Metodi con trasformate integrali

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WHY INTEGRAL TRANSFORMS ?

$$\boxed{t = K\,r}$$

EXPERIMENTAL PHYSICS

* extraction of an "observable" from the experimental data: the instrument "integrates" r with the shape of its "window" and gives t

THEORETICAL PHYSICS

Laplace Kernel

$$D(\tau) = \int < \Theta^\dagger(\tau, \vec{x})\, \Theta(0, \vec{0}) > d^3x = \int_0^\infty e^{-\tau\omega} A(\omega)\, d\omega$$

Eucledean Correlation Function ($D(\tau)$)   *Spectral Function* ($A(\omega)$)

* **Condensed Matter Physics** (*Electron Gas, Quantum Liquids, Spin Systems*)
- Θ = Density Operator or Spin Lowering Operator
- $A(\omega)$ = Dynamical Structure Function or Dynamical Susceptibility
- $D(\tau)$ by Montecarlo Simulations

* **Nuclear Physics**
- Θ = Charge Operator
- $A(\omega)$ = Longitudinal Response $R_L(\omega)$
- $D(\tau)$ by Montecarlo Simulations

* **Lattice QCD, QCD Sum Rules**
- Θ = Quark or Gluon Operator
- $A(\omega)$ = Hadron Spectral Function
- $D(\tau)$ Lattice Calculation or Operator Product Expansion (*In QCD Sum Rules also other kernels*)

INCLUSIVE RESPONSE FUNCTIONS

$$R(\omega) = \sum_n | < n|\Theta|0 > |^2 \, \delta(\omega - E_n + E_0)$$

$$H|n >= E_n|n >$$

Θ "probe" operator

* need knowledge of $|n >$

* ω of interest can be in the continuum

* very difficult if not impossible!

INTEGRAL TRANSFORMS

$$R(\omega) = \sum_n |<n|\Theta|0>|^2\, \delta(\omega - E_n + E_0)$$

$$\boxed{T(\sigma) = \int d\omega\, K(\sigma,\omega)\, R(\omega)}$$

$$= \sum_n |<n|\Theta|0>|^2\, K(\sigma, E_n - E_0)$$

$$= \sum_n < 0|\Theta^\dagger \underbrace{|n><n|}_{=1 \text{ completeness}} \hat{K}(\sigma, H - E_0)\Theta|0>$$

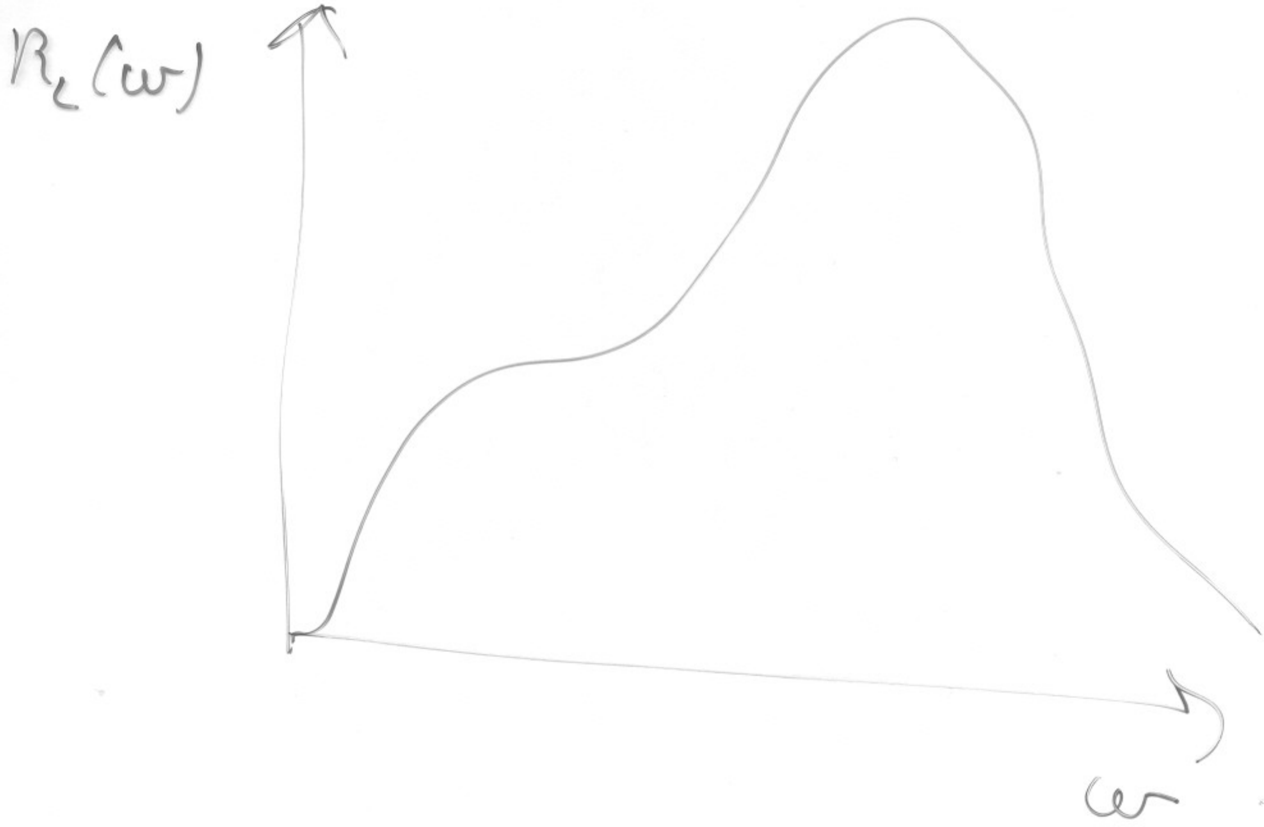
$$\boxed{T(\sigma) = <0|\Theta^\dagger \hat{K}(\sigma, H - E_0)\Theta|0>}$$

e.g. $\hat{K}(\sigma, H - E_0) \equiv e^{-\sigma(H-E_0)}$

QMC to evaluate $T(\sigma)$

$$K(\sigma,\omega) = \omega^{\sigma}$$

$$\langle 0 | O^{+} H^{\sigma} O | 0 \rangle$$



THE STIELTJES KERNEL

Efros '85

$$K_{\mathcal{S}}(\sigma,\omega) = \frac{1}{\omega - E_0 + \sigma} \quad \sigma > 0$$

$$(T_{\mathcal{L}} = \mathcal{L}R \qquad T_{\mathcal{S}} = \mathcal{L}\mathcal{L}R)$$

$$\boxed{T_{\mathcal{S}}(\sigma)} = < 0|\Theta^\dagger \frac{1}{H - E_0 + \sigma} \Theta|0 > = \boxed{< 0|\Theta^\dagger|\tilde{\Psi} >}$$

with

$$\boxed{(H - E_0 + \sigma)|\tilde{\Psi} > = \Theta|0 > \quad \sigma > 0}$$

Schrödinger like eq. with a *source*

Important properties:

