
Scuola di Fisica Nucleare “Raimondo Anni”, secondo corso
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Formazione e struttura delle stelle compatte
Struttura delle Stelle Compatte
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Star deaths and formation of dense stars

At end of all the possible nuclear "burning" processes (nuclear fusion reactions) a star ends his "life" as one of the following dense and compact configurations:

1) **White Dwarf** $0.1\ M_\odot < M_{prog} < 8\ M_\odot$
2) **Neutron Star** $8\ M_\odot < M_{prog} < (25 - 30)\ M_\odot$

$$\text{Fe-core} \xrightarrow{\text{collapse}} \text{SN explosion} \longrightarrow \text{NS}$$

3) **Black Hole** $M_{prog} > (25 - 30)\ M_\odot$

$$\text{Fe-core} \xrightarrow{\text{collapse}} \text{BH}$$

M_{prog} = mass progenitor star, $M_\odot = 1.989 \times 10^{33}$ g (mass of the Sun)

	Sun	White dwarf	Neutron Star	Black Hole
mass	$M_\odot$	1–1.4 $M_\odot$	1–2 $M_\odot$	arbitrary
radius	$R_\odot$	~ 10^{-2} $R_\odot$	~ 10 km	$2GM/c^2$
R/R_g	2.4 ´ 10^5	~ 2 ´ 10^3	~ 2 – 4	1
av. dens	~ 1 g/cm^3	~$10^{7\text{-}8}$ g/cm^3	2–9 ´ 10^{14} g/cm^3	=

R_g º $2GM/c^2$ (Schwarzschild radius)

x º R/R_g (compactness parameter)

$M_\odot$ = 1.989 ´ 10^{33} g $R_\odot$ = 6.96 ´ 10^5 km $R_{g\,\odot}$ = 2.95 km

r_0 = 2.8 ´ 10^{14} g/cm^3 (nuclear saturation density)

When x is "small" gravity must be described by the Einstein theory of General Relativity

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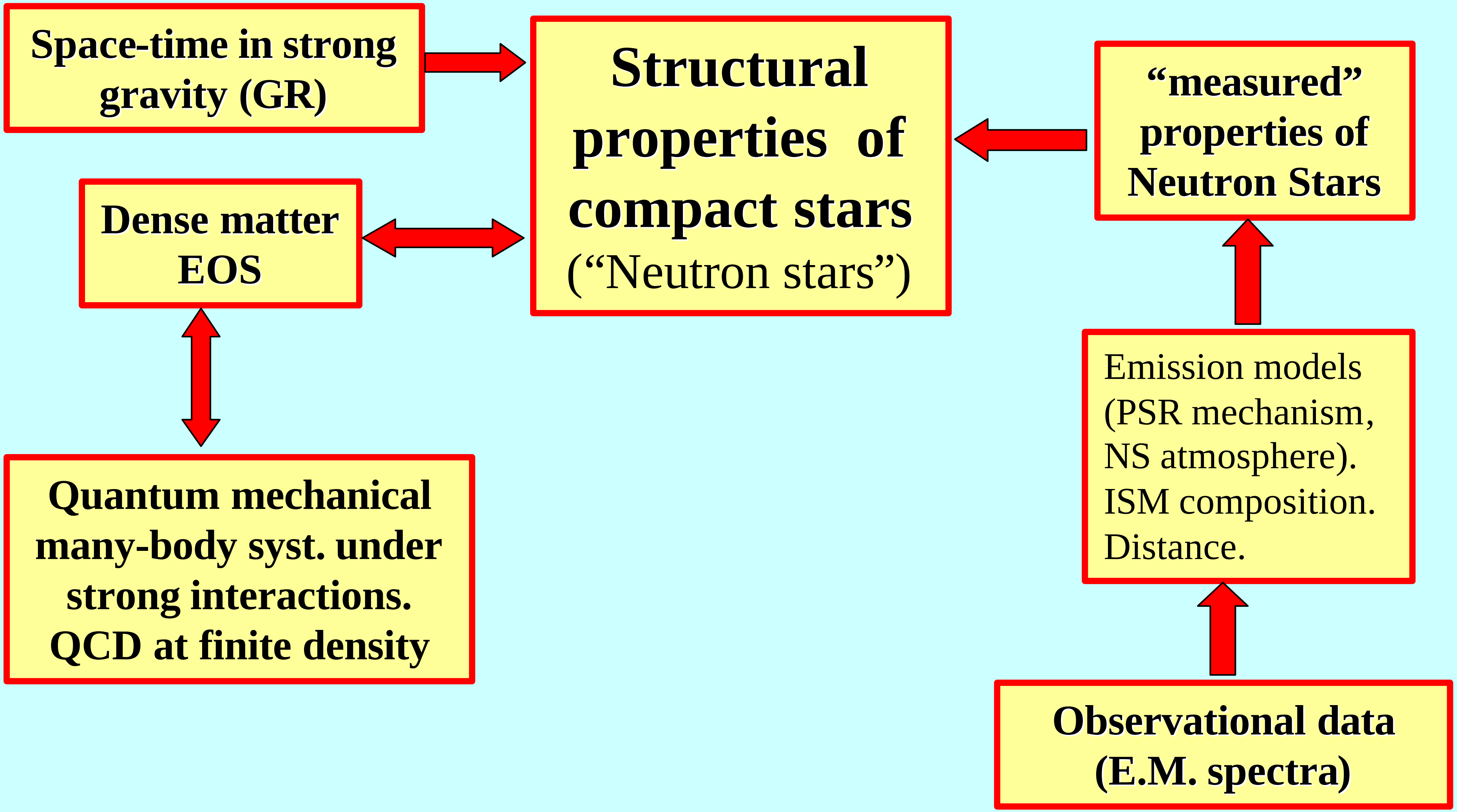

Space-time in strong gravity (GR)
Structural properties of compact stars ("Neutron stars")
"measured" properties of Neutron Stars
Dense matter EOS
Emission models (PSR mechanism, NS atmosphere). ISM composition. Distance.
Quantum mechanical many-body syst. under strong interactions. QCD at finite density
Observational data (E.M. spectra)

Structure equations for compact stars

Hydrostatic equilibrium in General Relativity **for a self-gravitating mass distribution**

Under the following assumpions:

- **Spherical symmetry**
- **Non-rotating configurations**
- **No magnetic field ("weak" magnetic field)**

The Einstein's field equations thake the form called the

Tolman – Oppenheimer – Volkov equations (TOV)

$$\frac{dP}{dr} = -G\frac{m(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{c^2\rho(r)}\right)\left(1+4\pi\frac{r^3P(r)}{c^2 m(r)}\right)\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2\rho(r)$$

Boundary conditions:

$$m(r=0) = 0$$

(Density is finite at the star center)

$$P(r=R) = P_{surf}$$

define the stellar surface (surface area $4\pi R^2$)

R = stellar radius

The solutions of the TOV eq.s depend parametrically on the central density

$$\rho_c = \rho(r=0)$$

$$\boxed{\begin{aligned} P &= P(r, \rho_c) \\ m &= m(r, \rho_c) \end{aligned}}$$

Role of the Equation of State (EOS)

The key input to solve the TOV equations EOS of dense matter.

In the following we assume matter in the Neutron Star to be a **perfect fluid** (this assumption has been already done to derive the TOV eq.) in a **cold (T = 0)** and **catalyzed** state (state of minimum energy per baryon)

$$\rho = \rho(n) = \rho_0 + \frac{\varepsilon'}{c^2} = \frac{\varepsilon}{c^2}$$

$$P = P(n) = n\left(\frac{\partial \varepsilon}{\partial n}\right) - \varepsilon$$

r = total mass density

r_0 = rest mass density

e = total energy density

e' = internal energy density (includes the kinetic plus the potential energy density due to interactions (not gravity))

The structure and the properties of a Neutron Star are determined by the (unknown) EOS of dense hadronic matter.

matter's constituents

interactions (particularly strong int.)

→ **EOS** → $\left\{ \begin{array}{l} M_{max} \\ R, \\ M(R), \dots \end{array} \right.$

Gravitational mass

$$M_G \equiv m(R) = \int_0^R 4\pi \; r^2 \rho\,(r)\,dr$$

M_G is the mass measured by a distant keplerian observer

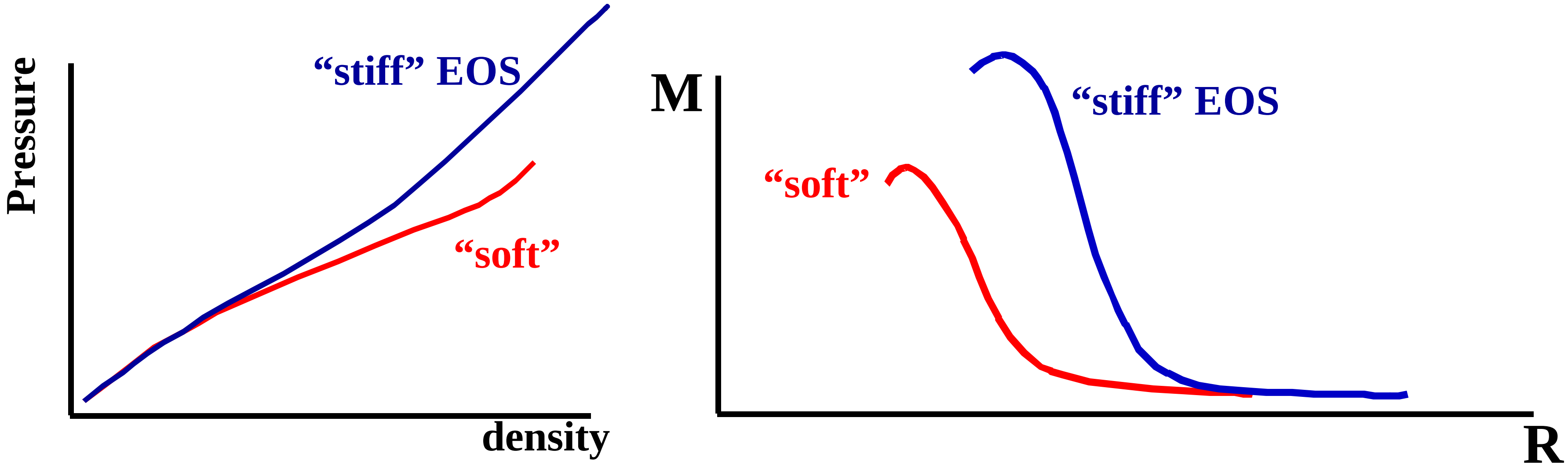
$M_G\, c^2$ = total energy in the star (rest mass + internal energy + gravitational energy)

The Oppenheimer-Volkoff maximum mass

There is a maximum value for the gravitational mass of a Neutron Star that a given EOS can support. This mass is called the Oppenheimer-Volkoff mass

$$M_{max} = (1.4 - 2.5)\ M_{\odot}$$

EOS dependent



The OV mass represent the key physical quantity to separate (and distinguish) Neutrons Stars from Black Holes.

