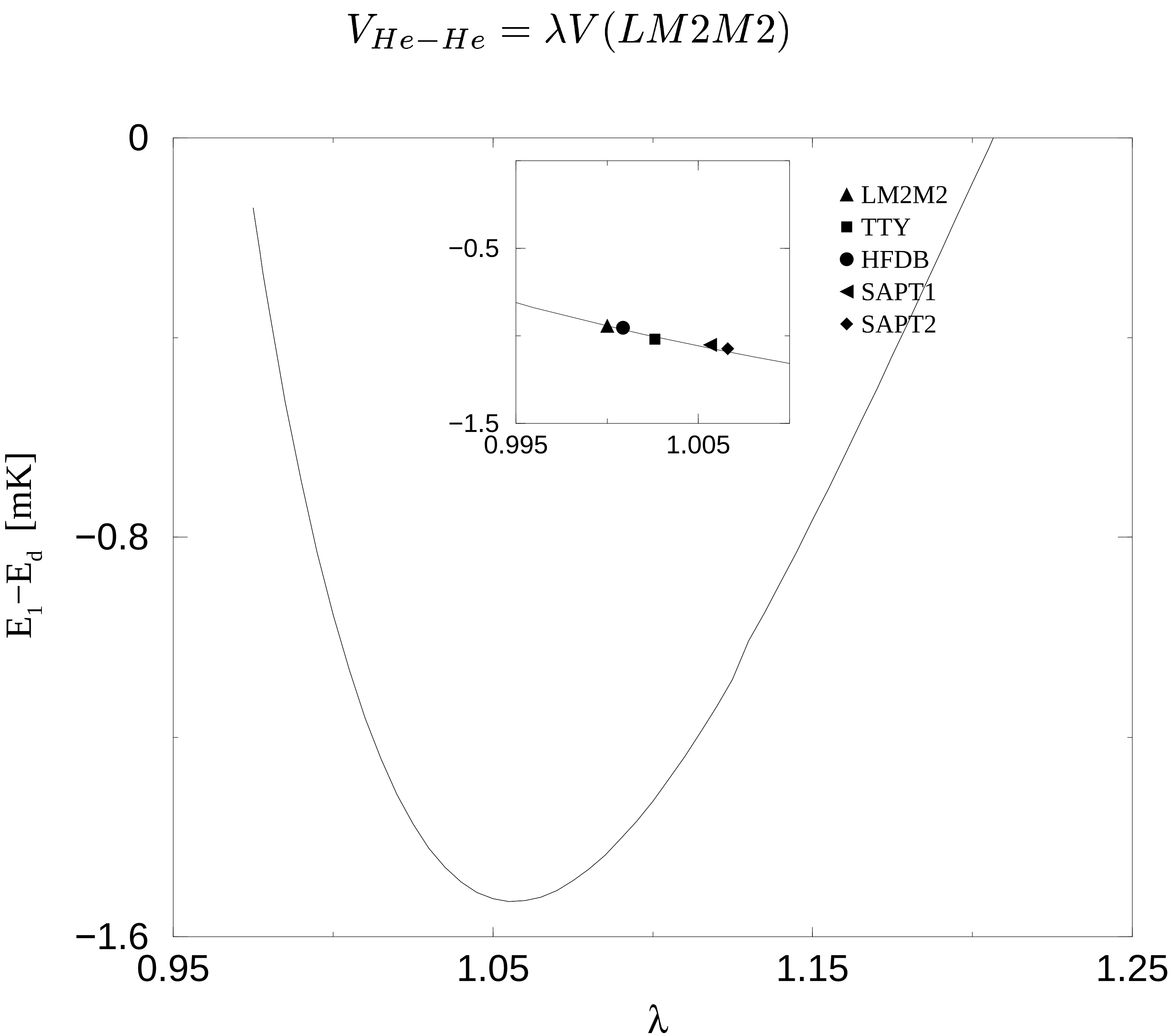




		HFDB	LM2M2	TTY	SAPT1	SAPT2	exp.
ϵ	(K)	10.95	10.97	10.98	11.05	11.06	
r_m	(a.u.)	5.599	5.611	5.616	5.603	5.602	
σ	(a.u.)	4.983	4.992	5.000	4.987	4.987	
$\lvert E_b \rvert$	(mK)	1.685	1.303	1.313	1.733	1.816	
$\langle T \rangle$	(mK)	112.2	99.43	99.93	115.0	117.8	
$-\langle V \rangle$	(mK)	113.9	100.7	101.2	116.8	119.6	
$\langle R \rangle$	(a.u.)	87.81	97.96	97.62	86.04	84.24	98 ± 8
$\sqrt{\langle R^2 \rangle}$	(a.u.)	119.0	132.9	132.5	116.5	114.0	
a	(a.u.)	170.5	191.4	190.7	166.9	163.2	
$2 < R >$	(a.u.)	175.6	195.9	195.2	172.1	168.5	$197 + 15 - 34$

Table 3: Characteristic values of the different potentials and their relative bound states. R represents the He-He distance, ϵ is the strength of the potential at its point of minimum r_m and σ is the distance at which the potential change sign.