* the solution *exists* and is *unique*

$$\tilde{\Psi}(\sigma) = \sum_n \frac{<n|\Theta|0>|n>}{(E_n - E_0 + \sigma)}$$

$$(H - E_0 + \sigma)(\tilde{\Psi}_1 - \tilde{\Psi}_2) = 0 \Rightarrow \tilde{\Psi}_1 - \tilde{\Psi}_2 = 0$$

* the solution is *localized*

$$\infty > T'_{\mathcal{S}}(\sigma) = -\int d\omega \frac{R(\omega)}{(\omega+\sigma)^2} = < \tilde{\Psi}|\tilde{\Psi} >$$

bound state techniques !

$T_{\mathcal{S}}(\sigma)$ *and the* $R(\omega)$ *moments*

$$T_{\mathcal{S}}(\sigma) = \sum_{n=0} (-)^n \, \frac{1}{\sigma^{(n+1)}} \, m_n$$

$$T_{\mathcal{S}}(\sigma) = \sum_{n=0} (-)^n \, \sigma^n \, m_{-n-1}$$

For very small σ $T_{\mathcal{S}}(\sigma)$ approaches the $\Theta-$polarizability of the system

$$\lim_{\sigma \to 0} T_{\mathcal{S}}(\sigma) = m_{-1} = 2\,\alpha_{\Theta}$$

$$m_k \equiv \int \omega^k \, R(\omega) \, d\omega = \langle 0 | \Theta^{+} H^{k} \Theta | 0 \rangle$$

Eh

TEST OF THE STIELTJES TRANSFORM

$$T_S(\sigma) = \int d\omega \, \frac{1}{(\sigma+\omega)} \, R(\omega)$$

$$T_{\mathcal{S}}(\sigma) \; = < 0|\Theta^\dagger|\tilde{\Psi} >$$

$$(H - E_0 + \sigma)|\tilde{\Psi} >= \Theta|0 > \quad \sigma > 0 \quad (*)$$

Program:

-1- solve (*) with good accuracy

-2- invert the transform (selection or regularization methods)

-3- compare with exact result

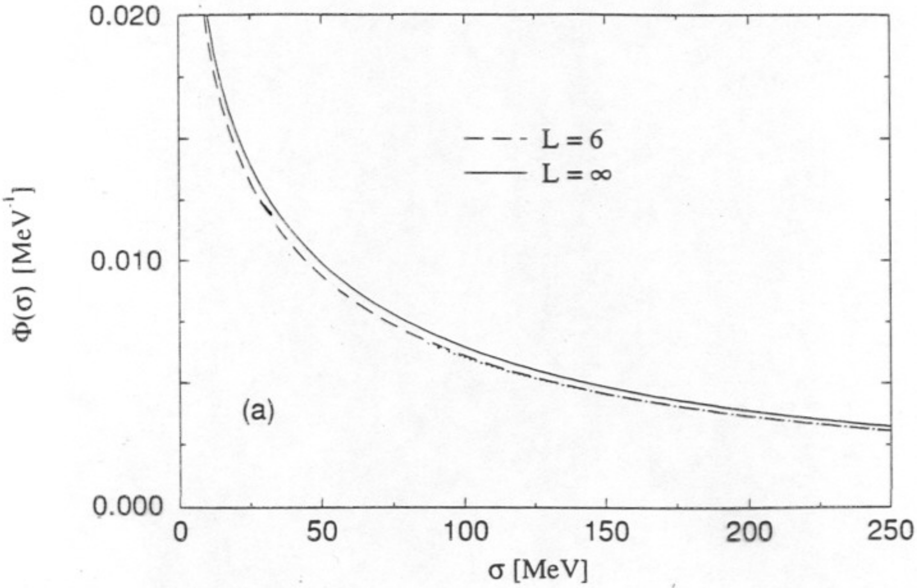
test on $R^L(q,\omega)$ in ^{2}H of (e,e') cross section

$$\Theta = \Theta(q) \equiv \sum_i^A \frac{1+\tau^3}{2} e^{i\mathbf{q}\cdot\mathbf{r}_i} \qquad \text{charge operator}$$

$$\frac{d\sigma}{d\omega d\Omega} = \sigma_M [\frac{q_\mu^4}{q^4} R^L(q,\omega) + (\frac{-2q_\mu^2}{2q^2} + tg^2\frac{\theta}{2}) R^T(q,\omega)]$$

* measured in many nuclei

^2H - Paris Potential



error ~ ‰

E6

INVERSION OF TRANSFORMS

(*) ill posed problems - instabilities!