Proper volume = volume of a spherical layer in the Schwarzschild metric

$$dV = 4\pi\, e^{\lambda(r)/2} r^2 dr = 4\pi \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2} r^2 dr$$

Gravitational mass

$$M_G \equiv m(R) = \int_0^R 4\pi\; r^2 \rho(r) dr$$

M_G is the mass measured by a distant keplerian observer

$M_G c^2$ = total energy in the star (rest mass + internal energy + gravitational energy)

Baryonic mass

is the **rest mass of the N_B baryons** (dispersed at infinity) which form the star

$$M_B = m_u \int n(r)\,dV$$

$$= m_u \int_0^R 4\pi\ r^2 n(r) \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2} dr = m_u N_B$$

$\boxed{N_B \sim 10^{57}}$

m_u = baryon mass unit (average nucleon mass)

$n(r)$ = baryon number density

Proper mass

is equal to the **sum of the mass elements** on the whole volume of the star, it includes the contributions of the **rest mass and internal energy** of the constituents of the star

$$M_P = \int \rho(r)\,dV$$

$$= \int_0^R 4\pi\ r^2 \rho(r) \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2} dr$$

- **Gravitational energy:** $E_G = (M_G - M_P)\, c^2 \le 0$

  Gravitational binding energy: $B_G = - E_G$

B_G **is the gravitational energy released moving the infinitesimal mass elements $\rho\, dV$ from infinity to form the star.**

In the *Newtonian limit*
$$E_G = -G \int_0^R 4\pi\, r^2 \frac{m(r)\,\rho(r)}{r}\, dr \equiv E_G^{Newt}$$

- **Internal energy:** $E_I = (M_P - M_B)\, c^2 = \int_0^R \varepsilon'(r)\, dV$

  Internal binding energy: $B_I = - E_I$

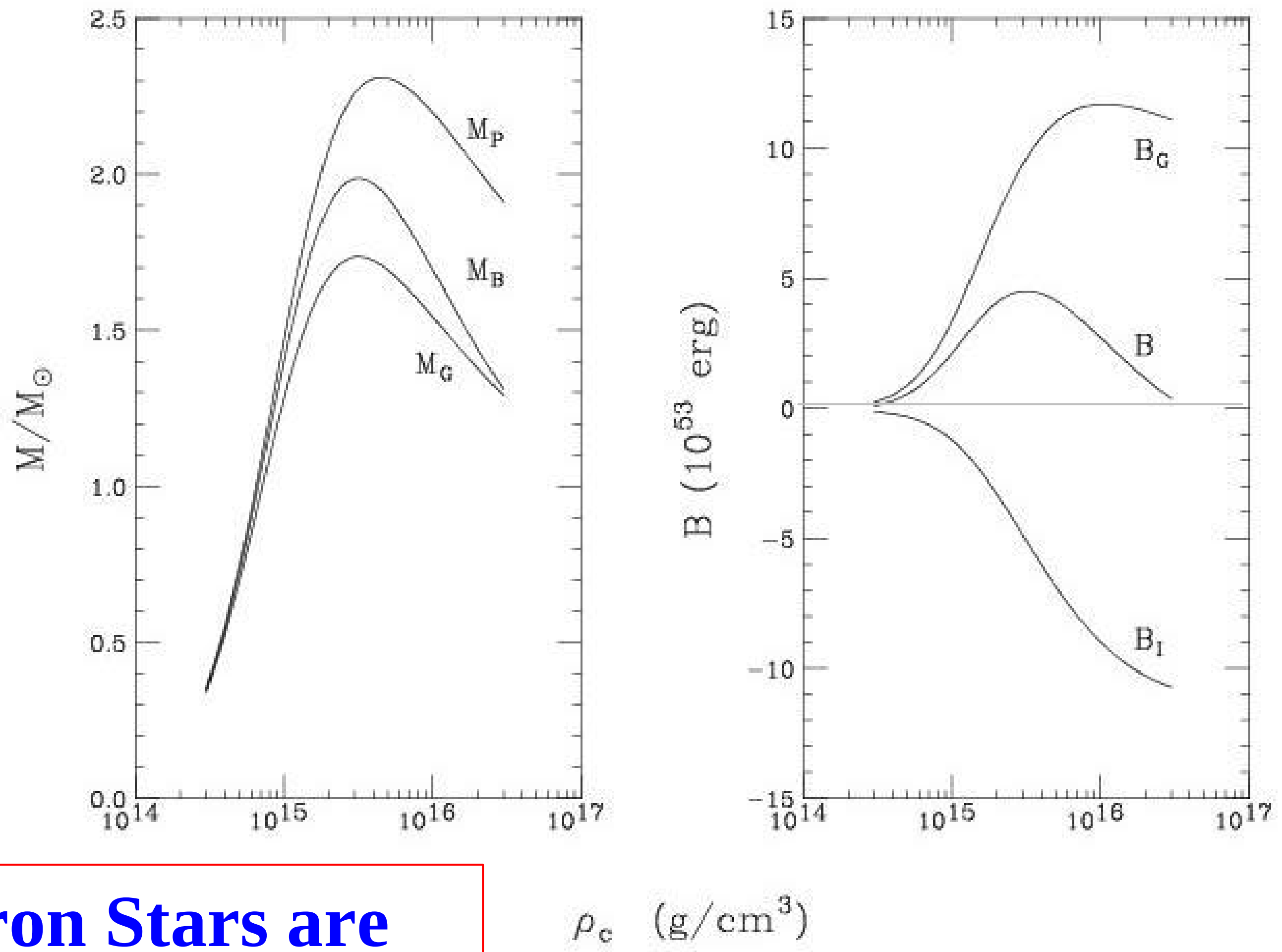
$\varepsilon' = (\rho - \rho_0)\, c^2$

- **Total energy:** $M_G c^2 = M_B c^2 + E_I + E_G = M_P c^2 + E_G$

  Total binding energy: $B = B_G + B_I = (M_B - M_G)\, c^2$

B is the total energy released during the formation of a static neutron star from a rarified gas of N_B baryons

Masses and binding energies of a Neutron Star



Bombaci (1995)

Neutron Stars are bound by gravity

Total binding energy: $\mathbf{B = B_G + B_I}$

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Stability of the solutions of the TOV equations

- The solutions of the TOV eq.s represent **static equilibrium configurations**
- **Stability** of the solutions of TOV eq.s **with respect to small perturbations**

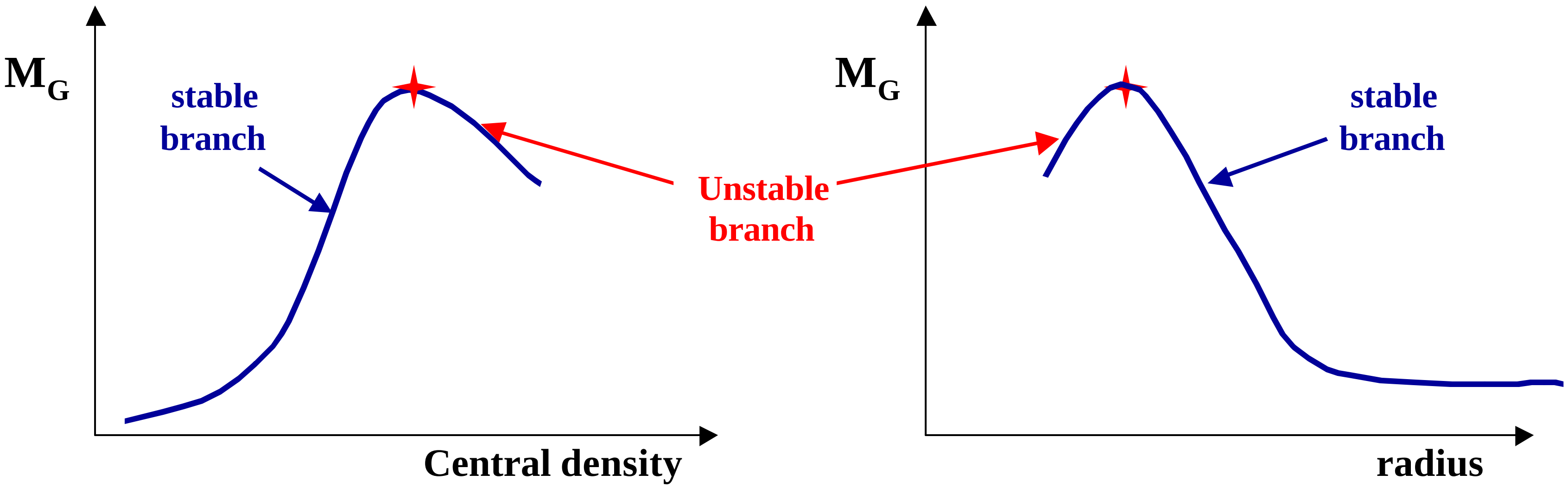
Assumption: the time-dependent stellar configuration, which undergoes small **radial perturbations**, could be described by the EOS of a **perfect fluid** in "chemical" equilibrium (**catalyzed matter**)

⇓

Stable configurations must have

$$dM_G/d\rho_c > 0$$

This is a **necessary** but **not sufficient** condition for stability



The first calculation of the Neutron Stars structure

- **Neutron ideal relativistic Fermi gas**
  (Oppenheimer, Volkoff, 1939).

$M_{max} = 0.71\ M_{\odot}$, $R = 9.5$ km , $n_c/n_0 = 13.75$

$$M_{max} < M_{PSR1913+16} = 1.4411 \pm 0.0007\ M_{\odot}$$

Too soft EOS: needs repulsions from nn strong interaction !

- **Role of the weak interaction**

$$n \text{ ® } p + e^{-} + \bar{n}_e$$

Some protons must be present in dense matter to balance this reaction.

The core of a Neutron Star can not be made of pure neutron matter

Before we start a systematic study of neutron star properties using different models for the EOS of dense matter, we want to answer the following question:

Is it possible to establish an upper bound for the maximum mass of a Neutron Star which does not depend on the deatils of the high density equation of state?

Upper bound on M_{max}

Assumpions:

(a) General Relativity is the correct theory of gravitation.

(b) The stellar matter is a perfect fluid described by a one-parameter EOS,
$P = P(\rho)$.

