Gravitational Wave Emission from Compact Stars
Lecture 1

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Outline

1.0 Motivation

1.1 Gravitational waves for beginners

1.2 Sources of gravitational waves

1.3 Detection of gravitational waves

Motivation

- “The day of the first undeniable detection of gravitational waves should not be far away” (Andersson & Kokkotas, 1998)
- Neutron (or, more generally, compact) stars are regarded as one of the most promising sources
- Observation of a gravitational wave signal can be used to infer with good accuracy different star properties, like its mass and (most important) radius.
- These data will provide strong additional constraints to the models od equation of state of strongly interacting (hadronic and quark) matter at large density and low temperature

- In Einstein's theory of gravitation, the invariant spacetime interval

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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- Consider a weak perturbation of the Minkowski (flat) spacetime

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) \quad , \quad |h_{\mu\nu}| \ll 1$$

Einstein equation

- The Einstein equation establishes the relation between matter and spacetime curvature, described by the Ricci tensor $R_{\mu\nu}$ (to be defined later)
- In vacuum the spacetime is flat, $g_{\mu\nu} = \eta_{\mu\nu}$ and the curvature vanishes, i.e.

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- For small perturbations, ignore $\delta^2 R_{\mu\nu}$ and higher order terms and solve

$$R_{\mu\nu} = \delta R_{\mu\nu}$$

Definitions

- Christoffel symbol, or affine connection. Plays the same role as the field strength tensor in Maxwell theory ($\partial_\mu = \partial/\partial x^\mu$)

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu} \right)$$

$$= \frac{1}{2} \eta^{\alpha\beta} \left(\partial_\mu h_{\beta\nu} + \partial_\nu h_{\beta\mu} - \partial_\beta h_{\mu\nu} \right) + O(h^2)$$

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- Only terms linear in $\Gamma^\alpha_{\mu\nu}$ will contribute at first order in $h_{\mu\nu}$

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Obtaining the wave equation

- As $\eta_{\mu\nu}$ is constant, derivatives only act on $h_{\mu\nu}$. At first order

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- Defining (note that at first order indeces can be raised and lowered with the flat metric tensor)

$$V_\alpha = \partial_\beta h^\beta_\alpha - \frac{1}{2}\partial_\alpha h^\beta_\beta \quad , \quad \Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$$

- The linearized Einstein equation in vacuum becomes

$$\delta R_{\mu\nu} = \frac{1}{2}\left(-\Box h_{\mu\nu} + \partial_\mu V_\nu + \partial_\nu V_\mu\right)$$

Obtaining the wave equation (continued)

- Choose a coordinate system where the harmonic gauge condition

$$g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu} = 0$$

is satisfied (it can be proved that this choice is always possible)

- At first order the above condition becomes

$$V_\alpha \equiv 0$$

- Hence, we are left with the wave equation

$$\Box h_{\mu\nu} = 0$$

with the gauge condition

$$\partial_\beta h^\beta_\alpha = \frac{1}{2}\partial_\alpha h^\beta_\beta$$

Plane wave solutions

$$h_{\mu\nu} = A_{\mu\nu} \mathrm{e}^{ik^\lambda x_\lambda} \quad , \quad k^\lambda k_\lambda = 0$$

$$A^\beta_\alpha \partial_\beta \mathrm{e}^{ik^\lambda x_\lambda} = A^\beta_\beta \partial_\alpha \mathrm{e}^{ik^\lambda x_\lambda} \Longrightarrow A^\beta_\alpha k_\beta = A^\beta_\beta k_\alpha$$

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- The harmonic gauge condition does not determine the gauge uniquely. It is preserved by infinitesimal coordinate transformations

$$x^\mu \rightarrow x'^\mu = x^\mu + \zeta^\mu$$

as long as ζ^μ satisfies

$$\Box \zeta^\mu = 0$$

- A gravitational wave has only two physical degrees of freedom, corresponding to two states of polarization

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- For a wave travelling in the x_3 direction $k \equiv (1, 0, 0, 1)$ and

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{+} & A_{X} & 0 \\ 0 & A_{X} & -A_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- $A_{11} = -A_{22} = A_{+}$: plus polarization; $A_{12} = A_{21} = A_{X}$: cross polarization

Einstein equations with sources

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4}\, T_{\mu\nu}$$

- At first order reduces to

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \quad , \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\lambda}_{\lambda}$$

$$\partial_\mu \bar{h}^{\mu}_{\lambda} = 0$$

▷ $R = R^{\beta}_{\beta}$: Ricci scalar

▷ $T_{\mu\nu}$: energy-momentum (or stress-energy) tensor

- Solution

$$\bar{h}_{\mu\nu}(\mathbf{x}, t) = \frac{4G}{c^4} \int d^3x' \, \frac{T_{\mu\nu}(\mathbf{x}', t - |\mathbf{x} - \mathbf{x}'|/c)}{|\mathbf{x} - \mathbf{x}'|}$$