	ground state				
k_{max}	5	10	15	20	
m_{max}	E_0(mK)	E_0(mK)	E_0(mK)	E_0(mK)	T_0(mK)
4	-120.376	-120.689	-120.726	-120.737	1769.481
8	-126.080	-126.274	-126.286	-126.288	1662.829
12	-126.143	-126.337	-126.348	-126.349	1660.232
16	-126.148	-126.342	-126.353	-126.354	1660.185
20	-126.149	-126.343	-126.354	-126.355	1660.186
24	-126.149	-126.343	-126.354	-126.355	1660.187
	excited state				
k_{max}	20	40	60	80	
m_{max}	E_1(mK)	E_1(mK)	E_1(mK)	E_1(mK)	T_1(mK)
8	-1.168	-1.579	-1.612	-1.622	139.535
12	-1.523	-2.097	-2.148	-2.160	127.139
16	-1.555	-2.150	-2.222	-2.237	123.526
20	-1.562	-2.157	-2.237	-2.257	122.247
24	-1.565	-2.160	-2.240	-2.262	121.943
28	-1.567	-2.161	-2.241	-2.264	121.927
32	-1.567	-2.162	-2.242	-2.265	121.935



$V_{He-He} = \lambda V(LM2M2)$
0
−0.8
−1.6
0.95
1.05
1.15
1.25
λ
E_1-E_d [mK]
−0.5
−1.5
0.995
1.005
LM2M2
TTY
HFDB
SAPT1
SAPT2

Recent experimental results in $A = 3, 4$ systems

Searching sensitivity to the three-nucleon force

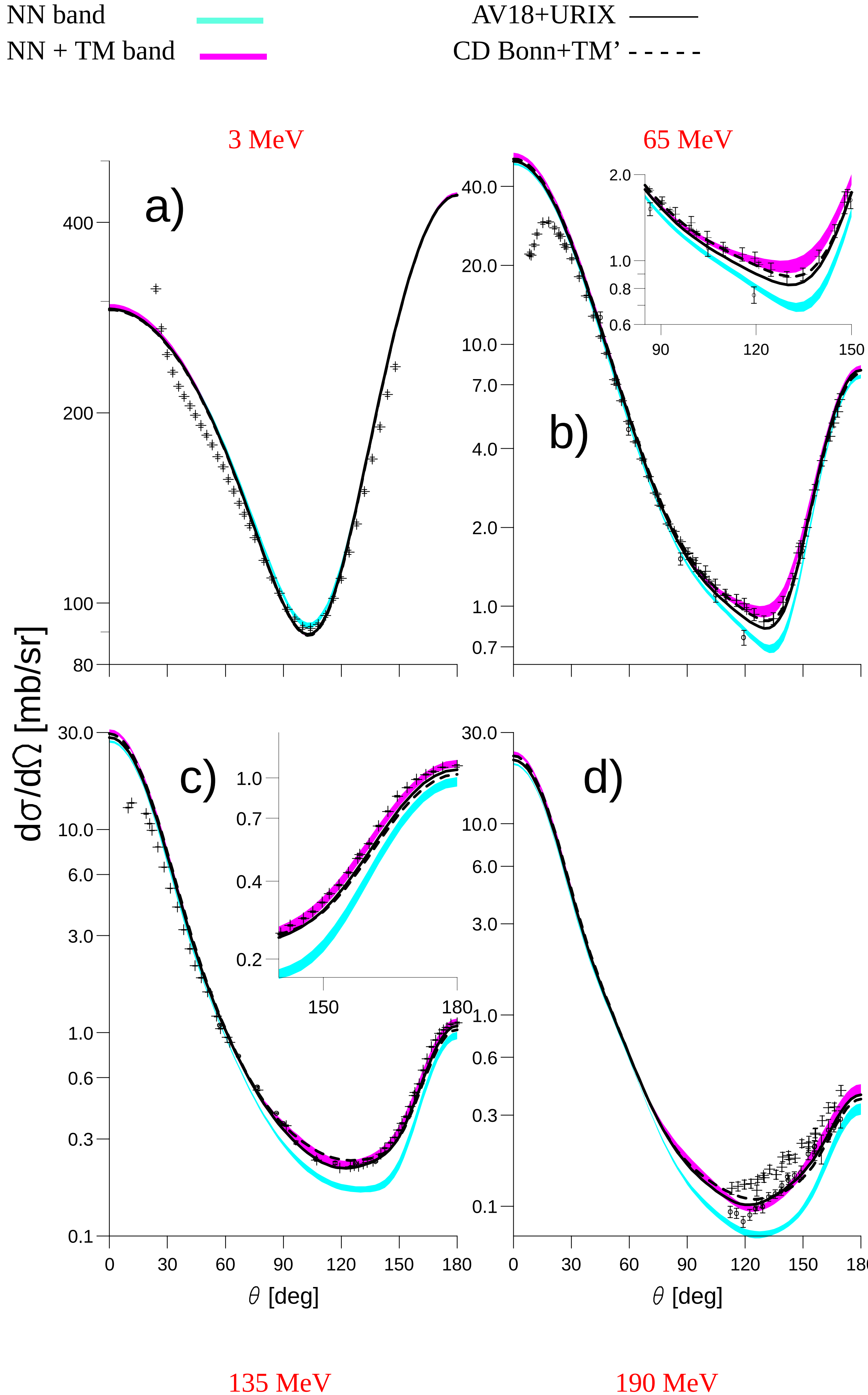
- Theoretical guidance for N-d reactions

In Witala *et al.* PRC 63, 024007 (2001) different elastic scattering observables have been calculated using realistic NN interactions: AV18, CD Bonn, NijmI, II and 93 and 3N interactions: TM and URIX