(c) $\rho \geq 0$ (gravity is attractive)

(d) “microscopic stability” condition: $dP/d\rho \geq 0$

(e) The EOS is known below some fiducial density ρ^*

(f) Causality condition

$$s = (dP/d\rho)^{1/2} \leq c$$

s = speed of sound in dense matter

Under the assumpions (a)—(f) is has been shown by **Rhoades and Ruffini**, **(PRL 32, 1974)** that:

- **The upper bound M^{upper} is independent on the details of the EOS below the fiducial density r ***

- **M^{upper} scales with r * as:**

$$\boxed{M^{upper} = 6.8 \left(\frac{10^{14} g/cm^3}{\rho^*} \right)^{1/2} M_{sun}}$$

r *	**M^{upper}/M_{sun}**
r_0	4.06
2 r_0	2.87

if $M > M^{upper}$

The compact star is a
Black Hole

r_0 = 2.8 ´ 10^{14} g/cm^3 = saturation density of nuclear matter

General features of a “realistic” EOS

Any “realistic” EOS must satisfy the following basic requirements:

(a) saturation properties of symmetric nuclear

$$n_0 = 0.16 \text{ — } 0.18 \text{ fm}^{-3} \qquad (E/A)_0 = -16 \pm 1 \text{ MeV}$$

(b) Nuclear Symmetry Energy $E_{sym}(n_0) = 28 \text{ — } 32$ **MeV,**

$E_{sym}(n)$ **“well behaved” at high density (see lecture by Prof. Di Toro)**

(c) Nuclear incompressibility $K_0 = 220 \pm 20$ **MeV**

(e) Causality condition:

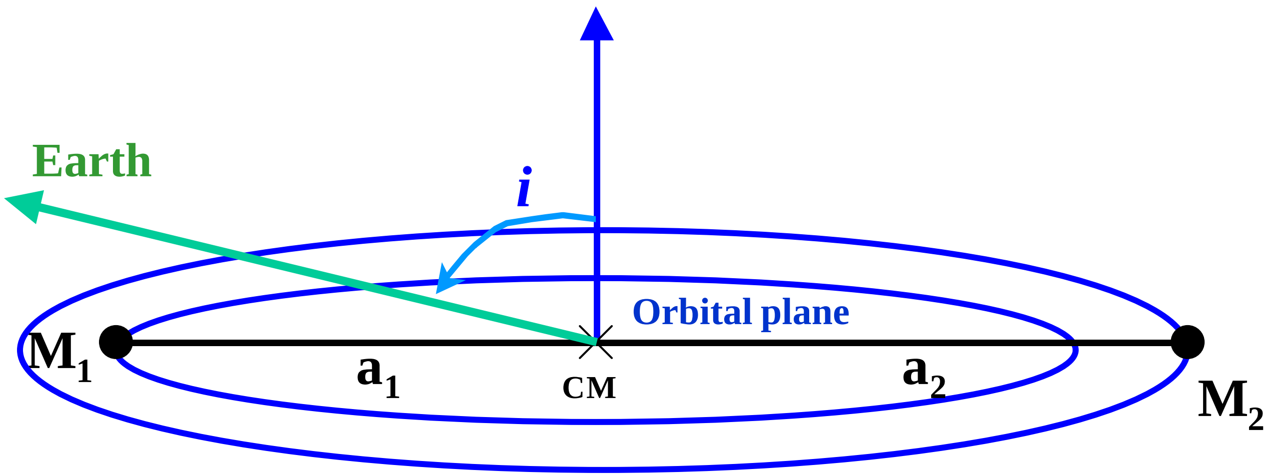
speed of sound **$s = (dP/d r)^{1/2}$ £ c**

Observational determination of the mass of Neutron Stars

Determination of the masses of neutron stars

1) X-ray binaries

The method makes use of the **Kepler's Third Law**.
Consider two spherical masses M_1 and M_2 in circular orbit around their center of mass (the method is valid in the general case of elliptic orbits).



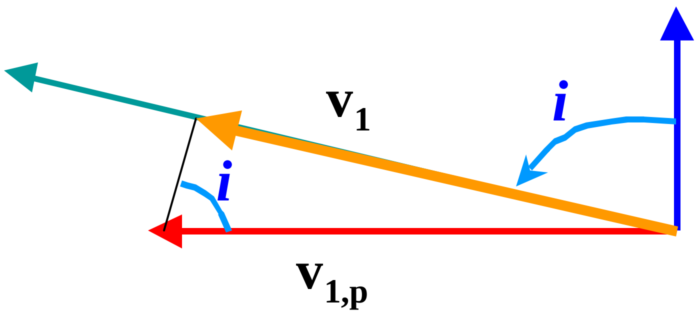
$$a = a_1 + a_2$$

In the CM frame:

$$M_1 a_1 = M_2 a_2$$

$v_{1,p} = 2\pi a_1/P_b$ = **velocity of M_1 in the orb. plane**

P_b = **orbital period**



$$\boxed{v_1 = v_{1,p} \; \sin i = \frac{2\pi}{P_b} a_1 \sin i}$$

Any **spectral feature** emitted by the star M_1 will be **Doppler shifted.**

measurig **P_b , v_1** **®** **$a_1 \sin i$**

Kepler's Third Law:

$$\boxed{G\frac{M_1+M_2}{a^3}=\frac{(2\pi)^2}{P_b^2}}$$

$$a=\frac{M_1+M_2}{M_2}a_1$$

Mass function for the star M_1

$$\boxed{f_1(M_1,M_2,\sin i)\equiv\frac{(M_2\sin i)^3}{(M_1+M_2)^2}=\frac{P_b v_1^3}{2\pi G}}$$

For some X-ray binaries its has been possible to measure **both the mass fuction for the optical companion star as well as the X-ray (NS) mass function**

$$
\begin{cases}
f_X \equiv \dfrac{(M_{op}\sin i)^3}{(M_X + M_{op})^2} \\[2ex]
f_{op} \equiv \dfrac{(M_X \sin i)^3}{(M_X + M_{op})^2}
\end{cases}
\quad \Longrightarrow \quad
\begin{cases}
q \equiv \left(\dfrac{f_{op}}{f_X}\right)^{1/3} = \dfrac{M_X}{M_{op}} \\[2ex]
M_X = \dfrac{f_X\, q\,(1+q^2)}{\sin^3 i}
\end{cases}
$$

The determination of the stellar masses depends on the value of **sin *i***.

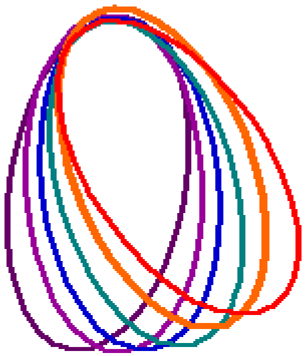
Geometrical constraints can be given on the possible values of **sin *i*:**
in some case **the X-ray component is eclipsed by the companion star** → ***i*** **~ 90º , sin *i* ~ 1**

2) Radio binary pulsar

Tight binary systems: $P_b =$ **a few hours.**

General Relativistic effects are crucial to describe the **orbital motion**

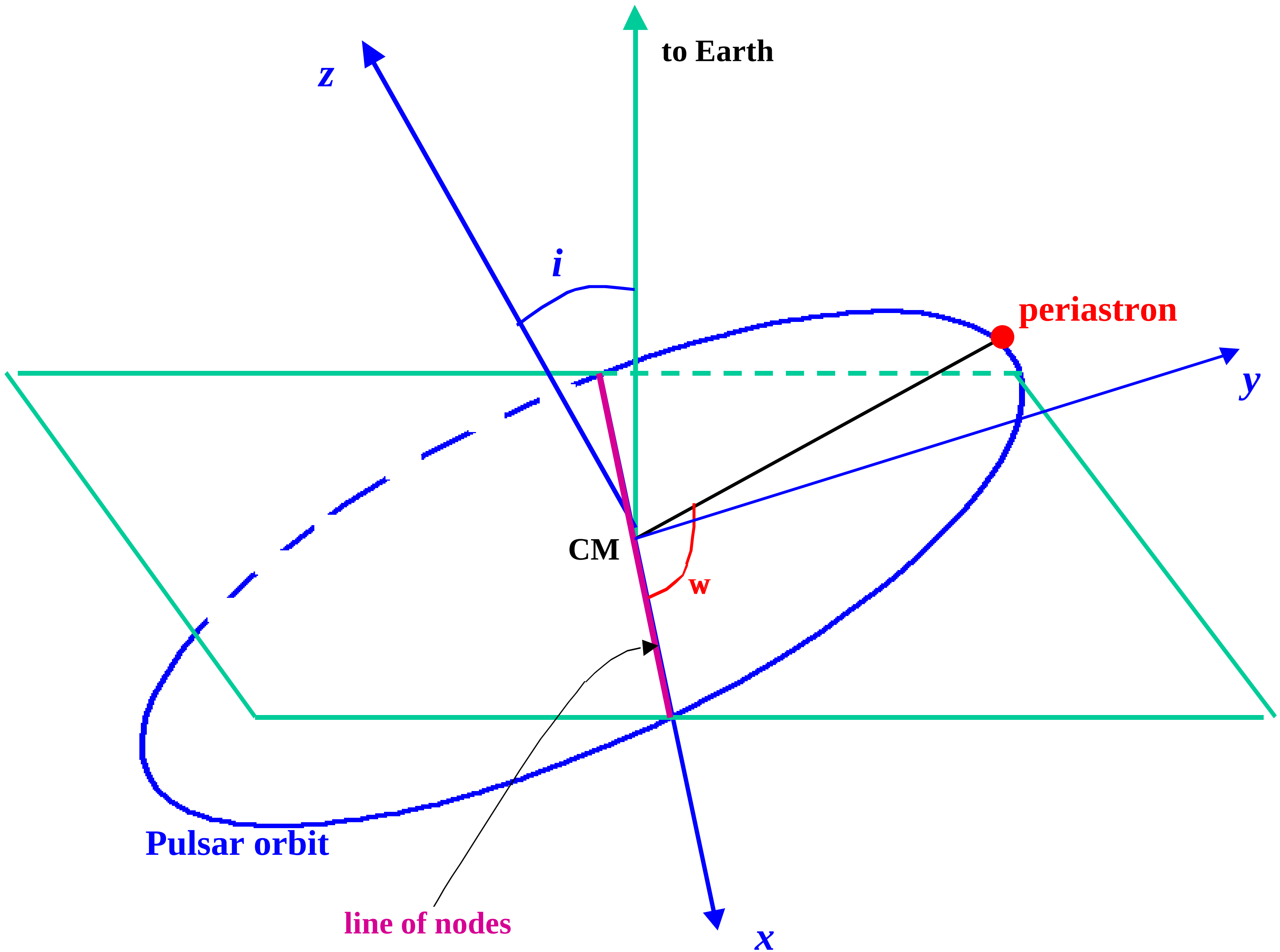
measure of post Keplerian parameters ⇒ **high precision NS mass determination**



Periastron advance : $\dot{\omega} \neq 0$

e.g. **Perielium advance for mercury,** $\dot{\omega}$ **= 43 *arcsec*/100 *yr***

