Improvement of $V^{[2]eff}$:

Beyond the Adiabatic Approach

Apply transformation to the non adiabatic Hamiltonian $\tilde{\mathcal{H}}^{[2]}$

$$\tilde{\mathcal{H}}^{[2]}(s) = T_s + \mathcal{H}^{[2]}(s)$$

$$\Rightarrow \tilde{\mathcal{H}}^{[2]eff}(s) = \underbrace{\mathcal{U}^{\dagger}(s)\, T_s\, \mathcal{U}(s)}_{T_s + \Delta T_s^{eff}} + \mathcal{H}^{[2]eff}$$

$$\tilde{v}^{[2]eff}(s) = v^{[2]eff} + \Delta T_s^{eff}$$

One finds : $\Delta T_s^{eff} \approx \frac{\partial \mathcal{U}^{\dagger}}{\partial s} \frac{\partial \mathcal{U}}{\partial s}$