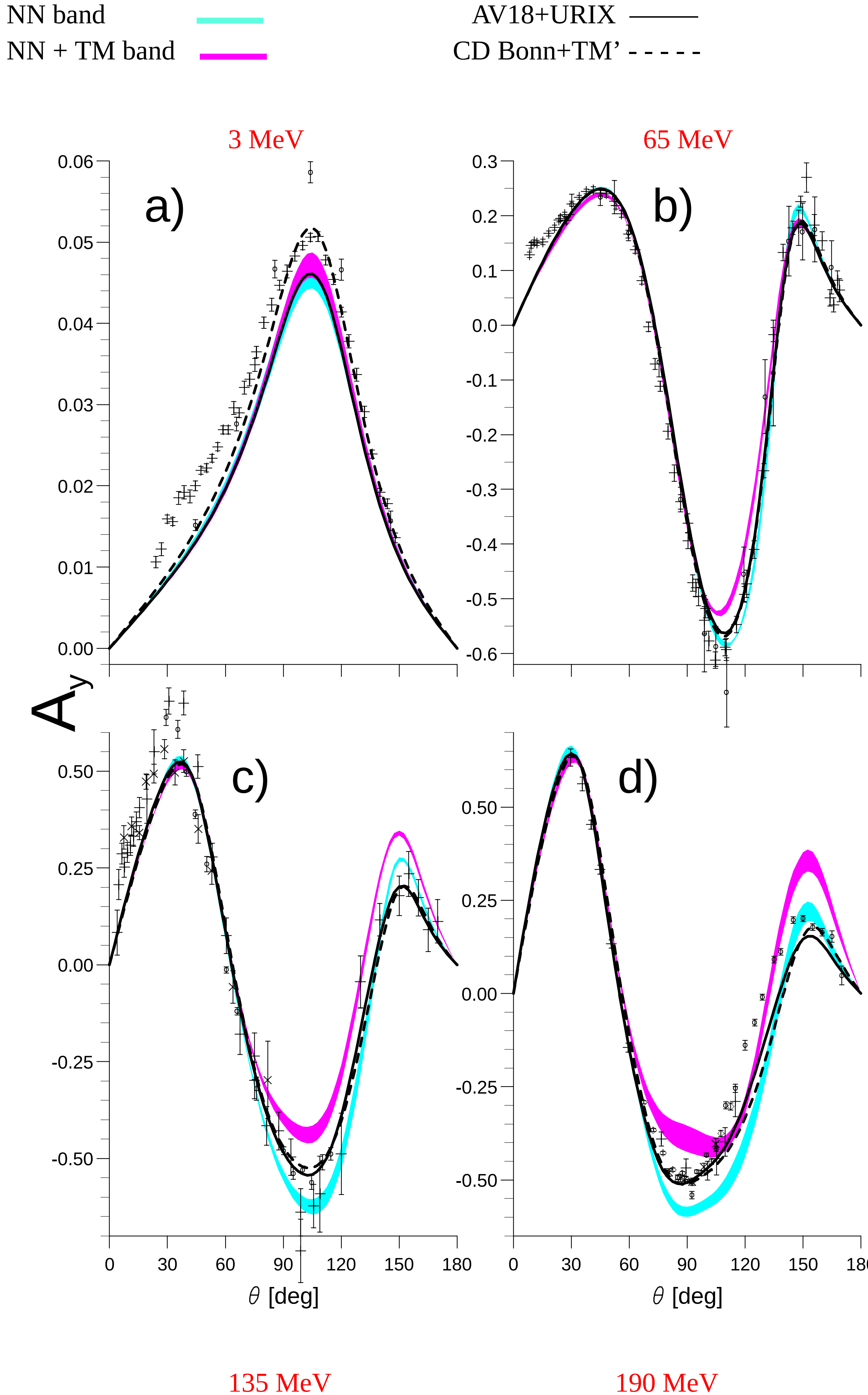
- The results have been presented in bands :

NN calculations
NN+TM calculations
AV18+URIX calculations

3N effect: Obs(NN) - Obs(exp.)