Far from source approximation

- As $|\mathbf{x}-\mathbf{x}'| = r \to \infty$, for a pointlike source ($R \ll \lambda_G$)

$$\bar{h}_{\mu\nu}(\mathbf{x},t) \approx \frac{4G}{c^4}\int d^3x'\, T_{\mu\nu}(\mathbf{x}', t - r/c)$$

- Exploiting local conservation of energy-momentum we find $(i,j=1,2,3)$

$$\bar{h}_{ij}(\mathbf{x},t) = \frac{2G}{r}\,\frac{d^2}{dt^2}q_{ij}(t)$$

where

$$q_{ij}(t) = \int d^3x x^i x^j\, T_{00}(\mathbf{x},t)$$

is the quadrupole moment tensor (T_{00} is the mass-energy density).

Summary

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- By a suitable choice of gauge, a small perturbation to the flat spacetime is found to satisfy a wave equation
- Its amplitude depends on the variation in time of the quadrupole moment, which is in turn defined in terms of the mass-energy of the source

Perturbation of a curved background

- Suppose we know the exact solution $g^0_{\mu\nu}$ describing the gravitational field of a shperically symmetric object, like a compact star or a black hole.
- If $h_{\mu\nu}$ is a perturbation to $g^0_{\mu\nu}$, it can be shown that the Einstein equation for a suitable combination of the metric function, say $\Phi_{\mu\nu}$, can be written in the form

$$\{\Box - V(x^\mu)\}\,\Phi_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \; ,$$

  where $\Box$ is the d'Alambert operator corresponding to flat spacetime and $V(x^\mu)$ is the “potential" generated by the spacetime curvature
- Perturbations of spherically symmetric, stationary gravitational fields are described by a Schrödinger-like equation.

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- Radial oscillations of spherically symmetric stars do not produce gravitational radiation

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▷ This process can lead to the excitation of non radial oscillation modes of the stars, leading to gravitational wave emission

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- The possibility of powerful gravitational wave emission occurs if the collapse to a neutron star or black hole is non spherically symmetric
- The intensity of the emitted radiation depends mostly on the amount of stellar mass converted into gravitational waves. Theoretical estimates put this efficiency ratio are in the broad range $10^{-10} - 10^{-3}$

- ▷ black holes: gravitational radiation can be emitted by a mass falling into a black hole, or by two black holes falling into one another. Gravitational radiation may be the most effective way of detection of black hole.