Orbital decay: $\dot{P}_b \neq 0$ ⇒ **evidence for gravitational waves**


to Earth
z
i
periastron
y
CM
w
Pulsar orbit
line of nodes
x

PSR 1913+16

(Hulse and Taylor 1974)

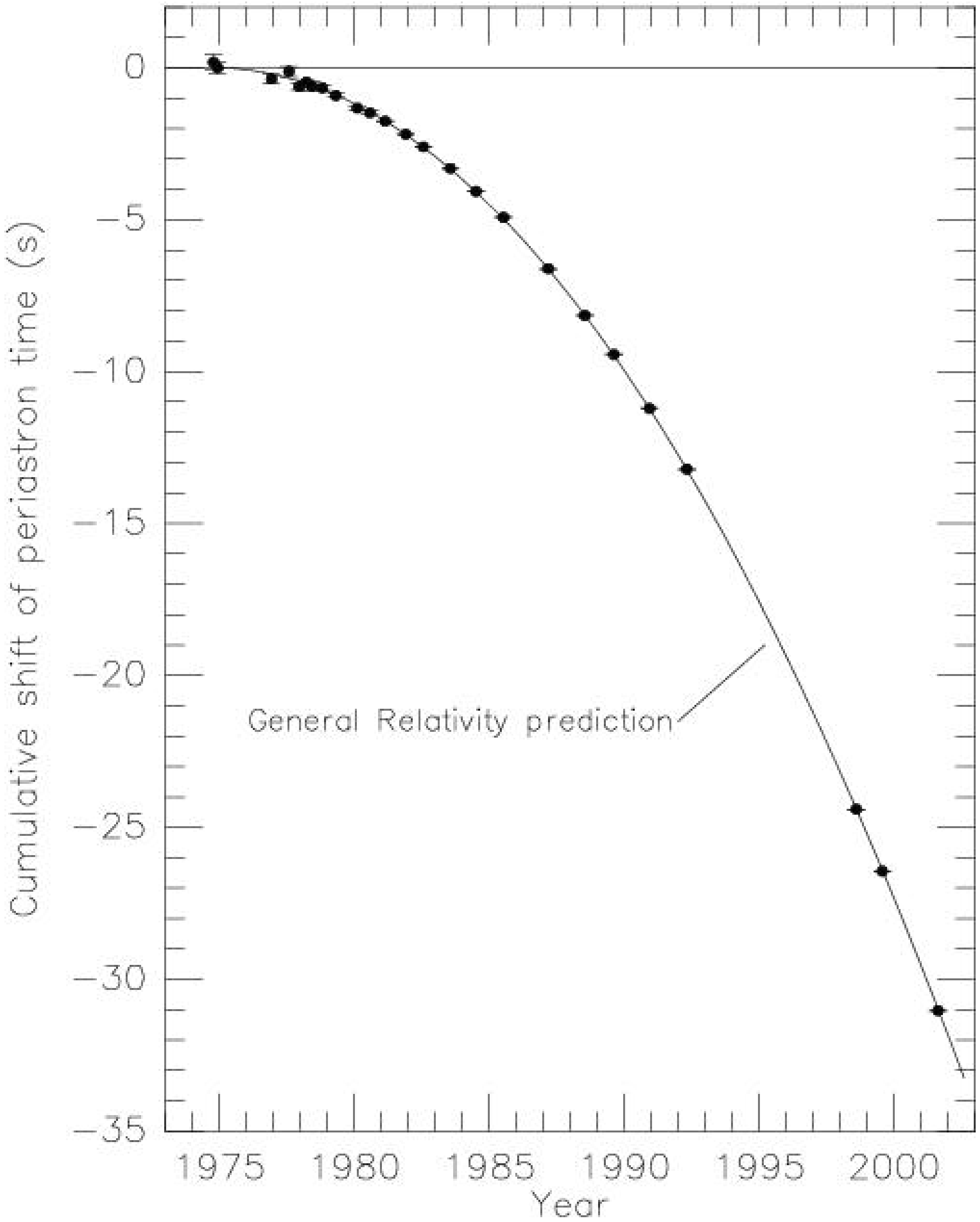
NS (radio PSR) + NS("silent")

P_{PSR} = 59 ms **P_b= 7 h 45 min** $\dot{\omega} = 4.22^0 / yr$

Parameter	Value
Orbital period P_b (d)	0.322997462727(5)
Projected semi-major axis x (s)	2.341774(1)
Eccentricity e	0.6171338(4)
Longitude of periastron ω (deg)	226.57518(4)
Epoch of periastron T_0 (MJD)	46443.99588317(3)
Advance of periastron $\dot{\omega}$ (deg yr^{-1})	4.226607(7)
Gravitational redshift γ (ms)	4.294(1)
Orbital period derivative $(\dot{P}_b)^{obs}$ (10^{-12})	$-2.4211(14)$

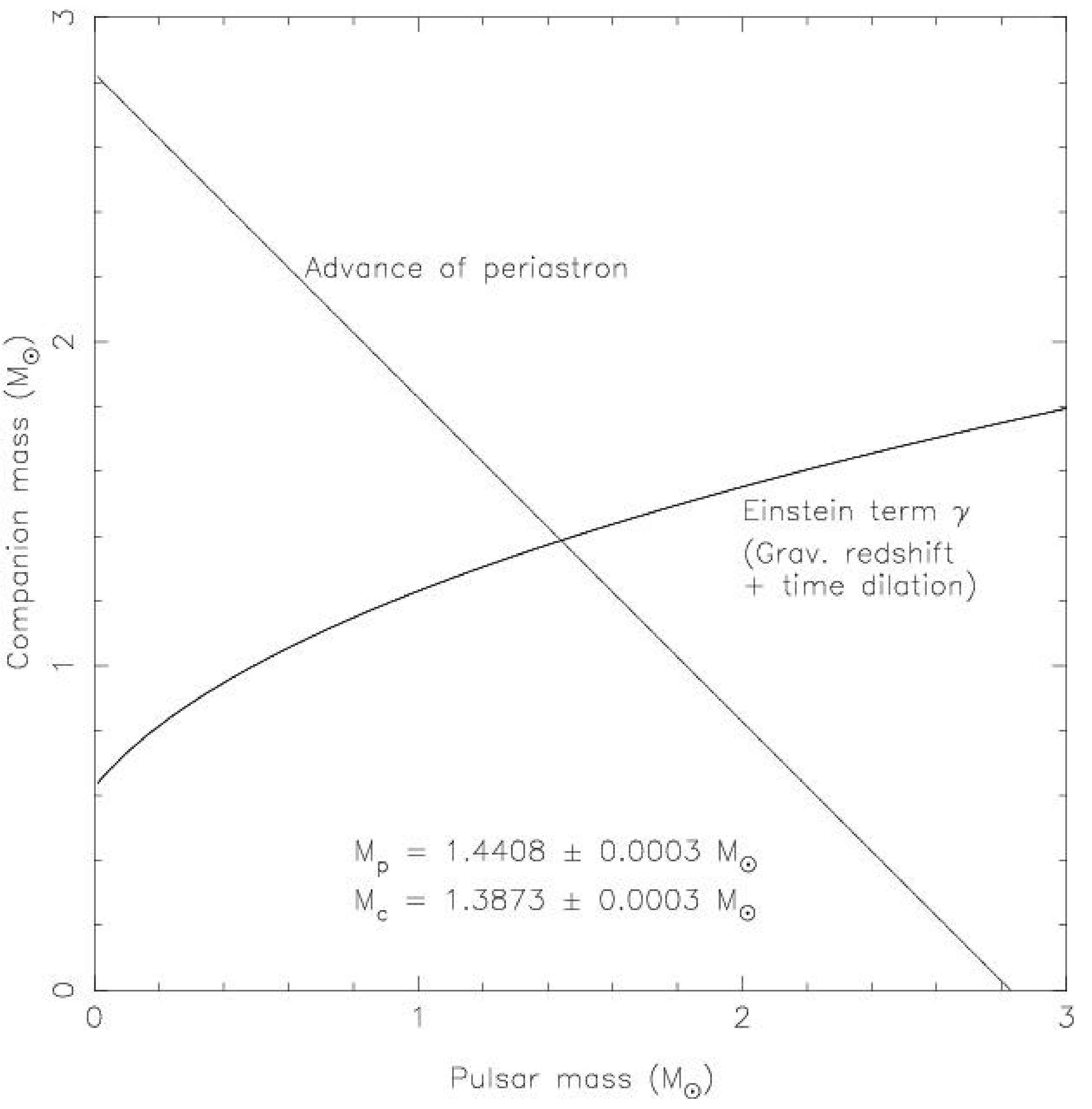
PSR 1913+16

Test of General Relativity and indirect evidence for gravitational radiation



The parabola indicates the predicted accumulated shift in the time of periastron caused by the **decay of the orbit**. The measured value at the epoch of periastron are indicated by the data points

PSR 1913+16


Advance of periastron
Einstein term γ
(Grav. redshift
+ time dilation)
$M_p = 1.4408 \pm 0.0003\ M_\odot$
$M_c = 1.3873 \pm 0.0003\ M_\odot$
Companion mass ($M_\odot$)
Pulsar mass ($M_\odot$)
0
1
2
3
0
1
2
3

PSR J0737-3039

(Burgay et al. 2003)

