

C.M. PROBLEM

JACOBI Coordinates

N particles

$$\mathbf{R} \equiv \underset{\uparrow}{\xi_1} = \frac{1}{M} \sum_i m_i \mathbf{r}_i \quad \text{where} \quad M = \sum_i m_i.$$

$$\xi_N = \mathbf{r}_{N-1} - \mathbf{r}_N,$$

$$\xi_{N-1} = \mathbf{r}_{N-2} - \mathbf{R}_{N,N-1},$$

$$\xi_{N-2} = \mathbf{r}_{N-3} - \mathbf{R}_{N,N-1,N-2},$$

...

...

$$\underset{\uparrow}{\xi_2} = \mathbf{r}_1 - \mathbf{R}_{N,N-1,\dots,2}.$$

where $\mathbf{R}_{N,N-1,\dots,N-j}$ is the c.o.m. of $j+1$ particles i.e.

$$\mathbf{R}_{N,N-1,\dots,N-j} = \left[\sum_{i=N-j}^N m_i \right]^{-1} \sum_{i=N-j}^N m_i \mathbf{r}_i$$