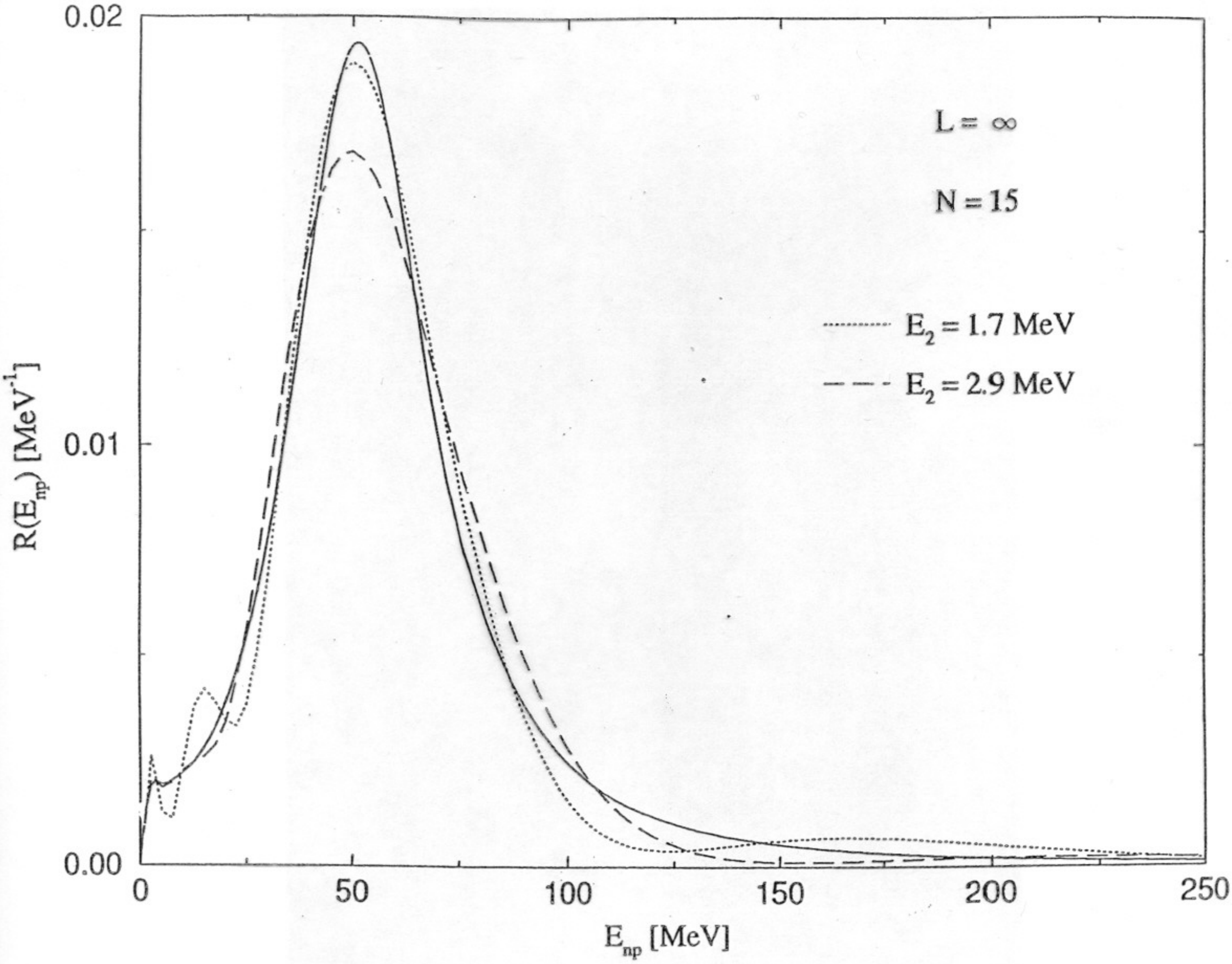
(*) if r_1 is a solution and $r_2 = r_1 + N sin(\nu\omega)$ then
$$t_2 = t_1 + N \int d\omega\, K(\omega, \sigma)\, sin(\nu\omega)$$

if ν is sufficiently large
$$||t_2 - t_1|| = |N|\; ||\int d\omega\, K(\omega, \sigma) sin(\nu\omega)||$$
sufficiently small, even with N big

(*) need supplementary information (qualitative - quantitative)

see A.N.Tikhonov and V.Y.Arsenin "The Solutions to Ill Posed Problems" in particular the *selection method* (quasisolutions) the *regularization method*

Electron Scattering Response Functions from Their Stieltjes Transforms



V. D. Efros[1], W. Leidemann[2], and G. Orlandini[2,]

Few-Body Systems 14, 151–170 (1993)

E7

LAPLACE

LEARNT FROM STIELTJES

- the kernels of Laplace and Stieltjes "spread" the information

- we need a kernel which "concentrates" information:

$$\text{if } K(\sigma, \omega) = \delta(\omega - \sigma) \Rightarrow \boxed{T(\sigma) = R(\sigma)}$$

- a "good" kernel must have a *bell shaped* form

Lorentz Kernel!

$$\boxed{L(\sigma_R, \sigma_I) = \int d\omega \, \frac{R(\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2}}$$

2 parameters:

$\longrightarrow$ σ_R samples region of interest

$\longrightarrow$ σ_I decides the "resolution"

HOW CALCULATE **LORENTZ** TRANS.

* extension of Stieltjes kernel to complex σ

If $\sigma = -\sigma_R + i\sigma_I \qquad \sigma_R, \sigma_I > 0$

$$\boxed{L(\sigma_R, \sigma_I)} = \int d\omega \frac{R(\omega)}{(\omega+\sigma)(\omega+\sigma)^*} \equiv \int d\omega \frac{R(\omega)}{(\omega-\sigma_R)^2+\sigma_I^2}$$

Lorentzian

$$=< 0|\Theta^\dagger \frac{1}{H-E_0-\sigma_R-i\sigma_I} \frac{1}{H-E_0-\sigma_R+i\sigma_I} \Theta|0 >$$

$$\boxed{=< \tilde{\Psi}|\tilde{\Psi} >}$$

with

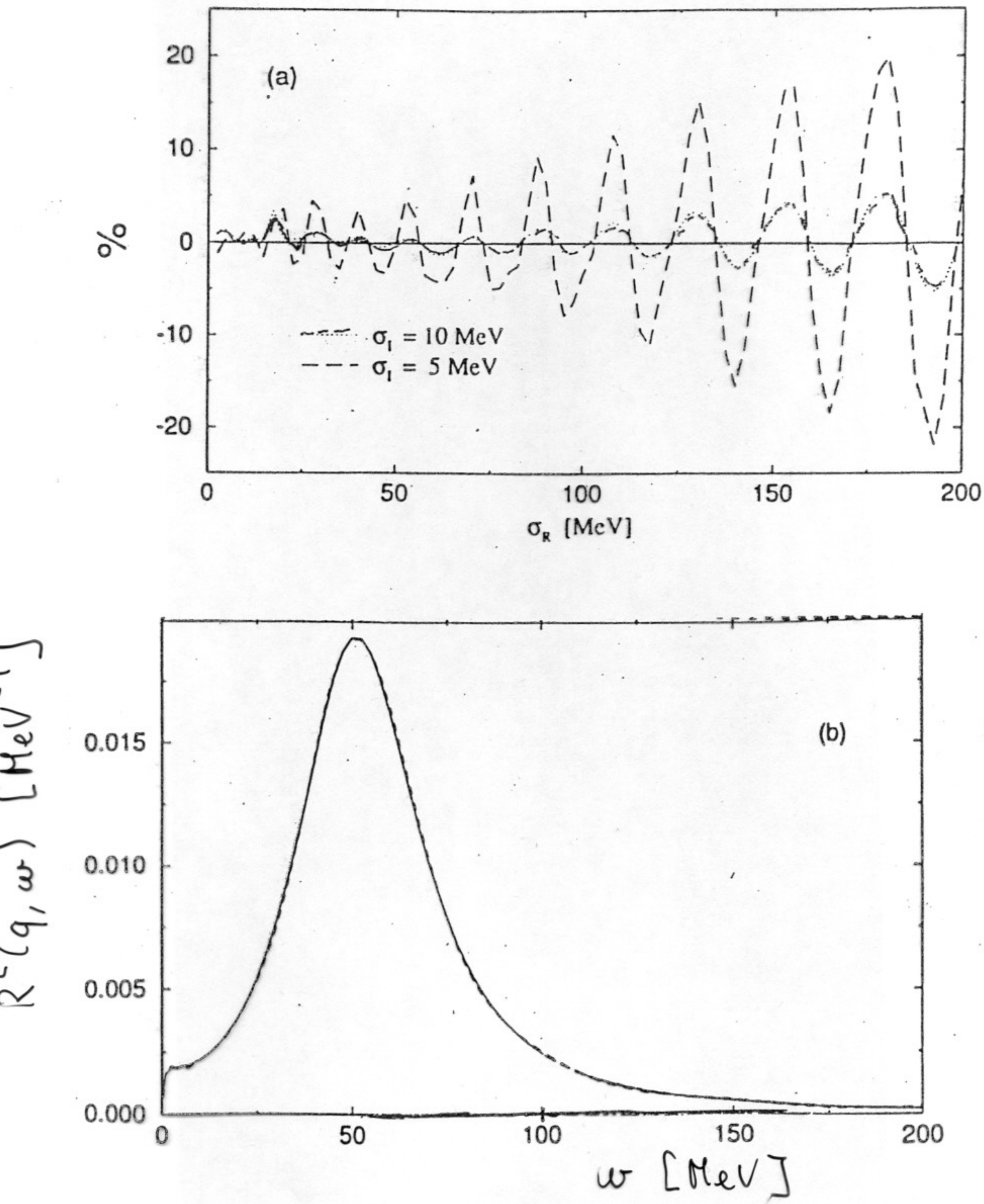
$$\boxed{(H - E_0 - \sigma_R + i\sigma_I)|\tilde{\Psi} >= \Theta|0 >}$$