NS(PSR) + NS(PSR) first **double pulsar**

P_{PSR1} = 22 ms **P_{PSR2} = 2.77 s**

P_b = 2 h 24 min

e ~ 0.088

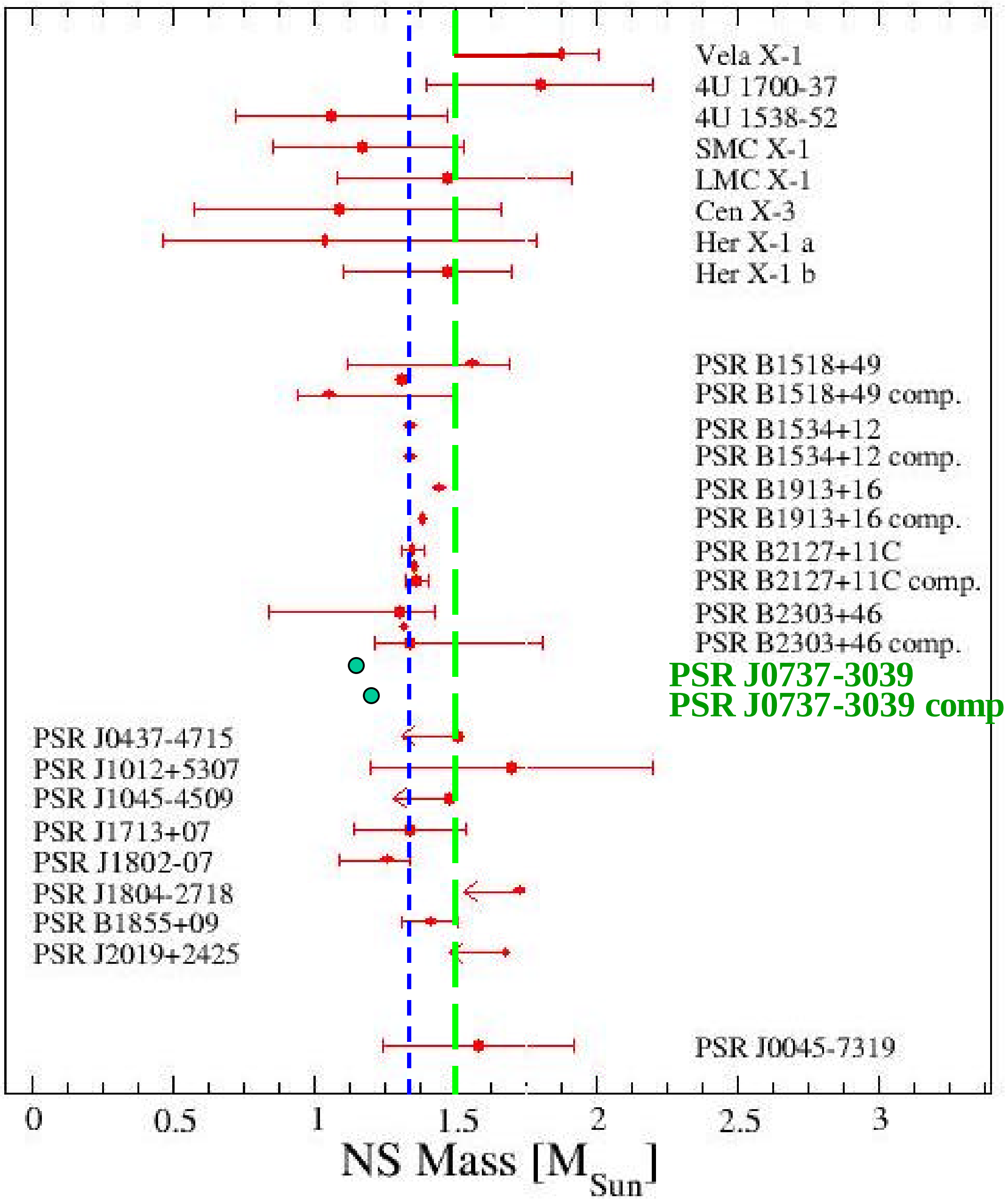
$$\boxed{\dot{\omega} = 16.9^0 / yr}$$

R_{orb} ~ 5.6 ´ 10^5 km **($R_\odot$ = 6.96 ´ 10^5 km)**

dP_b/dt = 0.88 ´ 10^{-15} , **T_{merg} ~ 85 Myr**

$$\boxed{M_1 = 1.34\ M_\odot \qquad M_2 = 1.25\ M_\odot}$$

Measured Neutron Star Masses



$M_{max}\ ^{3}\ M_{measured}$

$M_{max}\ ^{3}\ 1.44\ M_{\odot}$

"very soft" EOS are ruled out

$M_{max}\ ^{3}\ 1.57\ M_{\odot}$

$= M_{low}$(Vela X-1)

Quaintrell et al., 2003, Astron. & Astrophys., 401, 313

PRS J0751+1807: a heavy Neutron Star

NS – WD binary system	(He WD)
P_b = 6 hr	(orbital period)
P = 3.4 ms	(PRS spin period)
$\dot{P}_b = (-6.4 \pm 0.9)\ 10^{-14}$	(orbital decay)

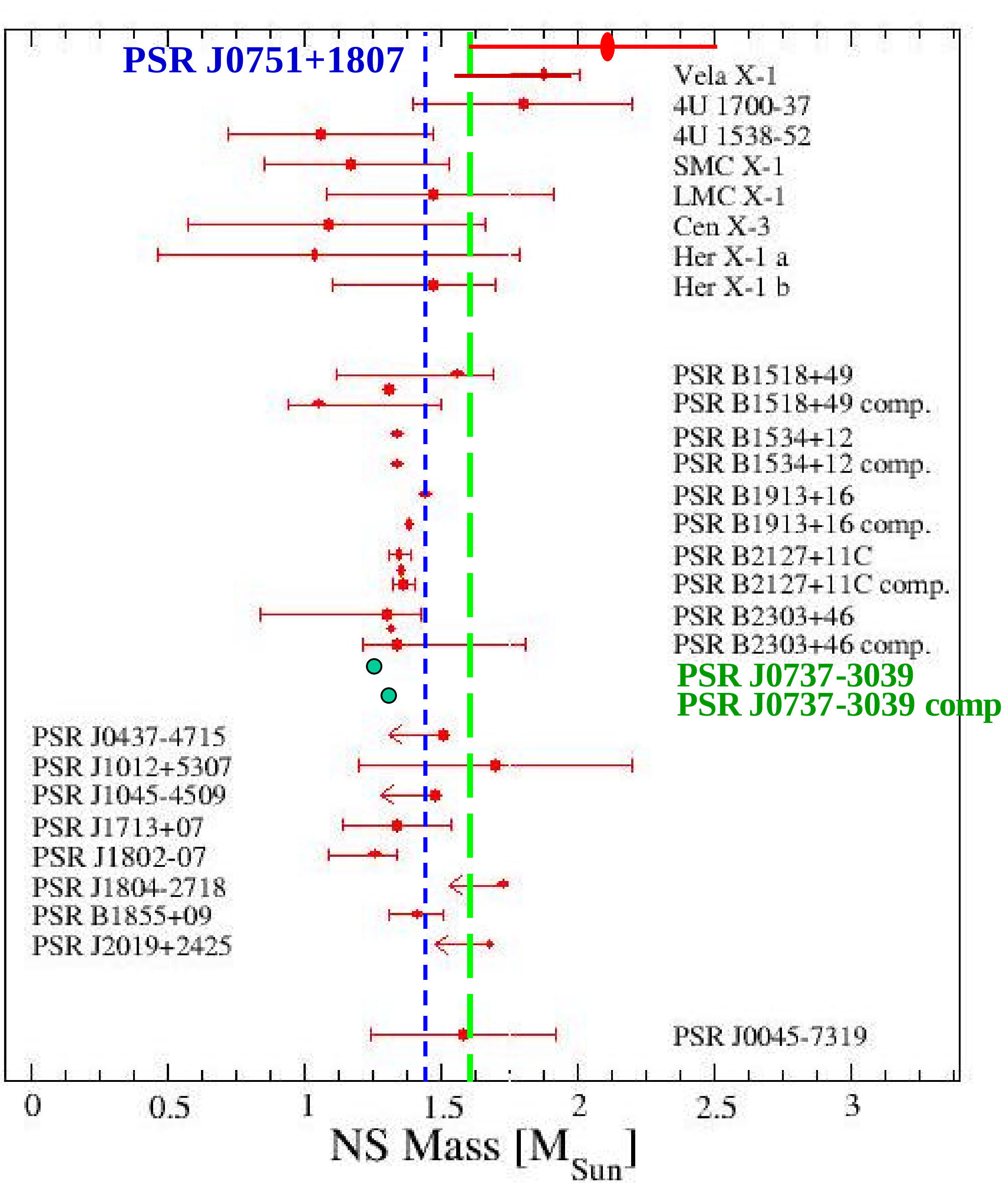
Orbital decay interpreted as due to gravitational wave emission in GR

$M_{NS} = 2.1 \pm 0.2\ M_\odot$ (68% conf. lev.)

$M_{NS} = 2.1 \pm_{0.5}^{0.4}\ M_\odot$ (95% conf. lev.)