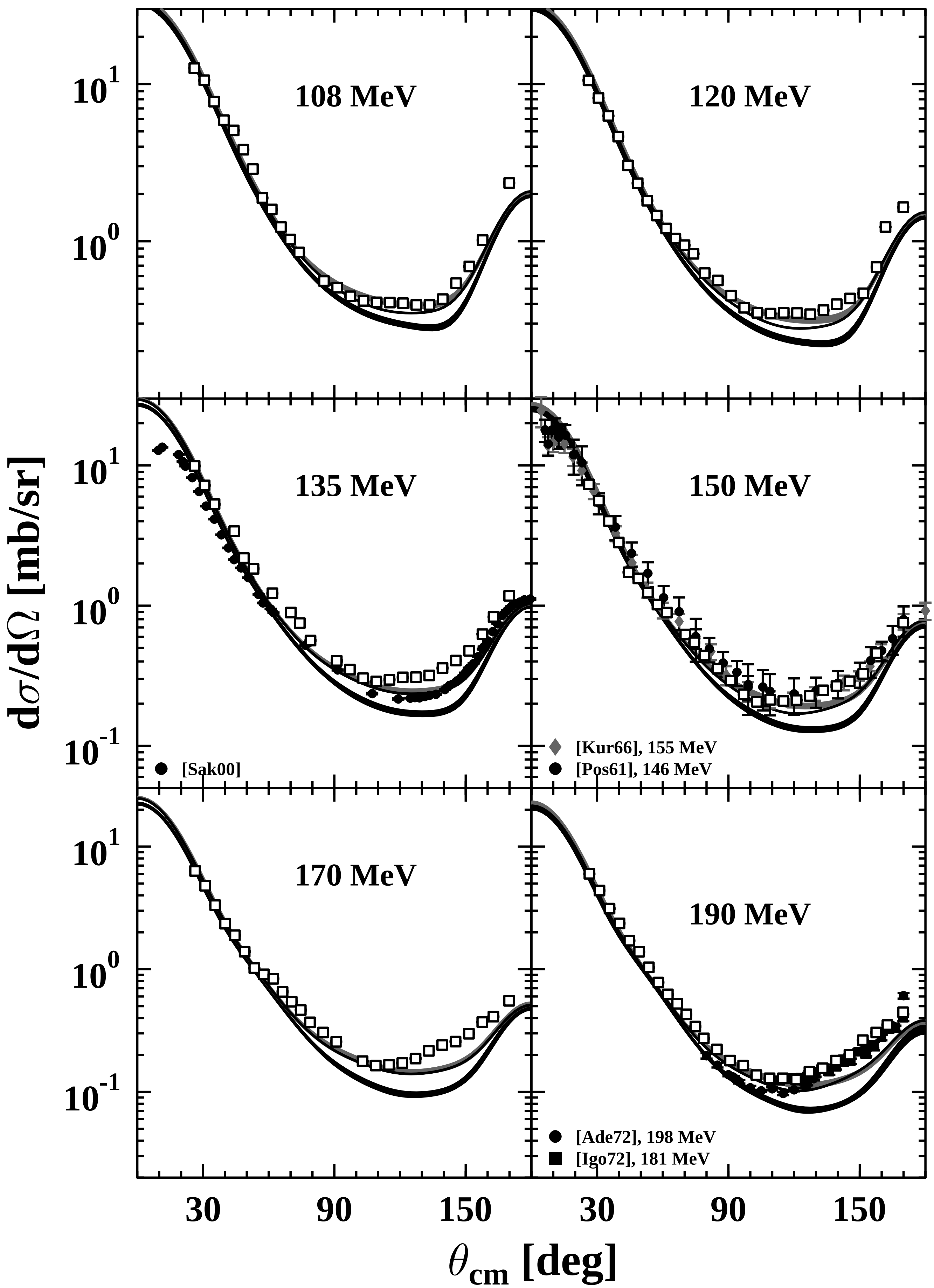


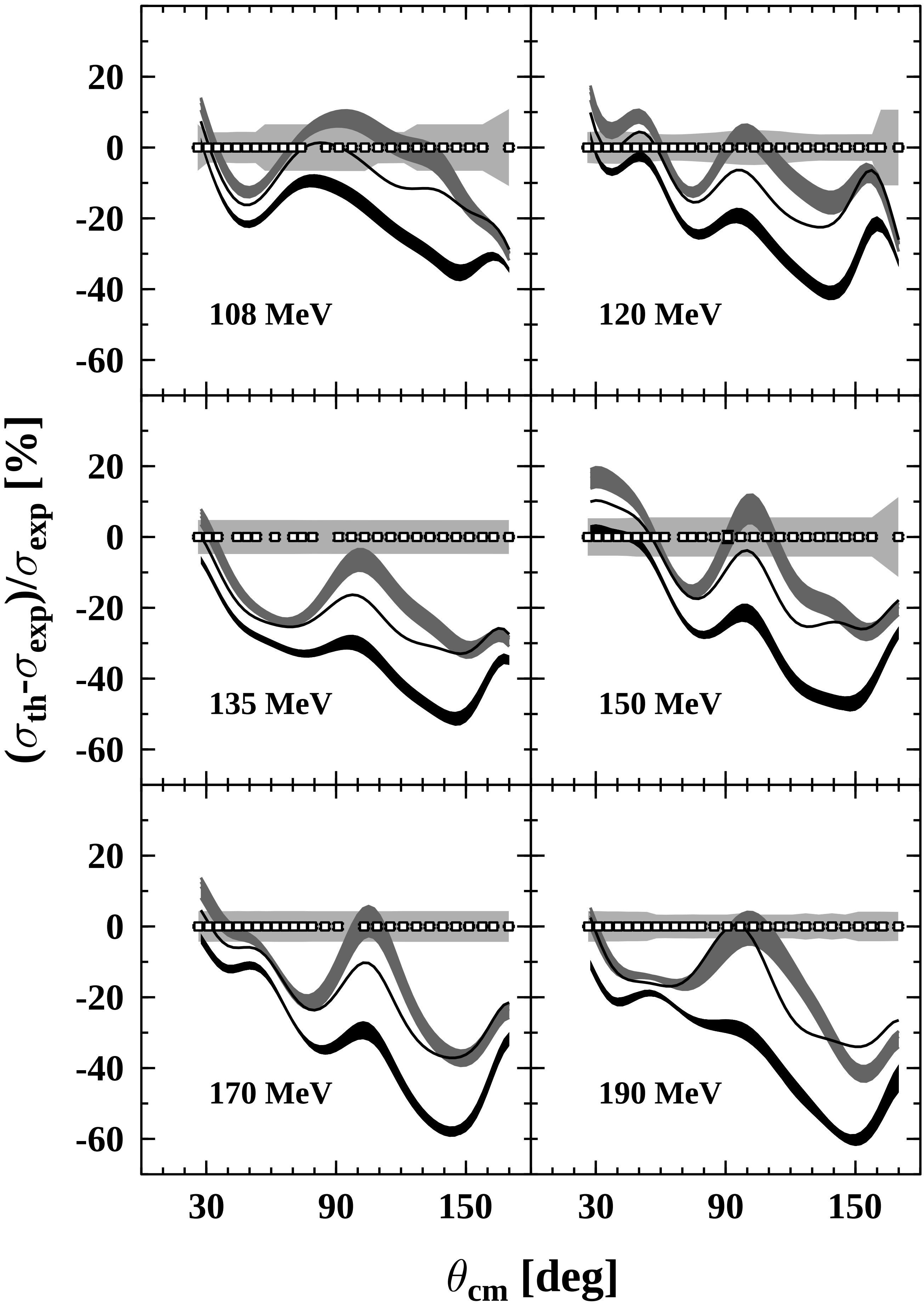
NN band
NN + TM band
AV18+URIX
CD Bonn+TM’
3 MeV
65 MeV
a)
b)
c)
d)
dσ/dΩ [mb/sr]
θ [deg]
θ [deg]
135 MeV
190 MeV