* one can proove that solution *exists*, is *unique* and is localized

* $i\sigma_I$ ensures the "good" asymptotic behaviour

bound state techniques!
information is concentrated!

Test of Lorentz Transform on deuteron



V. Efros, W. Leidemann and G.O.
Phys. Lett. B 338 (1994) 130

VARIATIONAL PRINCIPLE

$$\underbrace{\left(H - E_0 - \sigma_R + i\sigma_I\right)}_{L} \tilde{\Psi} = \underbrace{\vartheta \Psi_0}_{Q}$$

$$\boxed{L\tilde{\Psi} = Q}$$

Consider the functional

$$I(\Psi) \equiv \langle \Psi | L^+ L | \Psi \rangle - \langle \Psi | L^+ | Q \rangle - \langle Q | L | \Psi \rangle$$

$$I(\Psi) = \langle \Psi | L^+ L | \Psi \rangle - \langle \Psi | L^+ L \tilde{\Psi} \rangle - \langle \tilde{\Psi} | L^+ L | \Psi \rangle$$

$$I(\tilde{\Psi}) = \langle \tilde{\Psi} | L^+ \cancel{L} | \tilde{\Psi} \rangle - \langle \tilde{\Psi} | L^+ \cancel{L} \tilde{\Psi} \rangle - \langle \tilde{\Psi} | L^+ L | \tilde{\Psi} \rangle$$

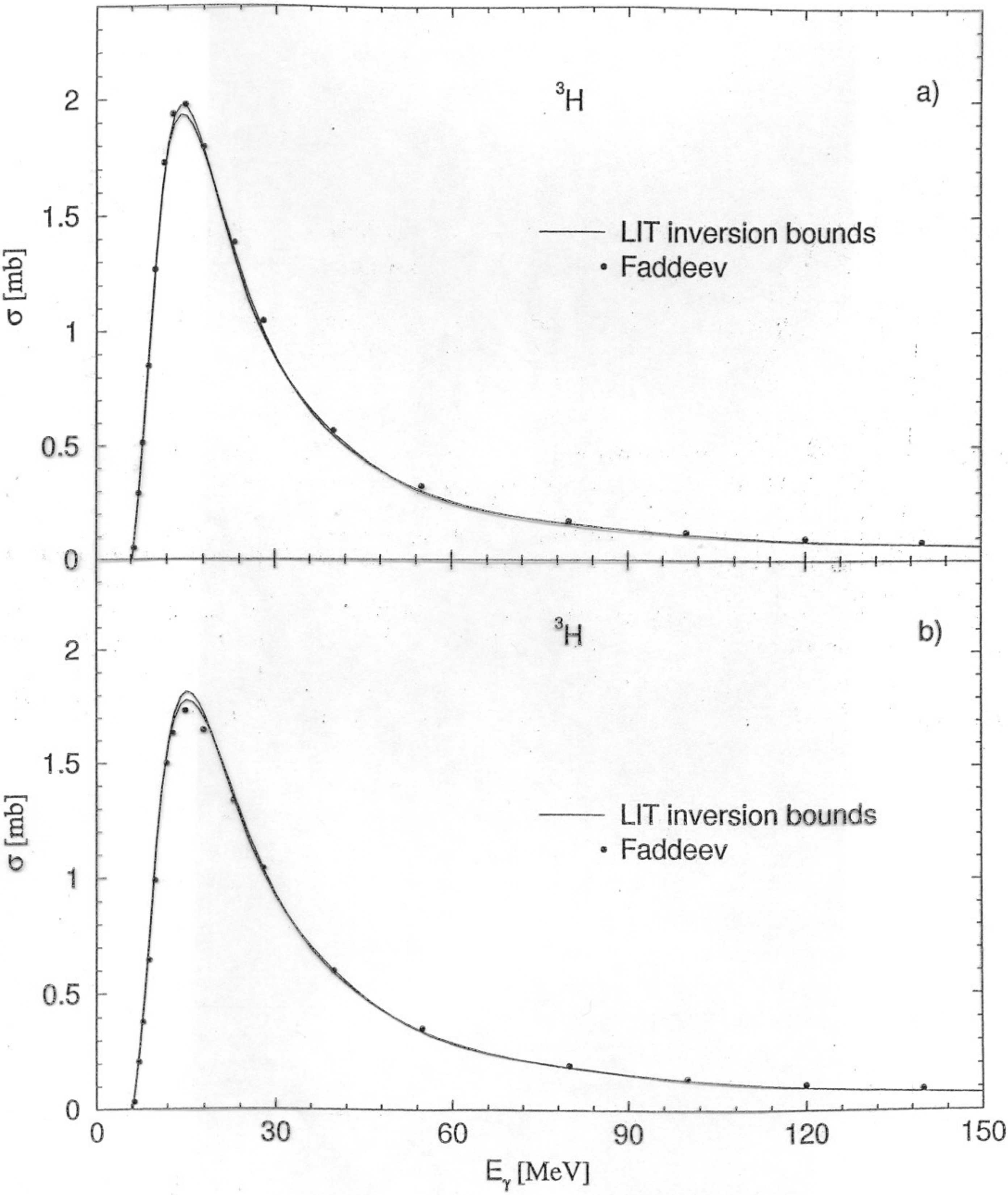
$$= - \langle \tilde{\Psi} | L^+ L | \tilde{\Psi} \rangle$$