D.J. Nice et al., 2005, Astrophys. Jour. 634, 1242

Measured Neutron Star Masses



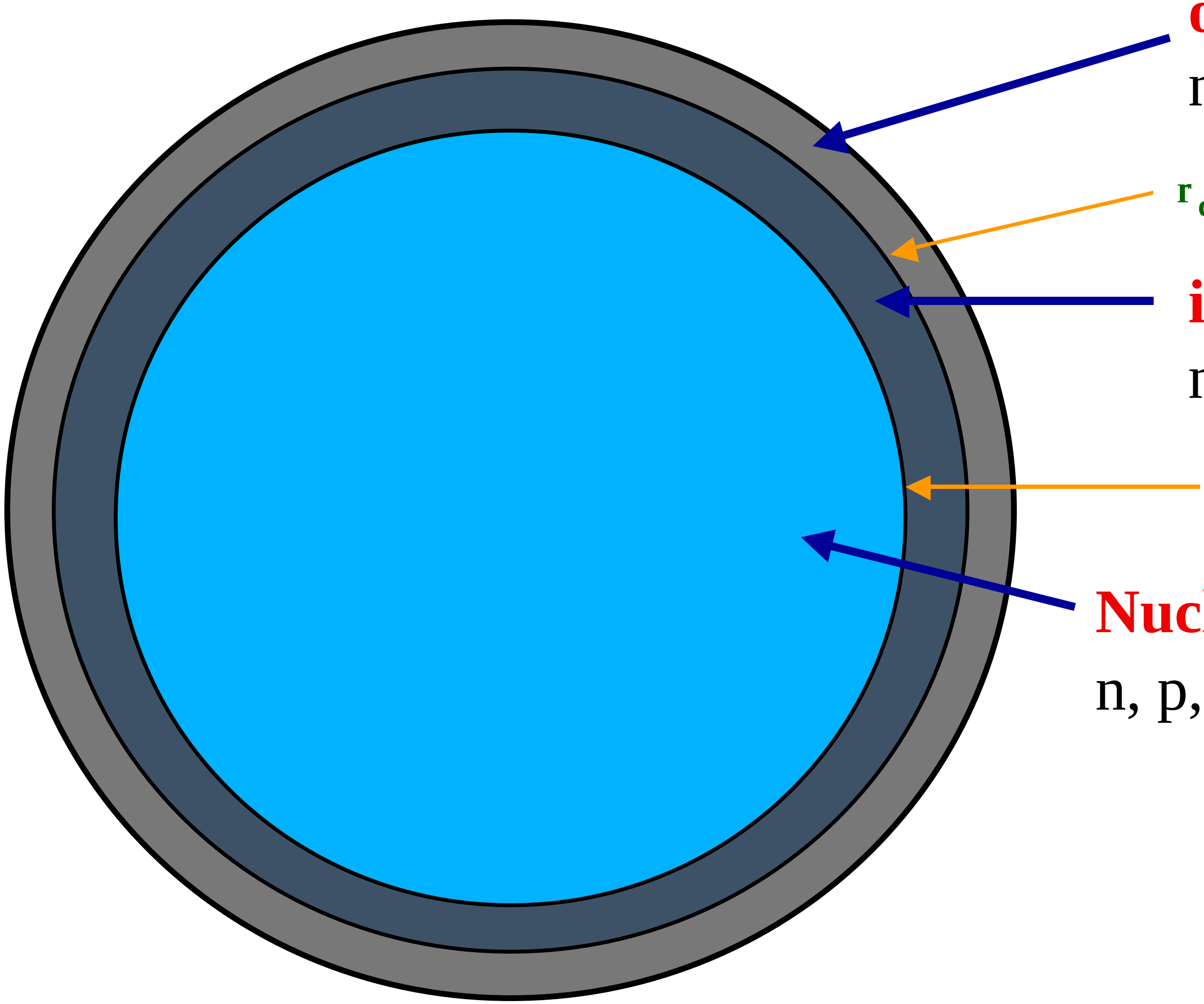
$M_{max} \geq M_{measured}$

$M_{max} \geq 1.6\ M_{\odot}$

$= M_{low}(J0751+1807)$

Nice et al., 2005, ApJ 634

Schematic cross section of a Neutron Star



$M \sim 1.4\, M_\odot$ **$R \sim 10$ km**

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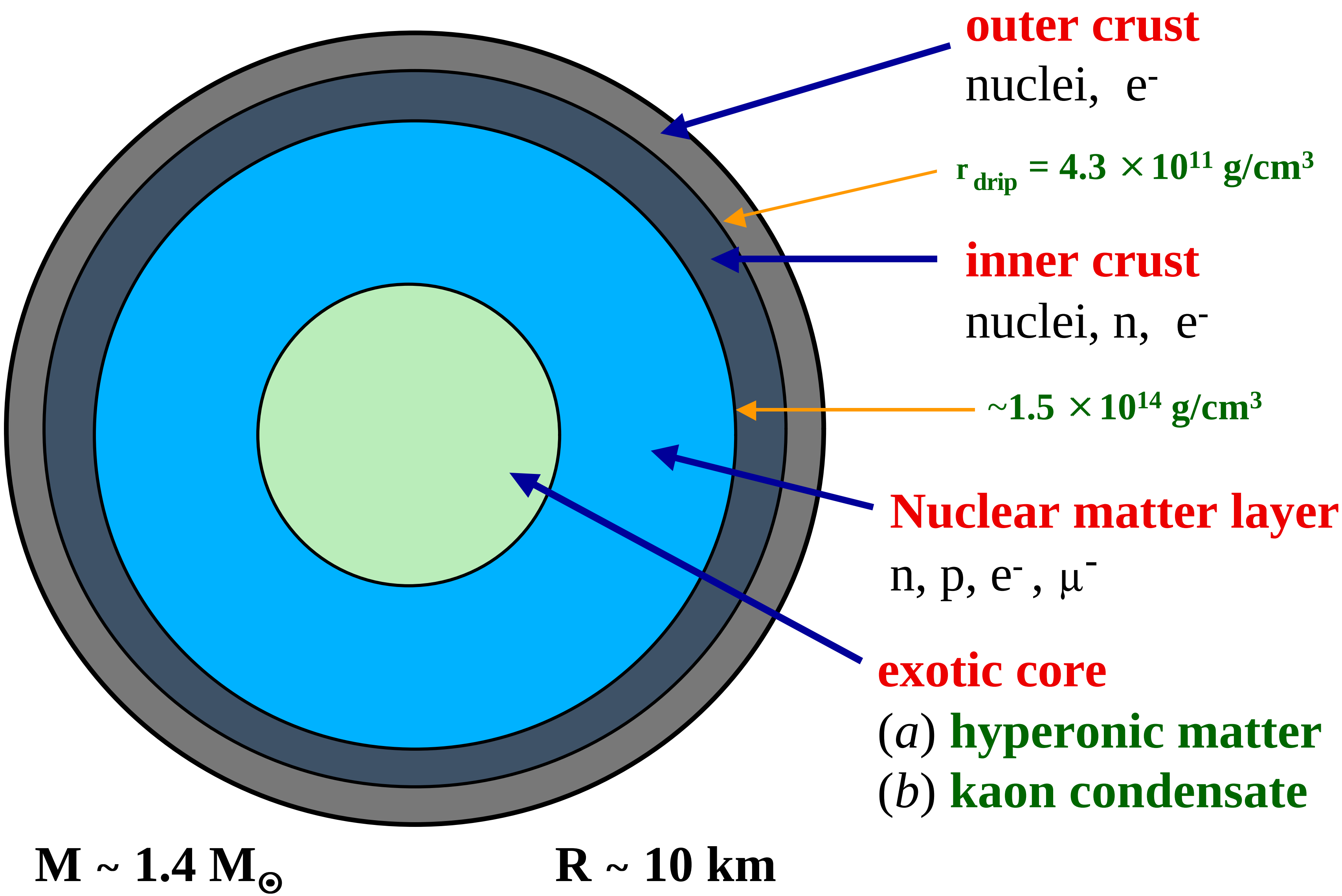
Schematic cross section of a Neutron Star



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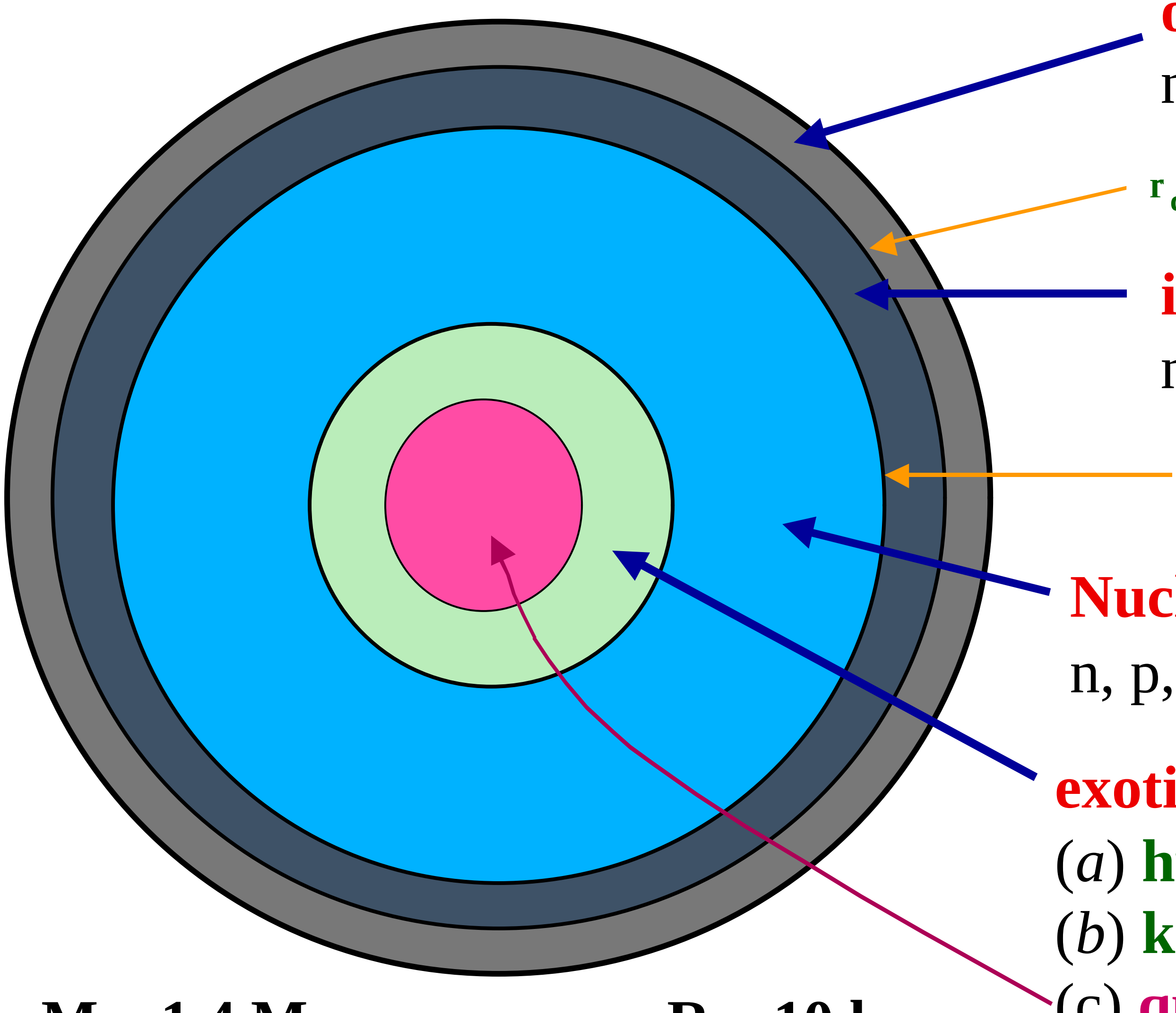
Schematic cross section of a Neutron Star



outer crust
nuclei, e^-

$r_{drip} = 4.3 \times 10^{11}$ g/cm^3

inner crust
nuclei, n, e^-

$\sim 1.5 \times 10^{14}$ g/cm^3

Nuclear matter layer
n, p, e^-, μ^-

exotic core
(*a*) **hyperonic matter**
(*b*) **kaon condensate**
(c) **quark matter**