NN band
NN + TM band
AV18+URIX
CD Bonn+TM'
3 MeV
65 MeV
135 MeV
190 MeV
a)
b)
c)
d)
A_y
θ [deg]

Recent experimental results at KVI and RIKEN

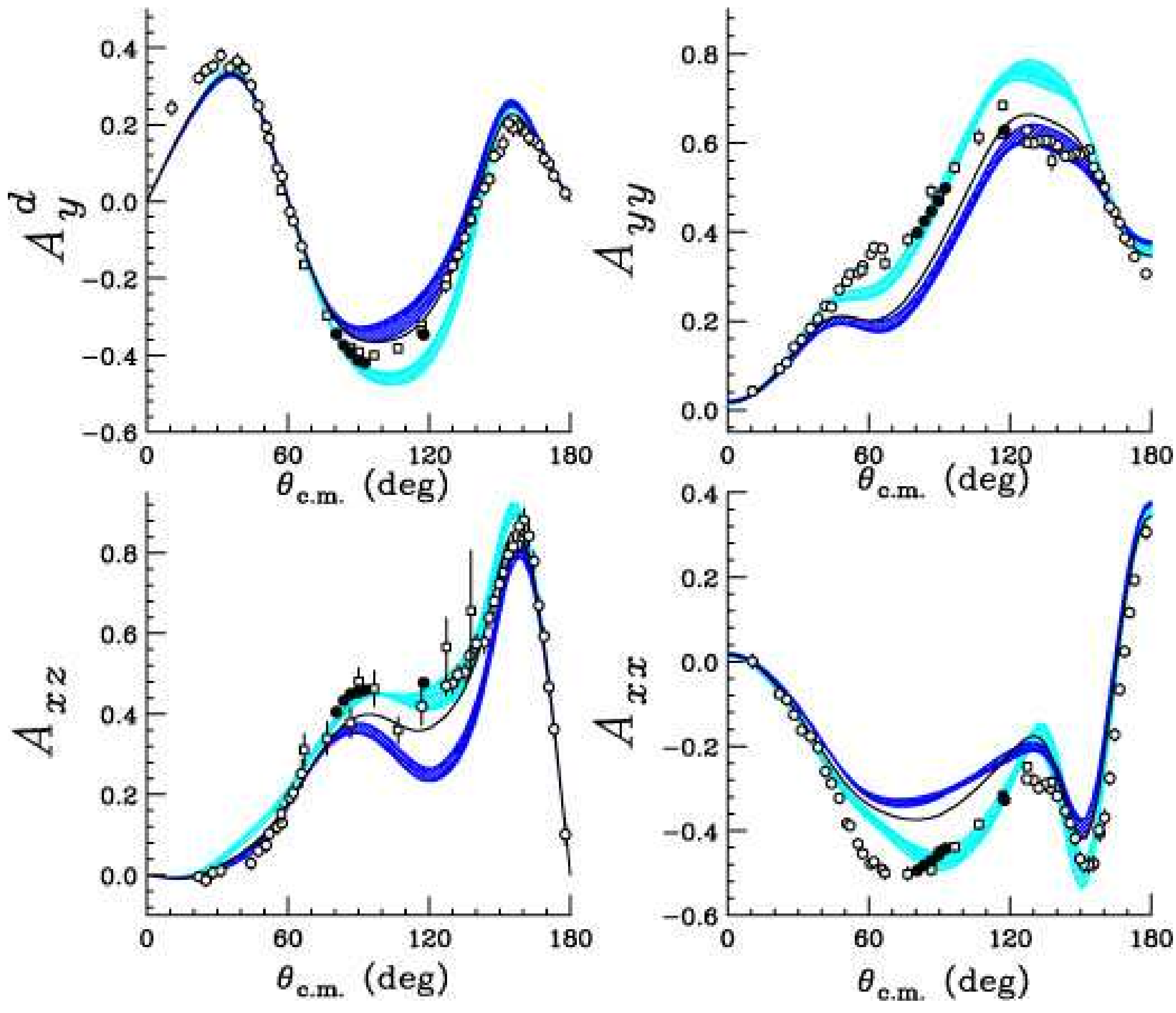




108 MeV
120 MeV
135 MeV
150 MeV
170 MeV
190 MeV
20
0
-20
-40
-60
30
90
150
$(\sigma_{th}-\sigma_{exp})/\sigma_{exp}$ [%]
θ_{cm} [deg]

Polarization observables measured at RIKEN at 135 MeV

NN band   NN + TM band   AV18+URIX



N-d scattering

Asymptotic state

$$\Omega^{+}_{LSJ}(\mathbf{x},\mathbf{y}) = \sum_{S'L'} \Big(F_{L'}(y)\delta_{LL'}\delta_{SS'} + {}^{J}T^{SS'}_{LL'} G_{L'}(y) \Big) \times \phi_d(\mathbf{x})[Y_{L'}(\hat{y}) \otimes [\chi_1 \otimes \chi_{\frac{1}{2}}]_{S'}]_{JJ_z}$$

Scattering state

$$\Psi^{+}_{LSJ} = \sum_{i=1,3} \left[\Psi_C(\mathbf{x}_i,\mathbf{y}_i) + \Omega^{+}_{LSJ}(\mathbf{x}_i,\mathbf{y}_i) \right]$$

$$\Psi_C(\mathbf{x}_i,\mathbf{y}_i) = \sum_{\alpha,n} u^{\alpha}_{n}(\rho)\mathcal{Y}^{n}_{\alpha}(\rho,\Omega)$$