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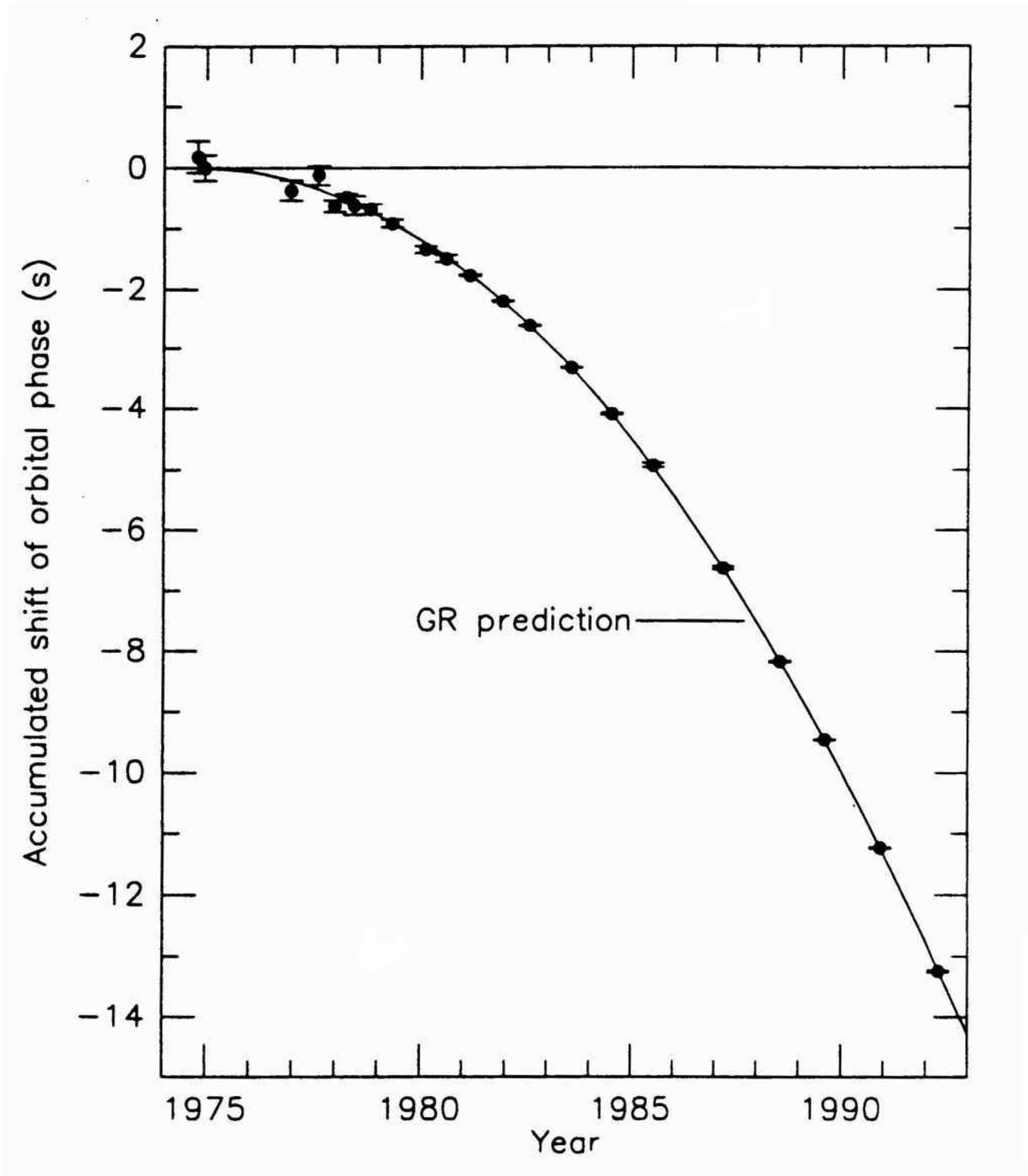
- In 1993 Hulse & Taylor were awarded the Nobel Prize in Physics for their observation of a decay rate of roughly 7 mm/day of the orbit of the binary pulsar PSR B1913+16. This measurement, in agreement of better than $\sim 0.3\%$ with the prediction of general relativity, is regarded as an indirect evidence of gravitational wave emission



Accumulated shift of orbital phase (s)
Year
GR prediction
2
0
−2
−4
−6
−8
−10
−12
−14
1975
1980
1985
1990

Detection of gravitational waves

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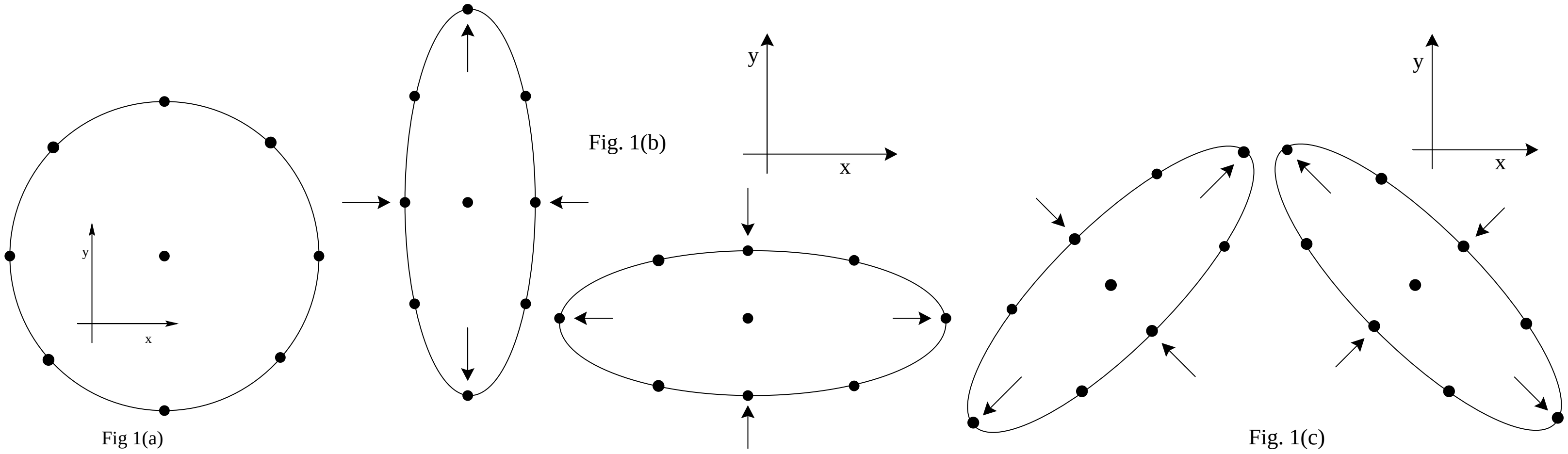
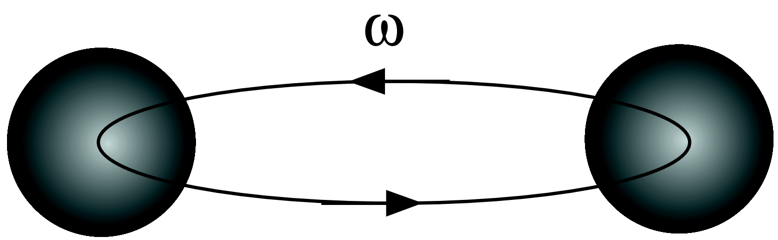
- Detection techniques are based on the observation of the relative motion of particles in the field produced by a gravitational wave
- Displacement of test particles



