The interaction and the e.m. current in nuclei

How are they related?

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Summary

- Introduction
- Gauge Invariance & current conservation
- Example: one pion exchange
- Realistic potentials
- Example: $p - d$ radiative capture
- Seminal works:
  Sachs, 1948
  Chemtob & Rho, 1971
  Buchmann, Leidemann, & Arenhövel, 1985
  Riska, 1985

Introduction

- Notation: quantum field theory: Bjorken & Drell

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- Charge density operator in coordinate space

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- We need

$$\langle N' | \int d^3x \; j^\mu(\boldsymbol{x}) A_\mu(\boldsymbol{x}) | N\gamma \rangle \sim$$

$$\begin{aligned} \sim \quad & \langle N' | \int d^3x j^\mu(\boldsymbol{x}) \epsilon_\mu \frac{e^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2\omega}} | N \rangle \\ = \quad & \langle N' | j^\mu(\boldsymbol{q}) | N \rangle \frac{\epsilon_\mu}{\sqrt{2\omega}} \end{aligned}$$

Introduction

- Fourier transform of the charge density operator in coordinate space

$$
\begin{aligned}
\langle \boldsymbol{r}'_i | \rho(\boldsymbol{q}) | \boldsymbol{r}_i \rangle &= \int d^3x \; \langle \boldsymbol{r}'_i | \rho(\boldsymbol{x}) | \boldsymbol{r}_i \rangle e^{\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{q}} \\
&= q_i e^{\mathrm{i}\boldsymbol{r}_i\cdot\boldsymbol{q}} \delta(\boldsymbol{r}_i - \boldsymbol{r}'_i)
\end{aligned}
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- Exercise: charge density operator in momentum space

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- time-dependence (Heisemberg picture)

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- Often we have to consider

$$\rho(\boldsymbol{x},\omega) = \int dt\ \rho(\boldsymbol{x},t)e^{-\mathrm{i}\omega t}$$

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- Then $\partial^\mu J_\mu(x_\mu) = 0$

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- Therefore, the Hamiltonian and the current must verify the

  current conservation relation (CCR)

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- For a given Lagrangian density $\mathcal{L}[\Psi^\ell, \partial_\mu \Psi^\ell, A_\mu]$
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$$J^\mu(x_\mu) = \mathrm{i} \sum_\ell \frac{\partial \mathcal{L}}{\partial\, \partial_\mu \Psi^\ell} q_\ell \Psi^\ell$$

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- $\mathcal{L}_0 \to \mathcal{L}_0 - J^\mu(x_\mu) A_\mu(x_\mu) + \mathcal{O}(A^2)$

Example

- Spin-0 isospin-1 field

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi_a(x) \partial^\mu \Phi_a(x) - \frac{1}{2} m^2 \Phi_a(x) \Phi_a(x)$$

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$$J^\mu(x) = -\mathrm{i}e[\Phi_-\partial^\mu\Phi_+ - \Phi_+\partial^\mu\Phi_-] = e\left(\vec{\Phi} \times \partial^\mu \vec{\Phi}\right)_z$$

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  fitted to reproduce the NN (and 3N) data set

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v_{ij} and V_{ijk} depend on *isospin* and *momentum*

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- Difficulties in treating e.m. transitions:
  there is no a recipe to derive J^μ
- In first approx. $\rho_i(\boldsymbol{q}) = q_i \frac{1+\tau_z(i)}{2} e^{\mathrm{i}\boldsymbol{r}_i\cdot\boldsymbol{q}}$
  ρ_i does not commute with v_{ij} and V_{ijk}

Definitions

- Let us define

$$\begin{aligned} \rho(\boldsymbol{q}) &= \sum_i \rho_i(\boldsymbol{q}) + \sum_{i<j} \rho_{ij}(\boldsymbol{q}) + \ldots \,, \\ \boldsymbol{j}(\boldsymbol{q}) &= \sum_i \boldsymbol{j}_i(\boldsymbol{q}) + \sum_{i<j} \boldsymbol{j}_{ij}(\boldsymbol{q}) + \ldots \,. \end{aligned}$$

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$$\boldsymbol{j}(\boldsymbol{q}) = \sum_i \boldsymbol{j}_i(\boldsymbol{q}) + \sum_{i<j} \boldsymbol{j}_{ij}(\boldsymbol{q}) + \dots\ .$$

- CCR:

$$\boldsymbol{q}\cdot\boldsymbol{j}_i(\boldsymbol{q}) = \left[\frac{\boldsymbol{p}_i^2}{2m}, \rho_i(\boldsymbol{q})\right],$$

$$\boldsymbol{q}\cdot\boldsymbol{j}_{ij}(\boldsymbol{q}) = \left[v_{ij}, \rho_i(\boldsymbol{q}) + \rho_j(\boldsymbol{q})\right],$$