$$\begin{aligned} \Delta I \equiv I(\Psi) - I(\tilde{\Psi}) &= \langle \Psi | L^+ L | \Psi \rangle - \langle \Psi | L^+ L | \tilde{\Psi} \rangle - \langle \tilde{\Psi} | L^+ L | \Psi \rangle \\ &\quad + \langle \tilde{\Psi} | L^+ L | \tilde{\Psi} \rangle \\ &= \langle \Psi - \tilde{\Psi} | L^+ L | \Psi - \tilde{\Psi} \rangle \\ &= \langle \delta\Psi | L^+ L | \delta\Psi \rangle \geq 0 \end{aligned}$$

$\Delta I = 0$ for $\Psi = \tilde{\Psi}$ i.e.

$I(\Psi)$ has a minimum for $\Psi = \tilde{\Psi}$

Fig. 1



Nucl. Phys. A707 (2002) 365

(e e')

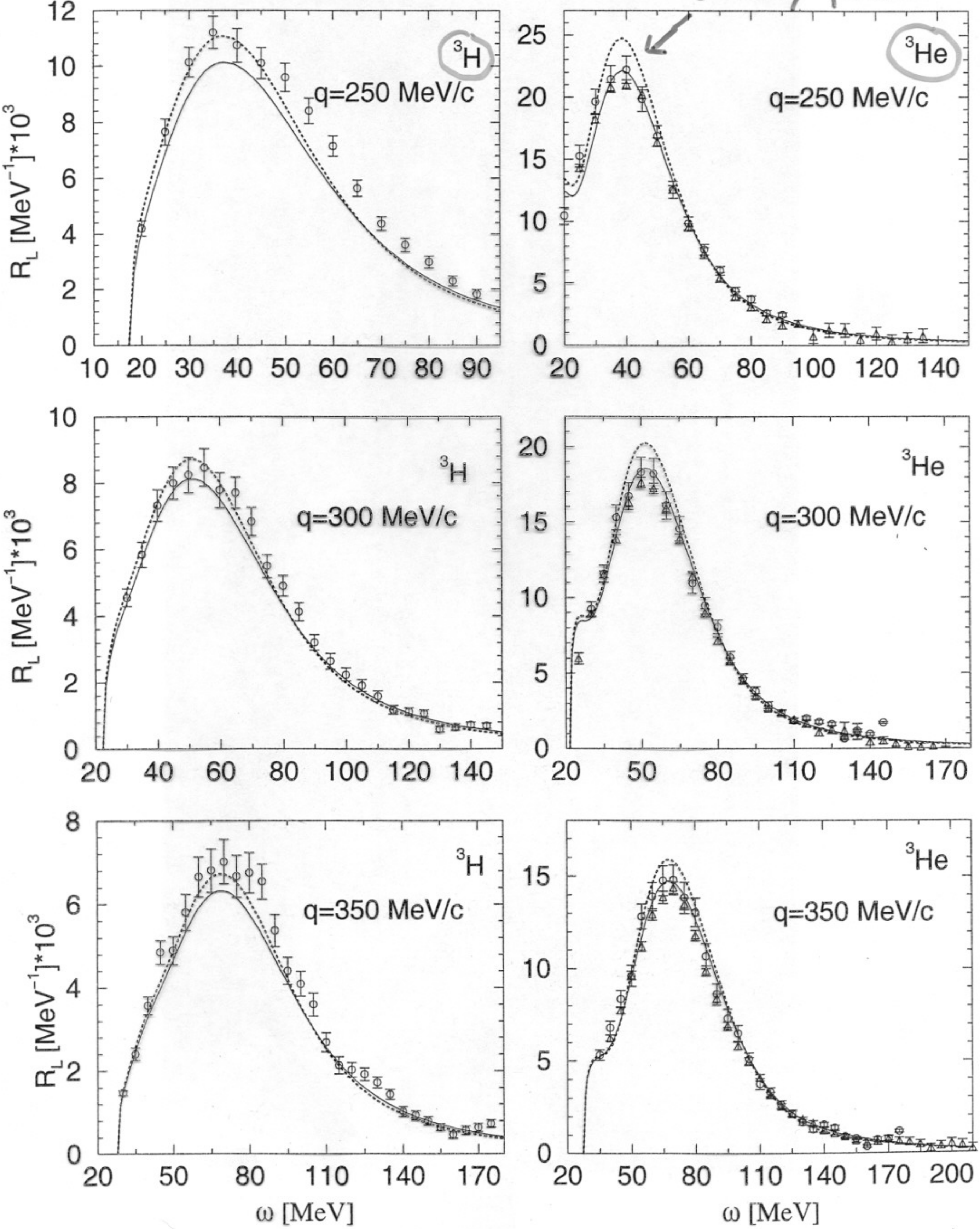
Dow et al. 1988

Marchand et al 1985

Potential: AV18 + $\overline{\text{UIX}}$