M ~ 1.4 $M_\odot$ **R ~ 10 km**

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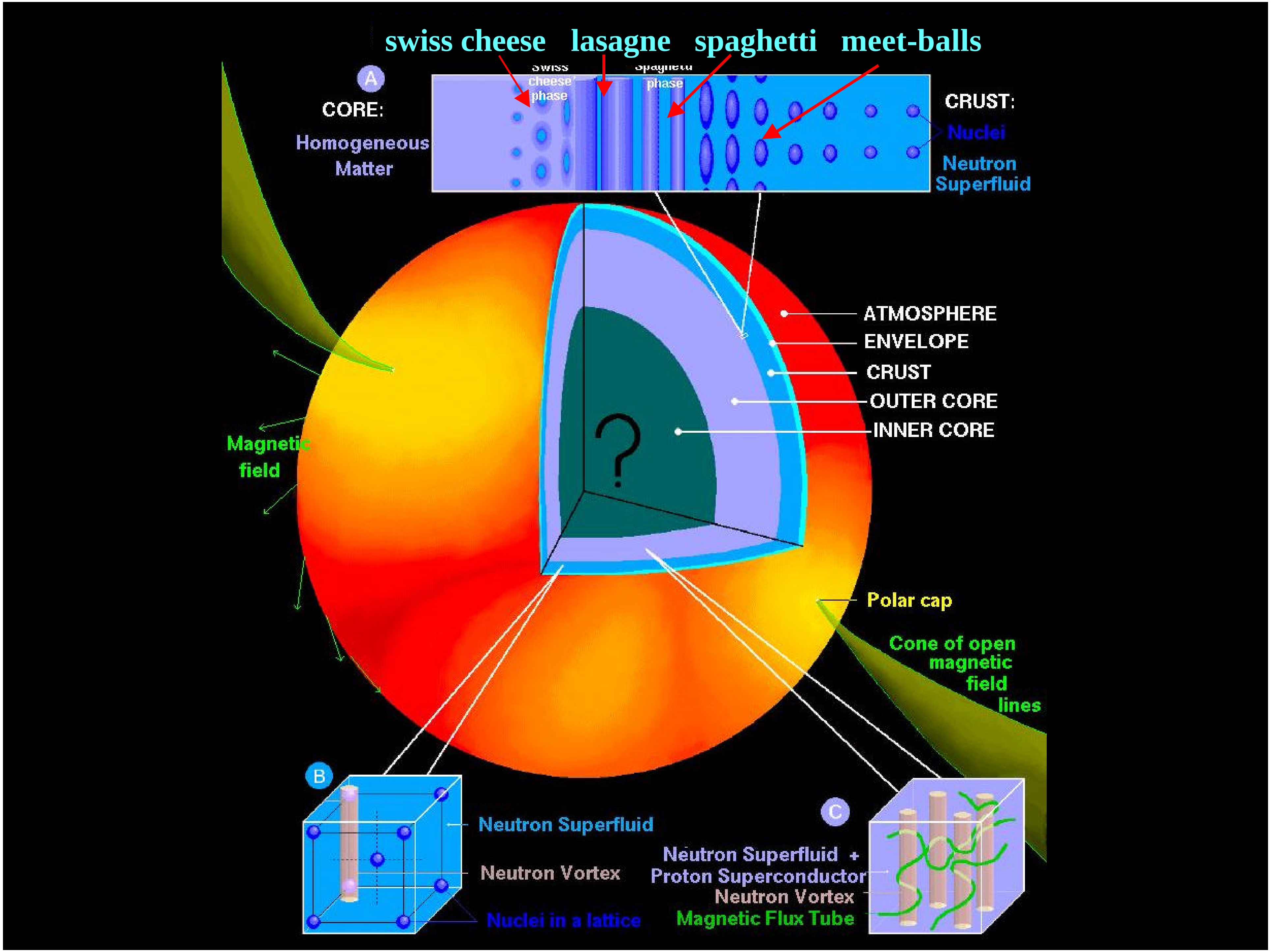
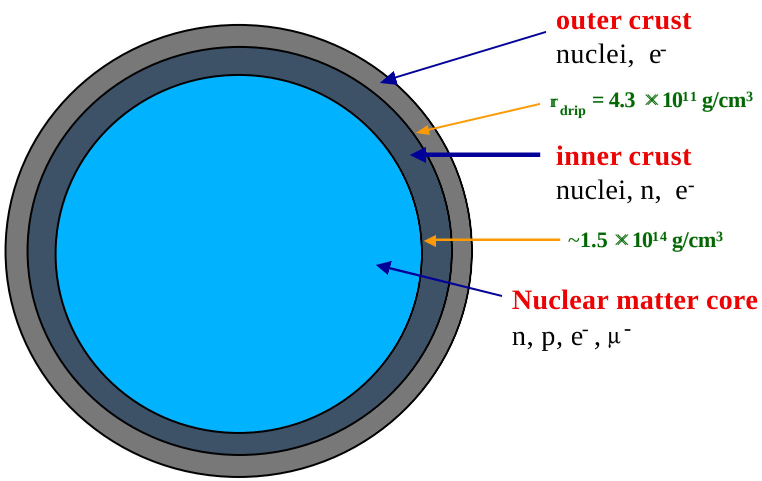

swiss cheese
lasagne
spaghetti
meet-balls
A
CORE:
Homogeneous
Matter
swiss
cheese
phase
spaghetti
phase
CRUST:
Nuclei
Neutron
Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic
field
Polar cap
Cone of open
magnetic
field
lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid +
Proton Superconductor
Neutron Vortex
Magnetic Flux Tube

Neutron Stars with a nuclear matter core

As we have already seen due to the weak interaction, the core of a Neutron Star can not be made of pure neutron matter.

Core constituents: n, p, e^-, m^-



b-stable nuclear matter

$$p + e^- \leftrightarrow n + \nu_e$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

$$if \quad \mu_e \geq m_\mu = 105.6 MeV$$

$$e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

$$p + \mu^- \leftrightarrow n + \nu_\mu$$

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$

neutrino-free matter

- **Equilibrium with respect to the weak interaction processes**

$$\mu_n - \mu_p = \mu_e$$

$$\mu_\mu = \mu_e$$

- **Charge neutrality**

$$n_p = n_e + n_\mu$$

To be solved for any given value of the total baryon number density n_B

Proton fraction in β-stable nuclear matter and role of the nuclear symmetry energy

$$\hat{\mu} \equiv \mu_n - \mu_p = -\frac{\partial (E/A)}{\partial x} = 2\frac{\partial (E/A)}{\partial \beta}$$

$$E_{sym}(n) \equiv \frac{1}{2}\frac{\partial^2 (E/A)}{\partial \beta^2}\Big|_{\beta=0}$$

$\beta = (n_n - n_p)/n = 1 - 2x$ asymmetry paramter

$n = n_n + n_p$ total baryon density

$x = n_p/n$ proton fraction

Energy per nucleon for asymmetric nuclear matter(*)

$$E(n,\beta)/A = E(n,\beta=0)/A + E_{sym}(n)\,\beta^2$$

$\beta = 0$ **symm nucl matter**

$\beta = 1$ **pure neutron matter**

$$E_{sym}(n) = E(n,\beta=1)/A - E(n,\beta=0)/A$$

$$\Rightarrow \quad \hat{\mu} = 4\,E_{sym}(n)\,[1-2x]$$

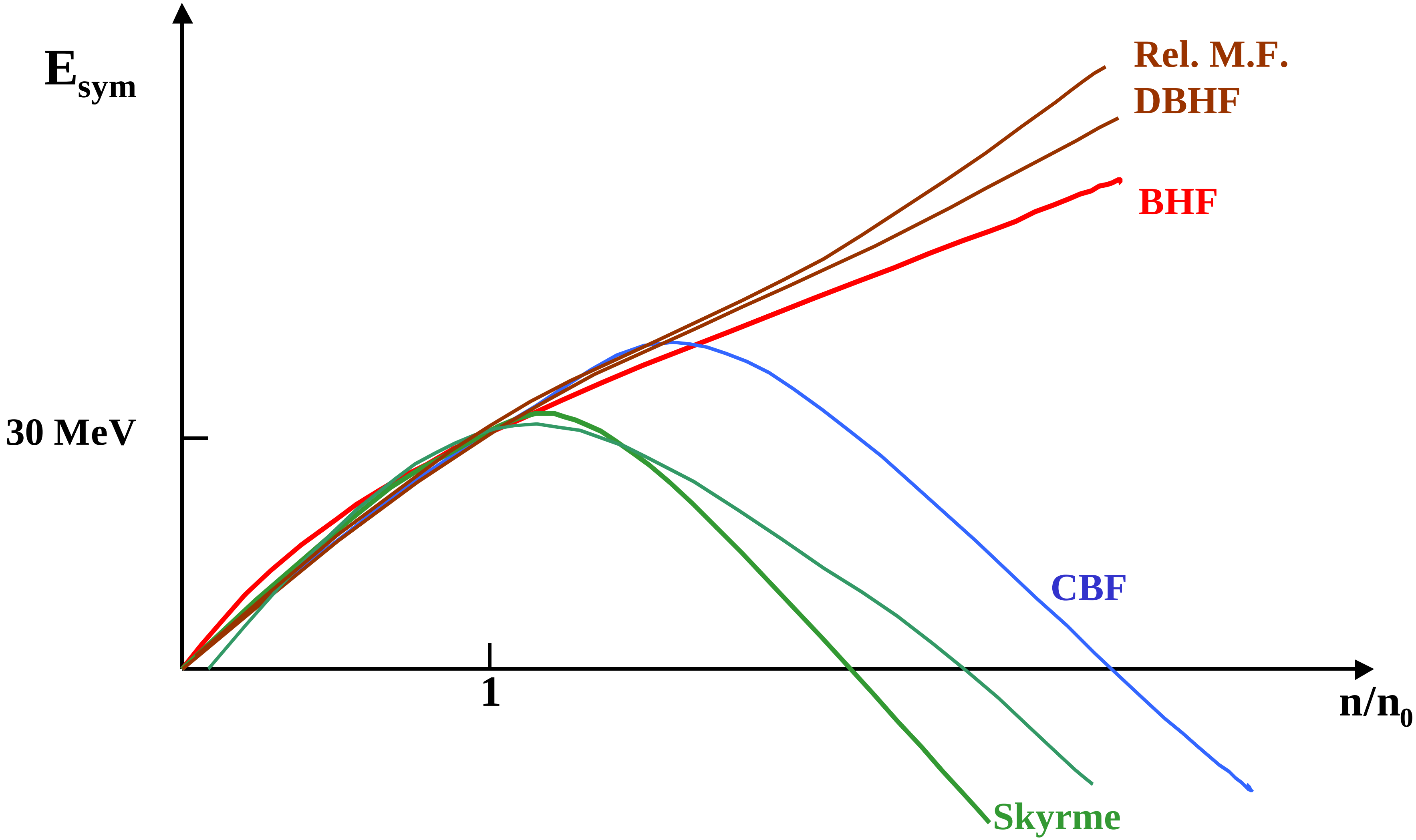
Chemical equil. + charge neutrality (no muons)

$$3\pi^2(\;\;c)^3\, n\, x(n) - [4\,E_{sym}(n)\,(1-2\,x(n))]^3 = 0$$

The composition of β-stable nuclear matter is strongly dependent on the nuclear symmetry energy.