Non-relativistic reduction

- The one-body charge density operator can be written as

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- $\rho_{i,\mathrm{RC}}(\boldsymbol{q})$, $\rho_{ij}(\boldsymbol{q})$, $\ldots$ turn out to be of the order $O(1/m^2)$
- At low energies, we must ask at least to verify the CCR for $\rho_i(\boldsymbol{q}) = \rho_{i,\mathrm{NR}}(\boldsymbol{q})$

Recipes

- Potential

  Feynman diagram $\longrightarrow$ NR reduction $\sim v$ or V in momentum space

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- Current/charge (example: γ absorption)

$$\langle \boldsymbol{p}'_1, \ldots, \boldsymbol{p}'_A| \int d^4x j^\mu(x) A_\mu(x) |\boldsymbol{p}_1, \ldots, \boldsymbol{p}_A; \gamma\rangle \sim$$
$$\langle \boldsymbol{p}'_1, \ldots, \boldsymbol{p}'_A| j^\mu(\boldsymbol{q}, \omega)\epsilon_\mu |\boldsymbol{p}_1, \ldots, \boldsymbol{p}_A\rangle$$

Feynman diagram $\longrightarrow$ take the operator multiplying ϵ_μ $\longrightarrow$ NR reduction $\longrightarrow$ (Fourier transform of the) current/charge operators
$\int d\omega/2\pi j^\mu(\boldsymbol{q}, \omega) \exp(\mathrm{i}\omega t) \longrightarrow j^\mu(\boldsymbol{q}, t)$

One-body current and charge

- Vertex: $e\bar{\Psi}(x)\gamma^{\mu}A_{\mu}(x)\Psi(x)$

$$\langle N,\boldsymbol{p}'|\int d^4x J^{\mu}(x)A_{\mu}(x)|N,\boldsymbol{p}\rangle|\gamma,\boldsymbol{q}\rangle \sim$$

$$\bar{u}(\boldsymbol{p}',s')\left[F_1(Q^2)\gamma^{\mu}+F_2(Q^2)\frac{\mathrm{i}\sigma^{\mu\nu}q_{\nu}}{2m}\right]u(\boldsymbol{p},s)(2\pi)^4\delta(p'_{\mu}p_{\mu}-q_{\mu})$$

F_1, F_2 Dirac and Pauli form factors
$Q^2=-q^{\mu}q_{\mu}$
$\sigma^{\mu\nu}=(\mathrm{i}/2)[\gamma^{\mu},\gamma^{\nu}]$

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- Vertex: $e\bar{\Psi}(x)\gamma^\mu A_\mu(x)\Psi(x)$

$$\langle N, \boldsymbol{p}'| \int d^4x J^\mu(x) A_\mu(x)|N, \boldsymbol{p}\rangle|\gamma, \boldsymbol{q}\rangle \sim$$

$$\bar{u}(\boldsymbol{p}', s')\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{\mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\right] u(\boldsymbol{p}, s)(2\pi)^4\delta(p'_\mu p_\mu - q_\mu)$$

F_1, F_2 Dirac and Pauli form factors
$Q^2 = -q^\mu q_\mu$
$\sigma^{\mu\nu} = (\mathrm{i}/2)[\gamma^\mu, \gamma^\nu]$

- 4-spinors

$$u(\boldsymbol{p}, s) = \left(\frac{E+m}{2m}\right)^{1/2} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m}\chi_s \end{pmatrix}$$

One-body current and charge

- NR reduction (retain the terms $O(1/m)$ only)

$$
\begin{aligned}
\rho^{(1)}_{NR}(\boldsymbol{q}) &= q_i \delta(\boldsymbol{p}' - \boldsymbol{p} - \boldsymbol{q}) \\
\boldsymbol{j}^{(1)}_{NR}(\boldsymbol{q}) &= \left[q_i \frac{\boldsymbol{p}' + \boldsymbol{p}}{2m} - \frac{\mathrm{i}}{2m} \mu_i \boldsymbol{q} \times \boldsymbol{p} \right] \delta(\boldsymbol{p}' - \boldsymbol{p} - \boldsymbol{q})
\end{aligned}
$$

$$
q_i = G^p_E(Q^2)\tfrac{1+\tau_z}{2} + G^n_E(Q^2)\tfrac{1-\tau_z}{2}
$$
$$
\mu_i = G^p_M(Q^2)\tfrac{1+\tau_z}{2} + G^n_M(Q^2)\tfrac{1-\tau_z}{2}
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$$
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2} F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
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\end{aligned}
$$

$$
\begin{aligned}
q_i &= G_E^p(Q^2) \tfrac{1+\tau_z}{2} + G_E^n(Q^2) \tfrac{1-\tau_z}{2} \\
\mu_i &= G_M^p(Q^2) \tfrac{1+\tau_z}{2} + G_M^n(Q^2) \tfrac{1-\tau_z}{2}
\end{aligned}
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G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2} F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
$$