Second order estime of the T-matrix

$$[{}^{J}T^{SS'}_{LL'}] = {}^{J}T^{SS'}_{LL'} + \frac{M}{2\sqrt{3}\hbar^2}\langle \Psi^{-}_{LSJ}|H-E|\Psi^{+}_{L'S'J}\rangle$$

Scattering observables can be described through the transition matrix

$$M_{\nu\nu'}^{SS'}(\theta) = f_c(\theta)\delta_{SS'}\delta\nu\nu'$$
$$+\frac{\sqrt{4\pi}}{k}\sum_{LL'J}^{L_{max}}\sqrt{2L+1}(L0S\nu|J\nu)(L'M'S'\nu'|J\nu)$$
$$\times\ \exp[i(\sigma_L+\sigma_{L'}-2\sigma_0)]\ {}^J T_{LL'}^{SS'}\ Y_{L'M'}(\theta,0)$$

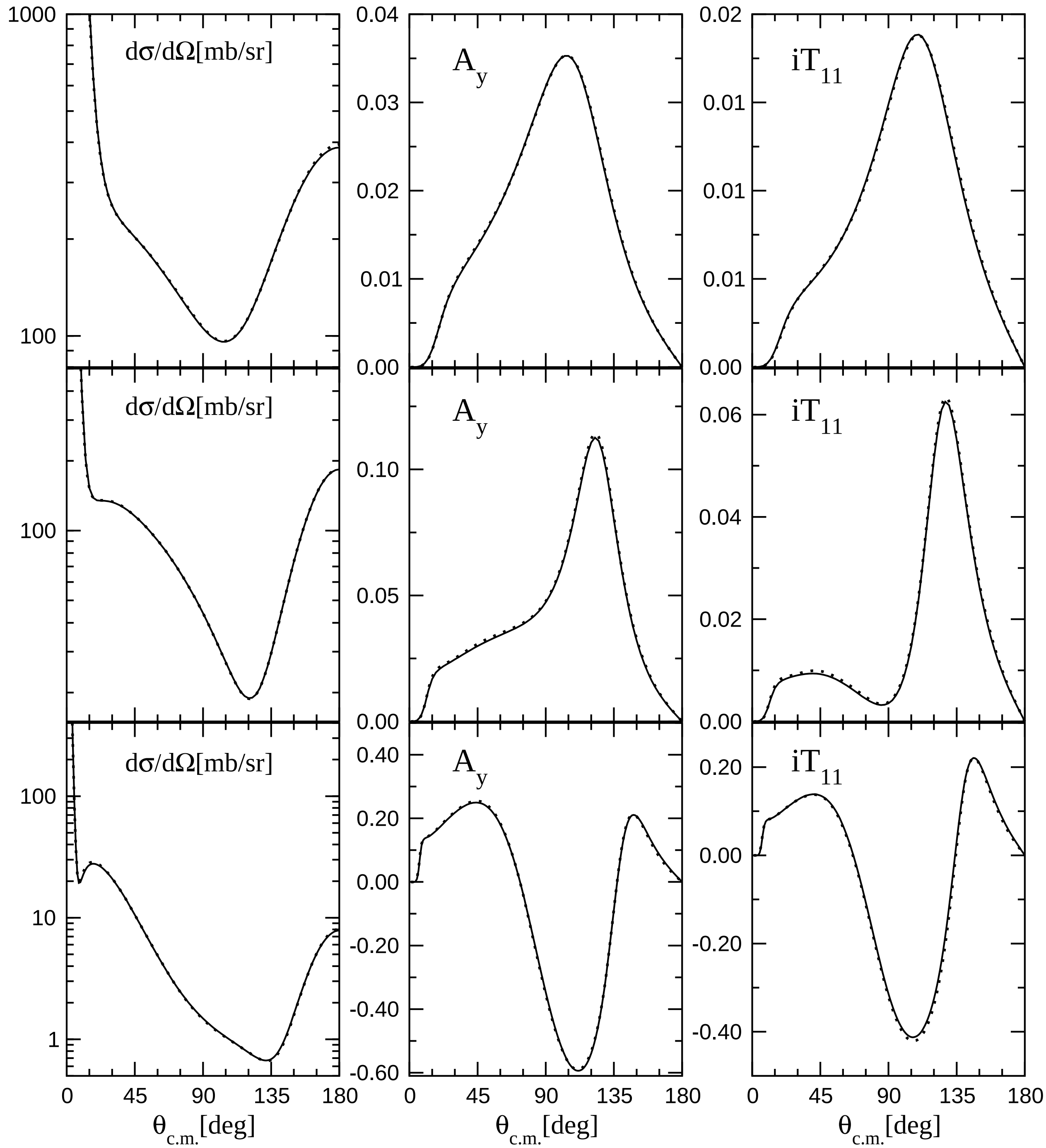
with the Coulomb amplitude

$$\begin{aligned} f_c(\theta) &= \sum_{L=0}^{\infty}(2L+1)(\mathrm{e}^{2i\sigma_L}-1)P_L(\cos\theta) \\ &= -2i\eta\frac{\mathrm{e}^{2i\sigma_0}}{1-\cos\theta}\mathrm{e}^{-i\eta\ln(\frac{1-\cos\theta}{2})} \end{aligned}$$

$$\sigma = \frac{tr(MM^\dagger)}{6}\ ,\quad A_y = \frac{tr(M\sigma_y M^\dagger)}{tr(MM^\dagger)}\ ,\quad T_{ij} = \frac{tr(MP_{ij}M^\dagger)}{tr(MM^\dagger)}$$