A very elusive signal

- Luminosity (power). Consider a binary system



$$L_G = \frac{G}{c^5} \sum_{ij} \left(\frac{d^3 q_{ij}}{dt^3} \right) \sim \frac{G}{c^5} M^2 L^4 \omega^6$$

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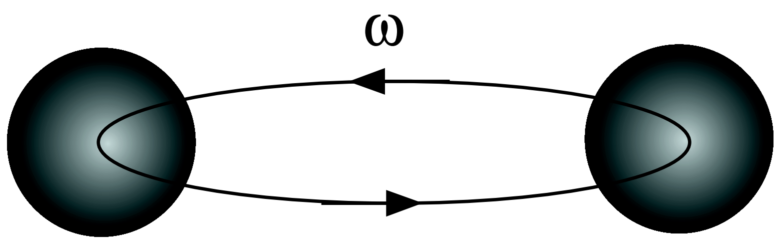

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- Strain amplitude. Consider two masses coalescing to form a black hole. For a system of $\sim 10\ M_\odot$ located at the galactic center one finds $h \sim 10^{-17}$ on Earth. A more likely figure is

$$h \sim 10^{-21}$$

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▷ Resonant bars are still in operation at CERN (Explorer), LNF (Nautilus), LNL (Auriga), Baton Rouge (Alegro), Perth (Niobe)

- Resonant bars have cylindrical shape and large mass M. They exploit the change of distance between two test masses produced by a gravitational wave, leading to the excitation of the bar fundamental vibrational mode.
- Time dependence of the bar response

$$g(t) \propto \mathrm{e}^{-t/\tau_d} \sin \Omega_0 t$$

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- Thermal noise sets a displacement limit to the bar sensitivity. At room temperature

$$\Delta x_{th} = \sqrt{\frac{k_B T}{M \Omega_0^2}} \sim 10^{-16} \text{ m}$$

- For a 1 m long bar the typical gravitational displacement is $\sim 10^{-21}$m. Must operate at cryogenic temperatures: $0.1 < T < 5\ ^\circ\mathrm{K}$

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- The light beam travels an extra distance $h_{11}dx/2$

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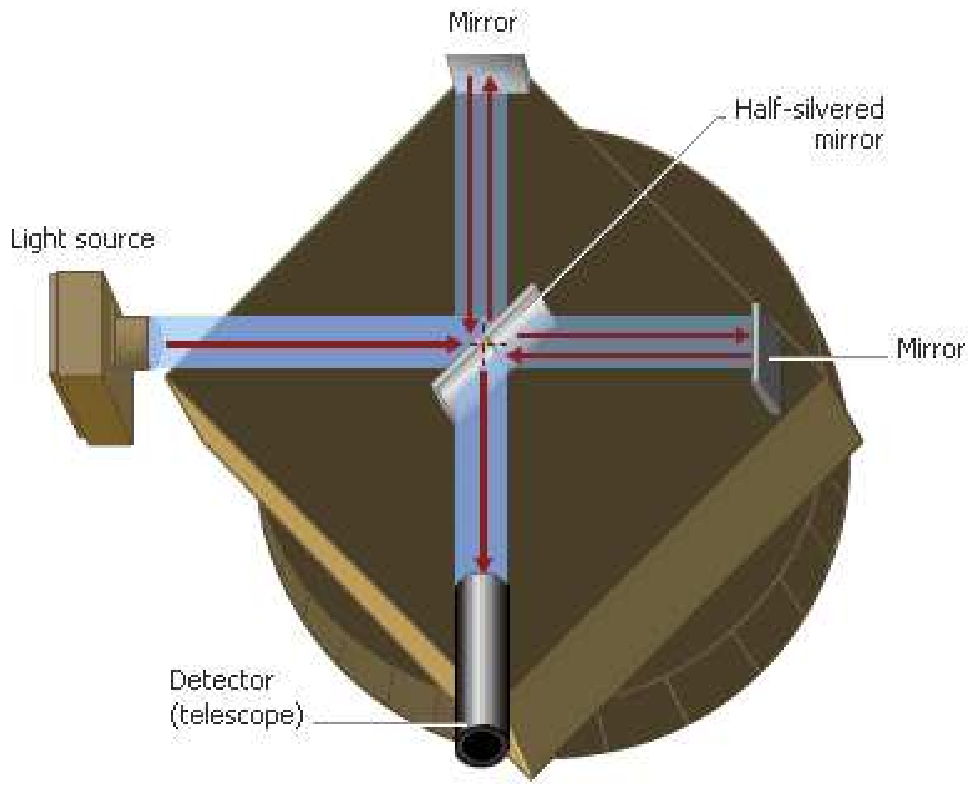
$$\phi(t) = h(t)\frac{4\pi L}{\lambda} \ ,$$

