Hyperspherical Harmonic Formalism

N-1 Jacobi coordinates : $\vec{\eta}_1, \cdots, \vec{\eta}_{N-1}$

spherical notation for $\vec{\eta}_i$: η_i, θ_i, ϕ_i

Define hyperradius : $\rho = \sqrt{\sum_{i=1}^{N-1} \eta_i^2}$

Define other N-2 angles :

3-body	4-body
$\eta_1 = \rho \sin\theta$	$\eta_1 = \rho \sin\theta$
$\eta_2 = \rho \cos\theta$	$\eta_2 = \rho \cos\theta \sin\gamma$
	$\eta_3 = \rho \cos\theta \cos\gamma$

Kinetic energy operator:

$$\hat{T} = -\left(\frac{\partial^2}{\partial\rho^2} + \frac{3N-4}{\rho}\frac{\partial}{\partial\rho} - \frac{\hat{K}^2}{\rho^2}\right)/2m$$

$\hat{K}$: grand angular momentum operator

Hyperspherical Harmonics are eigen states of $\hat{K}^2$

⇒ HH expansion of wave function

$$\psi = \sum_{K\nu} \overbrace{R_{K\nu}(\rho)\, H_{K\nu}(\Omega) \cdot \prod_{i<j} f(r_{ij}) \cdot \text{"spin-isospin"}}^{CHH}$$

- $R_{K\nu}(\rho)$ → hyperradial part
- $H_{K\nu}(\Omega)$ → hyperangular part
- $\prod_{i<j} f(r_{ij})$ → two-body correlation function $f(r_{ij})$