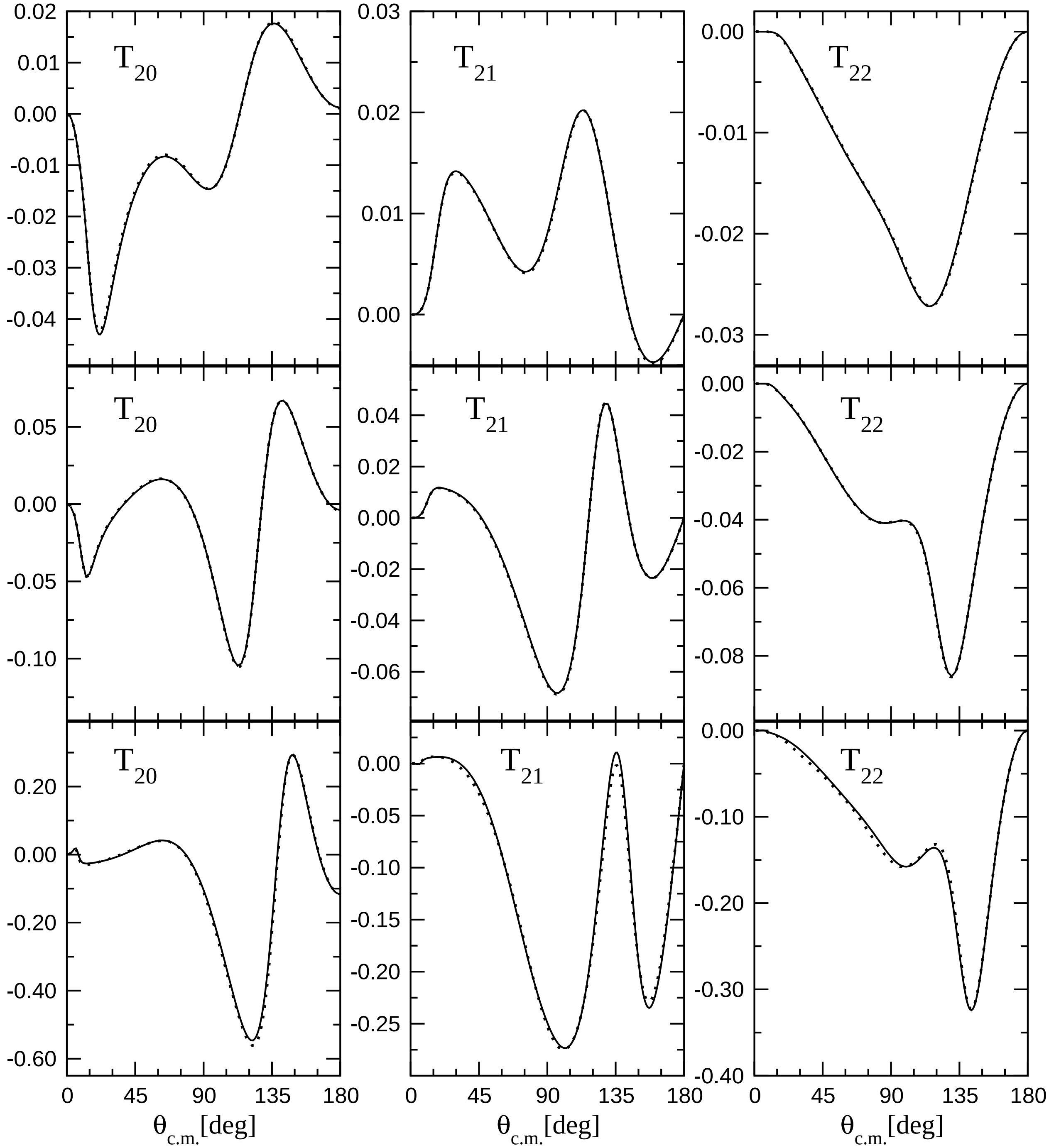
p-d scattering at 3 MeV, 10 MeV and 65 MeV

A. Deltuva et al. PRC in press



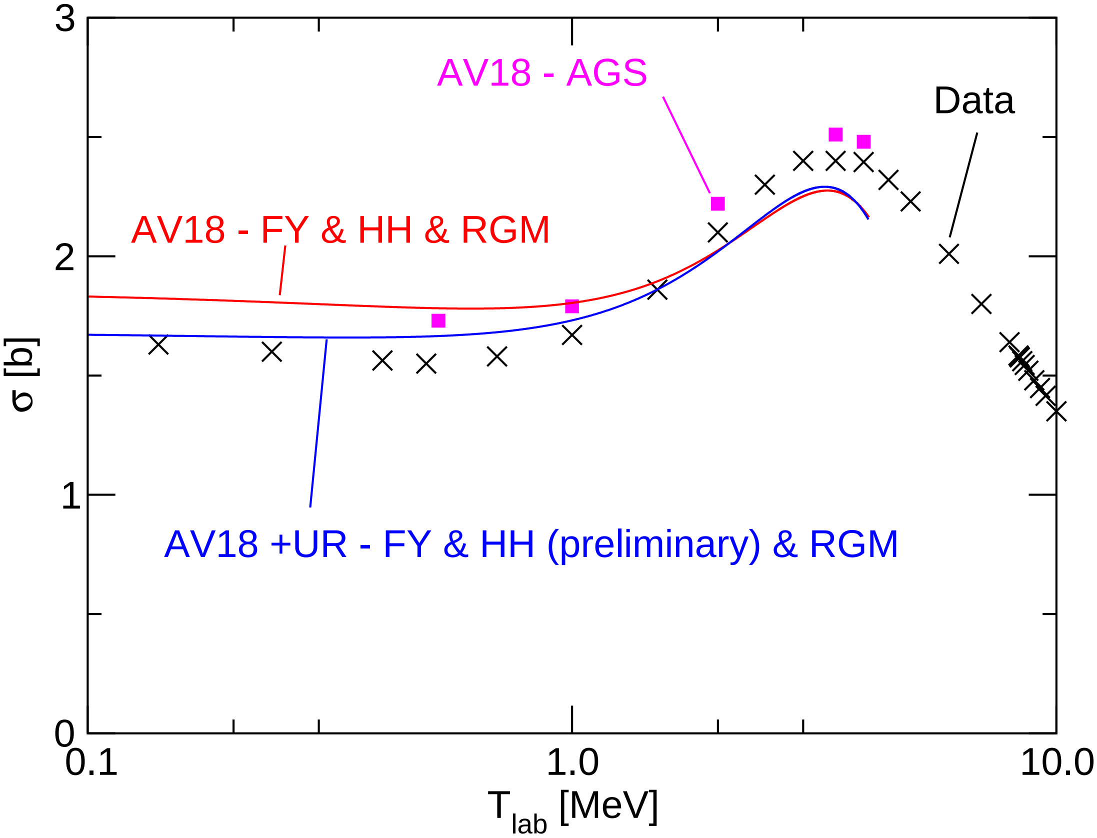
p-d scattering at 3 MeV, 10 MeV and 65 MeV