- Exercise: verify the CCR in this case

One-pion exchange potential

- Lagrangian

$$\mathcal{L}_I = \frac{f_{\pi NN}}{m_\pi} \bar{\Psi}(x)\gamma^\mu \gamma_5 \tau_a \Psi(x) \partial_\mu \Phi_a(x)$$

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- OPEP

$$\begin{aligned}
v_\pi(k) &= \frac{f^2_{\pi NN}}{3m^2_\pi}\frac{1}{k^2+m^2_\pi} \\
\langle \boldsymbol{p}'_1, \boldsymbol{p}'_2 | v | \boldsymbol{p}_1\, \boldsymbol{p}_2\rangle &= \Big[v_\pi(k)\, k^2\, (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + v_\pi(k) S_{12}(\boldsymbol{k})\Big]\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
&\qquad \times\delta(\boldsymbol{p}'_1+\boldsymbol{p}'_2-\boldsymbol{p}_1-\boldsymbol{p}_2) \\
S_{12}(\boldsymbol{k}) &= 3(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) - k^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)
\end{aligned}$$

$\boldsymbol{k} = \boldsymbol{p}'_1 - \boldsymbol{p}_1$

$f^2_{\pi NN}/4\pi \approx 0.075$

One-pion exchange current & charge

- Lagrangian with e.m. field

$$
\begin{aligned}
\mathcal{L}_I &= \frac{f_{\pi NN}}{m_\pi}\bar{\Psi}(x)\gamma^\mu\gamma_5\tau_a\Psi(x)\partial_\mu\Phi_b(x)\epsilon_{abz}A_\mu(x) \\
&+e\Phi_a \times \partial^\mu\Phi_b\epsilon_{abz}A_\mu(x)
\end{aligned}
$$

One-pion exchange current & charge

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$$+e\Phi_a \times \partial^\mu\Phi_b\epsilon_{abz}A_\mu(x)$$

- NR reduction of the 3 Feynman diagrams [$\boldsymbol{k}_1 = \boldsymbol{p}'_1 - \boldsymbol{p}_1$]

$$\langle \boldsymbol{p}'_1, \boldsymbol{p}'_2 | \boldsymbol{j}_{f_{\pi NN}, m_\pi}(\boldsymbol{q}) | \boldsymbol{p}_1\, \boldsymbol{p}_2 \rangle =$$
$$= 3\mathrm{i}\, G_E^V(q_\mu^2)(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \Big[v_\pi(k_2)\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}_2) - v_\pi(k_1)\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{k}_1)$$
$$+\frac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{k_1^2 - k_2^2}\left[v_\pi(k_1) - v_\pi(k_2)\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{k}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}_2)\Big]$$

$$G_E^V(q_\mu^2) = G_E^p(q_\mu^2) - G_E^n(q_\mu^2)$$

One-pion exchange current & charge

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$$= 3\mathrm{i}\, G_E^V(q_\mu^2)(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \Big[v_\pi(k_2)\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}_2) - v_\pi(k_1)\boldsymbol{\sigma}_2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}_1)$$
$$+ \frac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{k_1^2 - k_2^2}\left[v_\pi(k_1) - v_\pi(k_2)\right](\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}_1)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}_2)\Big]$$

$$G_E^V(q_\mu^2) = G_E^p(q_\mu^2) - G_E^n(q_\mu^2)$$

- Exercise: verify the CCR with OPEP

Realistic potentials

- but for a realistic potential?

  simple prescription!

  Buchmann, Leidemann, & Arenhövel, 1985

  Riska, 1985

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- Suppose that (in momentum space)

$$v_{ij}(\boldsymbol{k}) = \left[v_{PS}(k)\, k^2\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + v_{PS}(k) S_{ij}(\boldsymbol{k}) \right] \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$v_{PS}(k)$ generic function fixed by comparison with experiment

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- Suppose that (in momentum space)

$$v_{ij}(\boldsymbol{k}) = \left[v_{PS}(k)\, k^2\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + v_{PS}(k) S_{ij}(\boldsymbol{k}) \right] \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

  $v_{PS}(k)$ generic function fixed by comparison with experiment

- Let us consider a family of pseudo-scalar mesons of mass m_r and coupling constant with the nucleon f_r

  $r = 1\ m_1 = m_\pi,\ f_1 = f_{\pi NN}$

  $r > 1\ m_r > m_\pi,\ f_r =$ free parameters

Realistic potentials

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- It has been generalized also for the 3N interaction

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- $p + d \rightarrow {}^3\mathrm{He} + \gamma$

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- Tiny effects of $\boldsymbol{j}_{ijk}$ (but visible for some observables)