where L is the length of the arm and λ is the light wave-length



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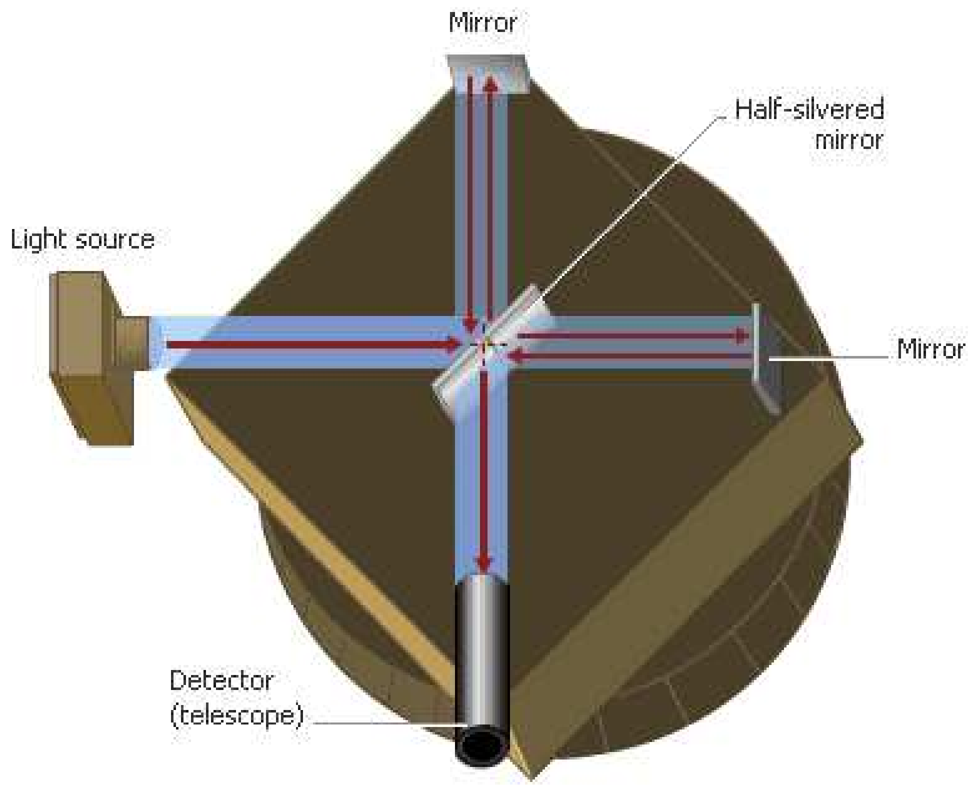

- The larger the arm length L the higher is the sensitivity

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- Earth curvature and budget limit L to few km. To overcome this problem the light path is folded into an optical cavity. Multiple reflections extend the effective optical length (for example, for VIRGO $L = 3$ km and the effective length is $L_{\text{eff}} = 120$ km).

Operating (ground-based) interferometers

- LIGO (Hanford, WA & Livingston, LU): $L = 4$ km
- VIRGO (Cascina, Italy): $L = 3$ km
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- The next generation of detectors will be space-borne. No seismic noise, $L \sim 4.8 \times 10^6\ \mathrm{km}$, optimized for very low frequencies. LISA (Laser Interferometer Space Antenna)

Detectors vs sources