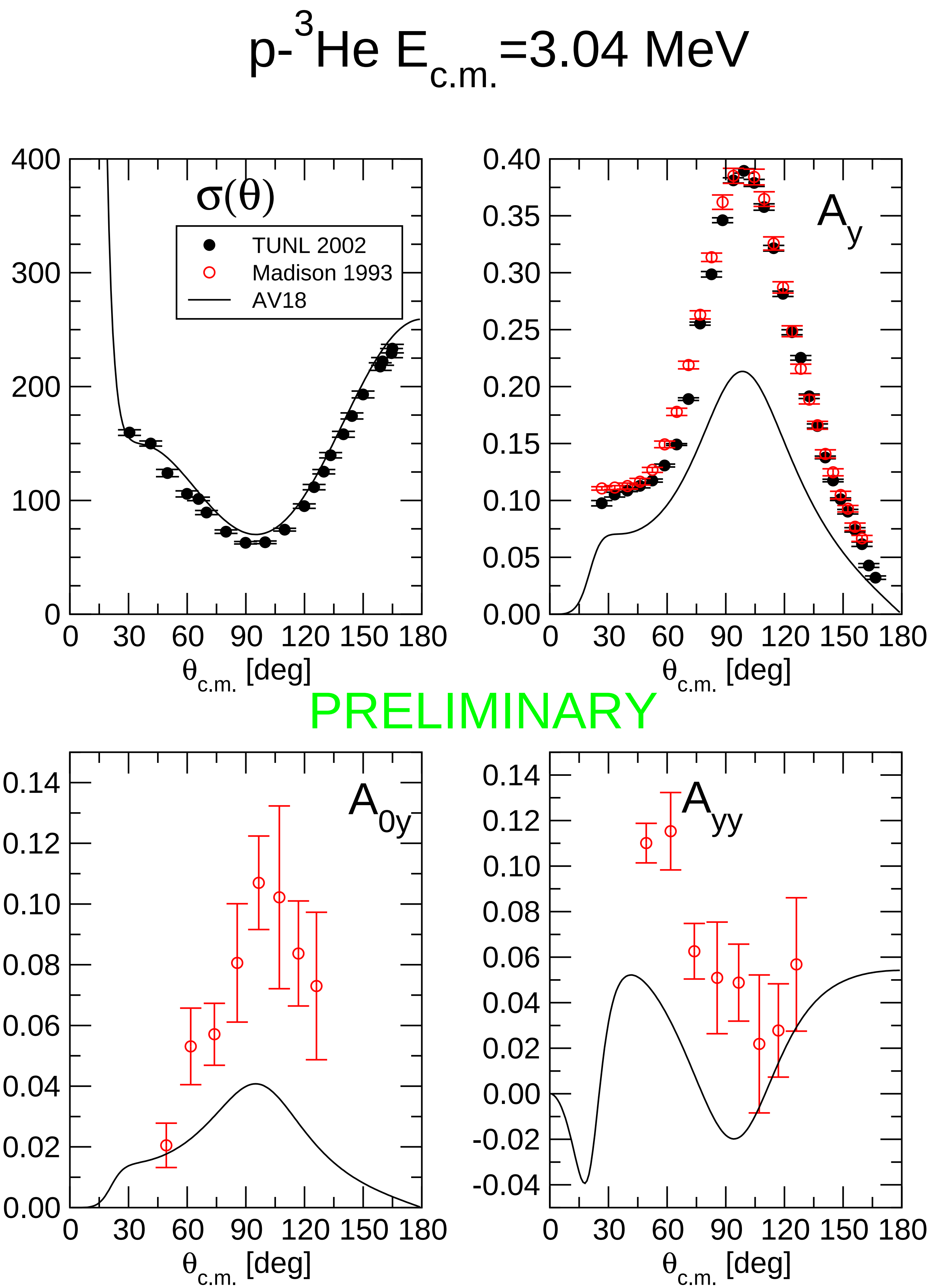
A. Deltuva et al. PRC in press



The four–nucleon continuum

- $n{-}^{3}$H scattering at low energies





p-^{3}He $E_{c.m.}$=3.04 MeV
σ(θ)
TUNL 2002
Madison 1993
AV18
A_y
PRELIMINARY
A_{0y}
A_{yy}
$\theta_{c.m.}$ [deg]