Fig. 6

3-body force



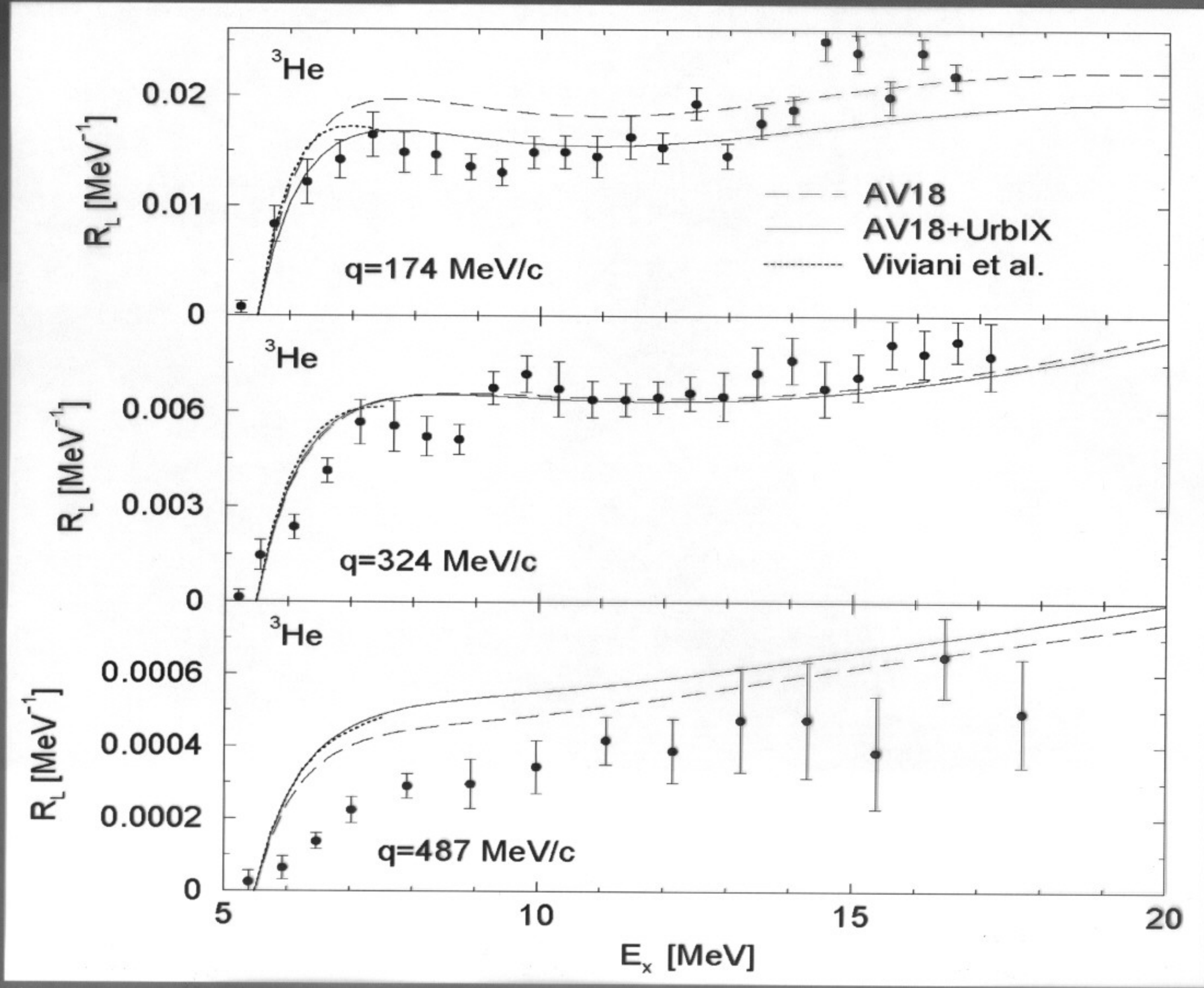
3-Body total photodisintegration
Role of 3-Nucleon force

Low energy

AV18

AV18 + UIX

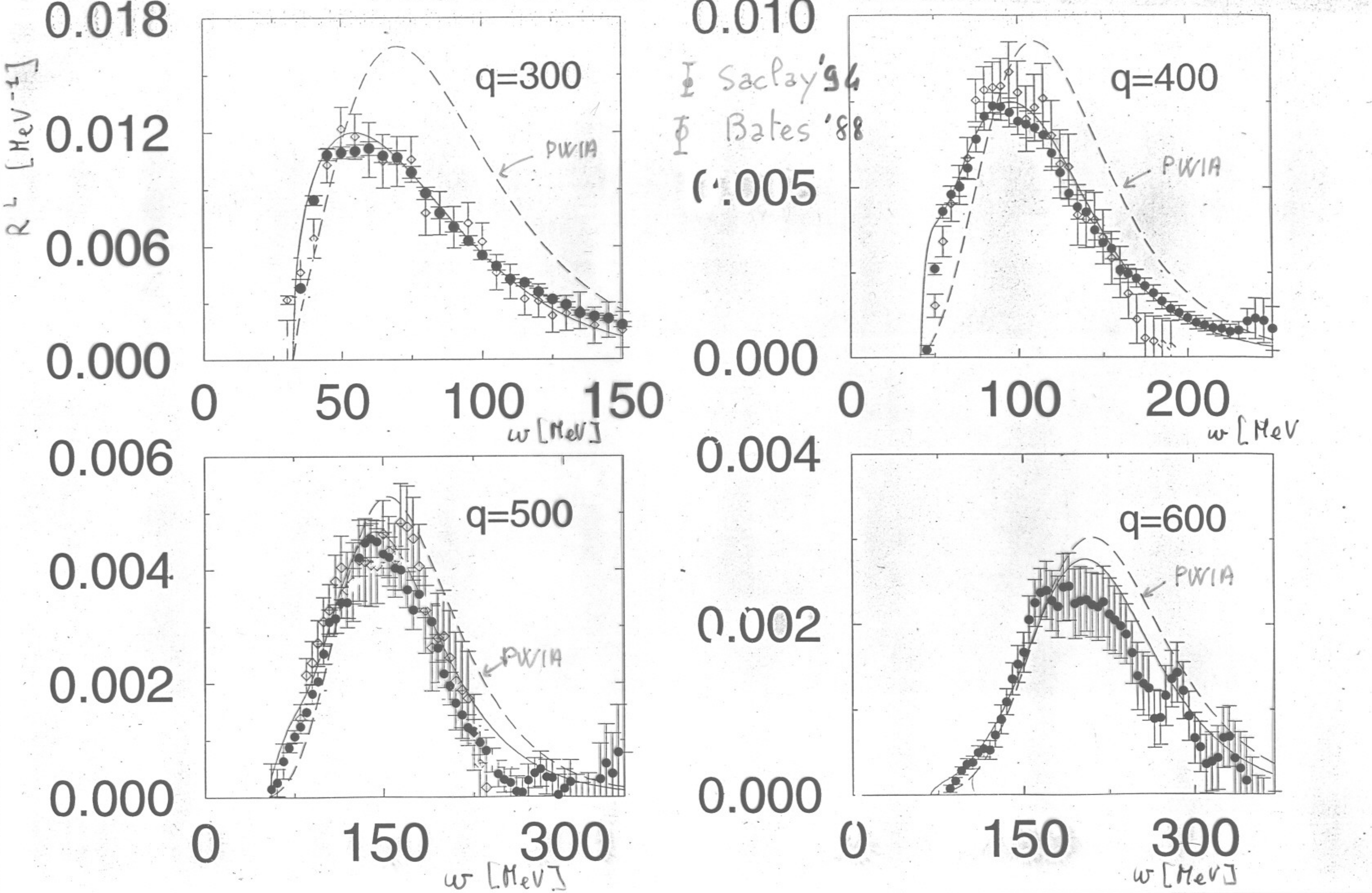
CHH



$^4He(e,e')$ Longitudinal Response

P.R.L. 78 (1997) 432

Malfliet Tjon Potential



Lorentz I.T.M for EXCLUSIVE cross section

$$T_{fi}(E_f) = \left\langle \Psi_f \left| \hat{O} \right| \Psi_0 \right\rangle = T_{fi}^{Born}(E_f) + T_{fi}^{FSI}(E_f)$$

$$\left\langle \Phi^{Born} \left| \widehat{O} \right| \Psi_0 \right\rangle$$

$$\left\langle \Phi^{Born} \left| \widehat{V} \frac{1}{E_f + i\varepsilon - H} \widehat{O} \right| \Psi_0 \right\rangle$$

(use closure)

$$\int dE \left\langle \Phi^{Born} \left| \widehat{V} \right| \Psi(E) \right\rangle \left\langle \Psi(E) \left| \frac{1}{E_f + i\varepsilon - \widehat{H}} \widehat{O} \right| \Psi_0 \right\rangle$$

$$= \int dE \frac{F_{fi}(E)}{E_f + i\varepsilon - E},$$

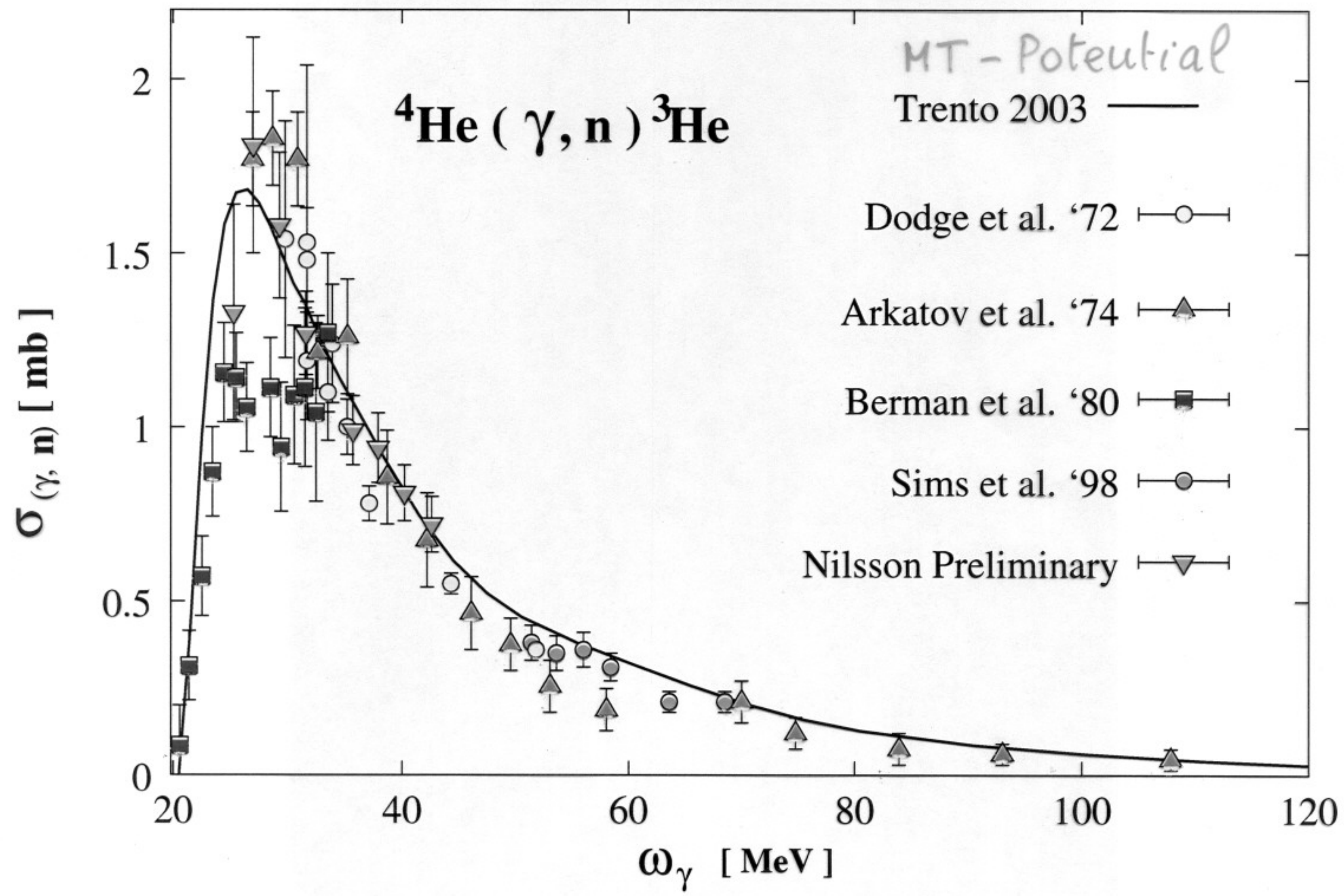
$$F_{fi}(E) = \left\langle \Phi^{Born} \left| \widehat{V} \right| \Psi(E) \right\rangle \left\langle \Psi(E) \left| \widehat{O} \right| \Psi_0 \right\rangle,$$

$$\boxed{L(\sigma) =} \int_{E_0}^{\infty} dE \frac{F_{fi}(E)}{(E + \sigma^*)(E + \sigma)}, \qquad \sigma_R > 0 : \quad \sigma = -\sigma_R + i\sigma_I$$

(use closure) $= \boxed{\left\langle \widetilde{\Psi}_2(\sigma) \middle| \widetilde{\Psi}_1(\sigma) \right\rangle.}$

1) $\boxed{\left(\widehat{H} - \sigma_R + i\sigma_I\right) \left| \widetilde{\Psi}_1(\sigma) \right\rangle = \widehat{O} \left| \Psi_0 \right\rangle,}$ ← idem inclusive

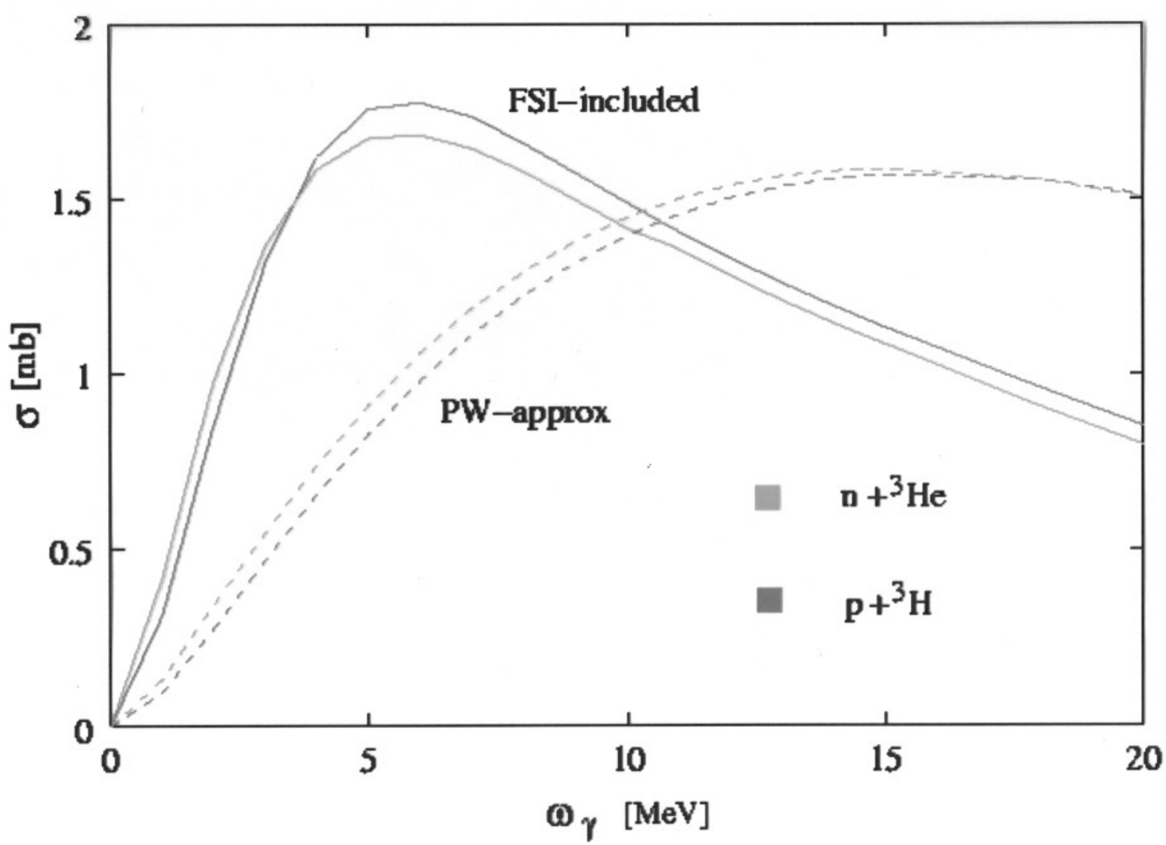
2) $\boxed{\left(\widehat{H} - \sigma_R + i\sigma_I\right) \left| \widetilde{\Psi}_2(\sigma) \right\rangle = \widehat{V} \left| \Phi^{Born} \right\rangle,}$



→ Sofia Quaglioni Monday h. ~~15.19~~ 15-01

Session 5

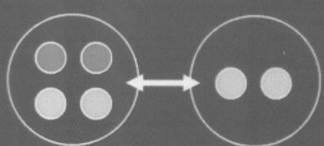
$^4He(\gamma,p)^3H$

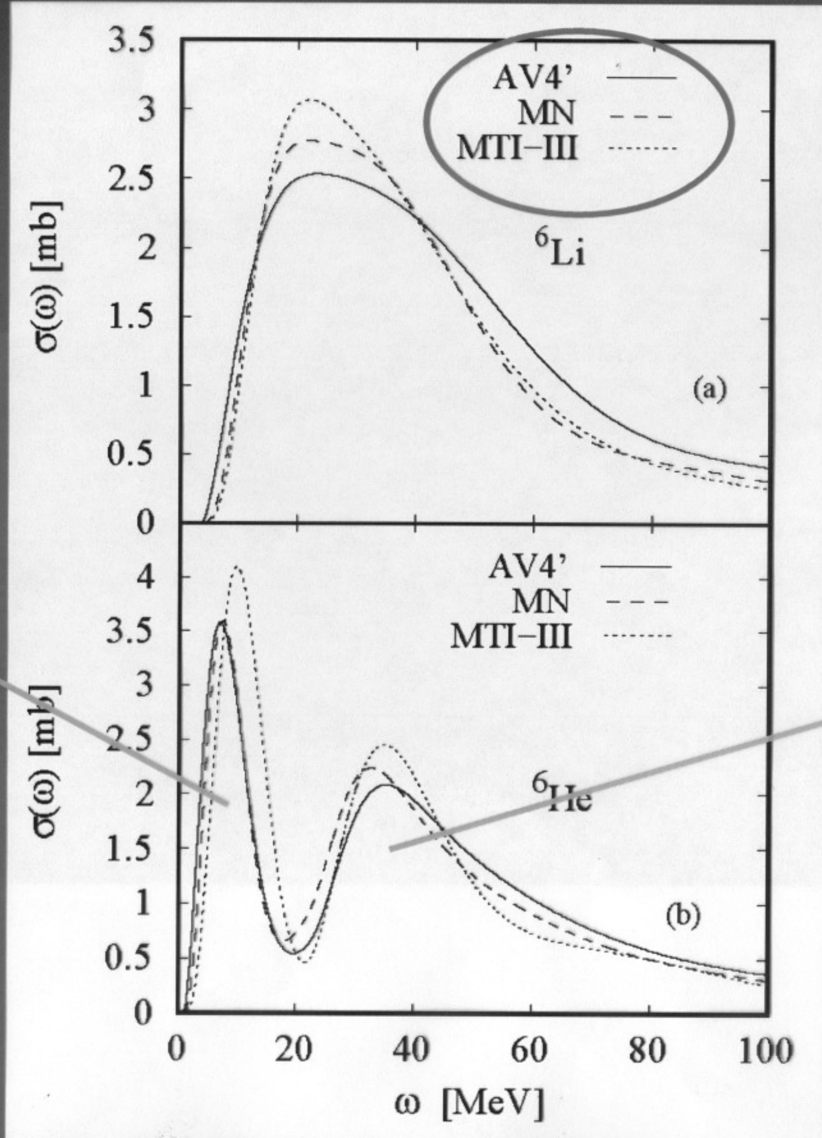


6-Body total photodisintegration
Appearance of collective motion

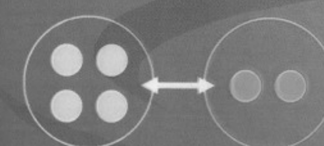
EIHH

Soft mode
"pigmy resonance"





GT mode



7-Body total photodisintegration

EIHH