(*) Bombaci, Lombardo, Phys. Rev: C44 (1991)

Schematic behaviour of the nuclear symmetry energy



Microscopic EOS for nuclear matter: Brueckner-Bethe-Goldstone theory

$$G_{\tau\tau'}(\omega) = V + V \sum_{k_a k_b} \frac{|k_a k_b\rangle Q_{\tau\tau'} \langle k_a k_b|}{\omega - e_\tau(k_a) - e_{\tau'}(k_b)} G_{\tau\tau'}(\omega)$$

$$e_\tau(k) = \frac{\hbar^2 k^2}{2M} + U_\tau(k)$$

$$U_\tau(k) = \sum_{\tau'} \sum_{k'} \langle \vec{k}\vec{k}' | G_{\tau\tau'}(e_\tau + e_{\tau'}) | \vec{k}\vec{k}' \rangle$$

V is the **nucleon-nucleon interaction** (*e.g.* **the Argonne v14, Paris, Bonn potential**) **plus a density depependent Three-Body Force (TBF) necessary to reproduce the empirical saturation on nuclear matter**

- **Energy per baryon in the Brueckner-Hartree-Fock (BHF) approximation**

$$\frac{E}{A} = \frac{1}{A} \sum_{\tau} \sum_{k} \frac{\hbar^2 k^2}{2M} + \frac{1}{2A} \sum_{\tau} \sum_{k} U_\tau(k)$$

The EOS for b-stable matter

Pressure:

$$P_{nucl}(n) = n^2 \frac{d(E/A)}{dn}$$

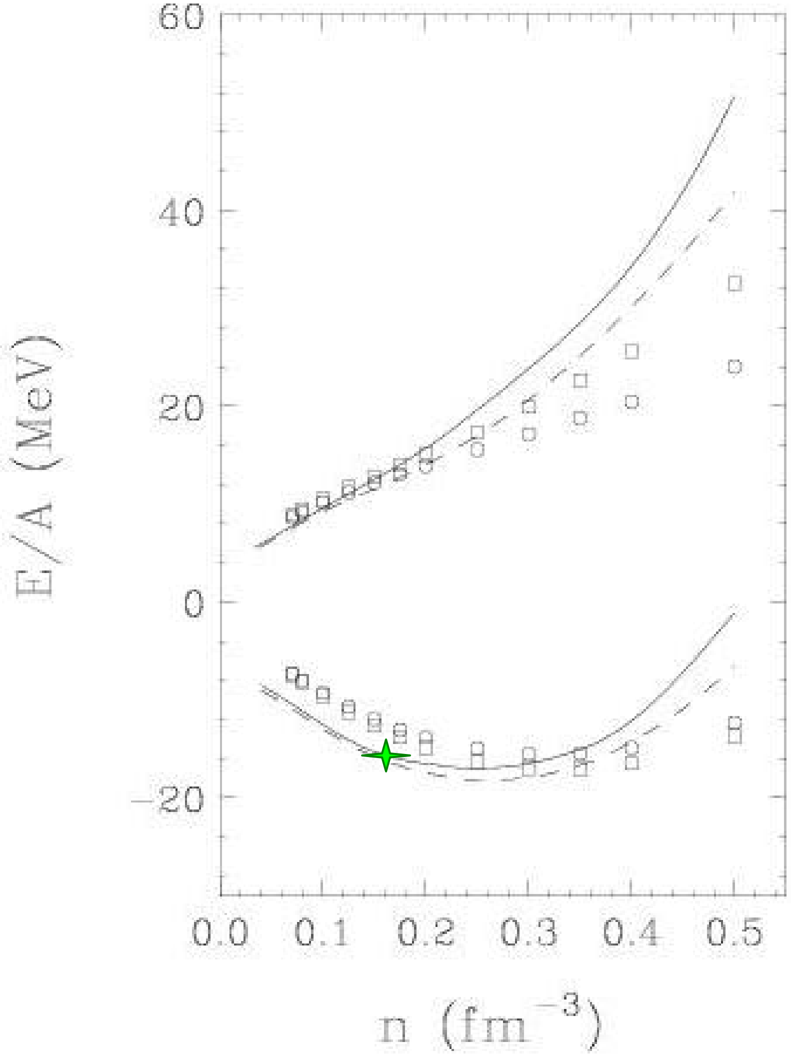
$$P = P_{nucl} + P_{lep}$$

Mass density:

$$\rho \;=\; \frac{1}{c^2}\left(\varepsilon_{nucl} + \varepsilon_{lep}\right) \;=\; \frac{1}{c^2}\left(n\frac{E}{A} + m_N c^2 n + \varepsilon_{lep} \right)$$

Leptons are treated as **non-interacting relativistic fermionic gases**

Energy per baryon
(two body forces only)



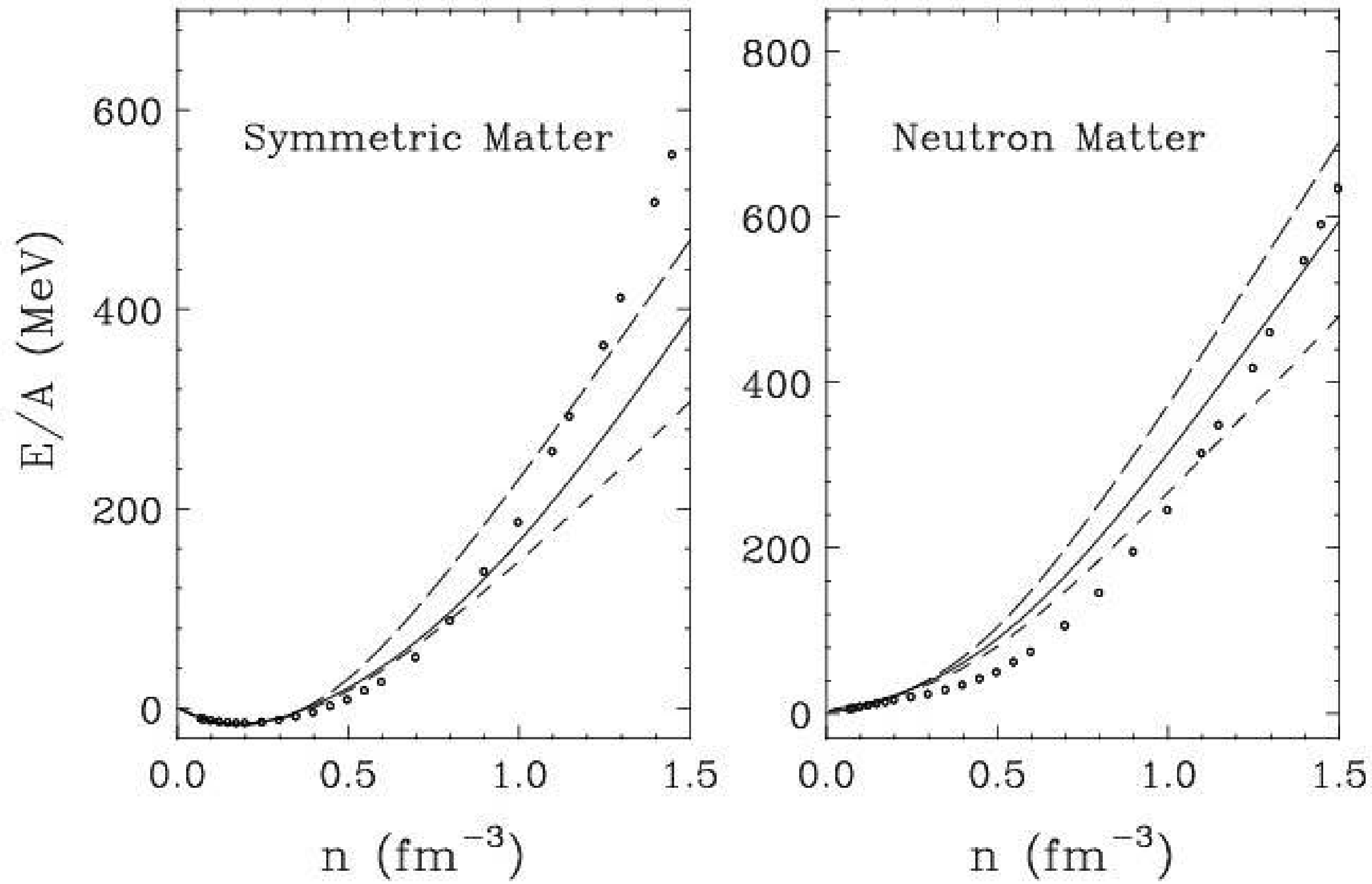
Upper curves: neutron matter
lower curves: symmetric nuclear matter

Empirical saturation point

BHF with A14
BHF with Paris
WFF: CBF with U14
WFF: CBF with A14

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Energy per baryon



BBB1: BHF with A14+TBF

BBB2: BHF with Paris+TBF

DBHF: Bonn A

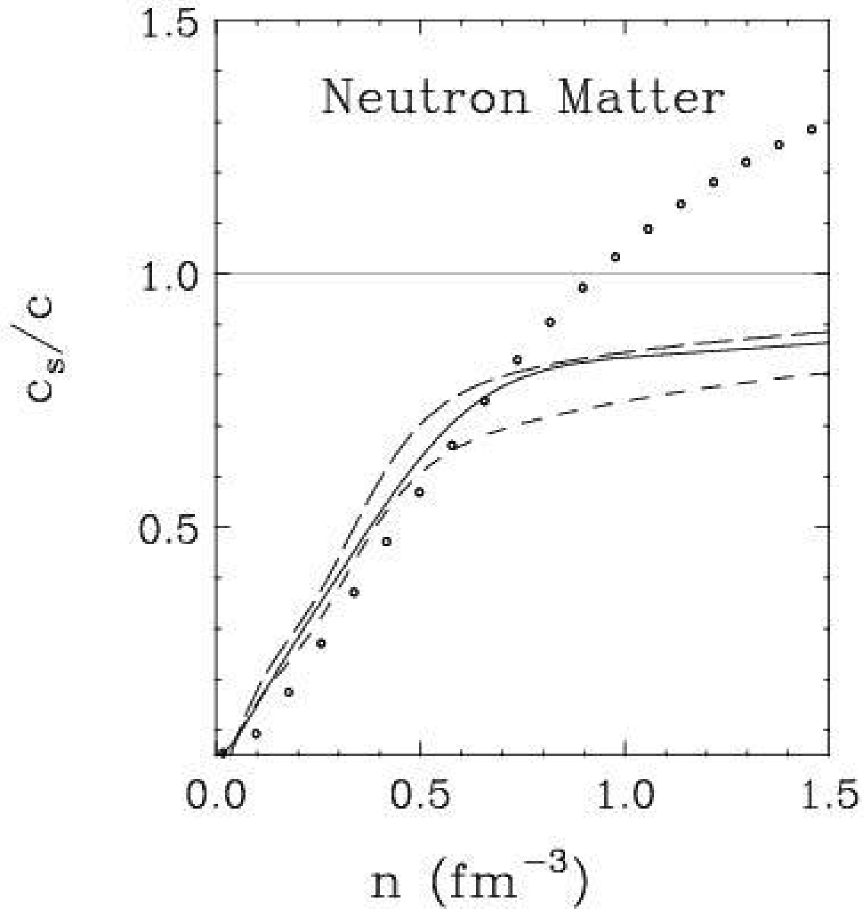
WFF: CBF with A14+TBF

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Saturation properties BHF EOS (with TBF)

EOS	n_0 (fm^{-3})	E_0/A (MeV)	K (MeV)
A14+TBF	0.178	-16.46	253
Paris+TBF	0.176	-16.01	281
empirical saturation	0.17 ± 0.1	– 16 ± 1	220 ± 20

Speed of sound



BBB1: BHF with A14+TBF
BBB2: BHF with Paris+TBF
DBHF: Bonn A
WFF: CBF with A14+TBF

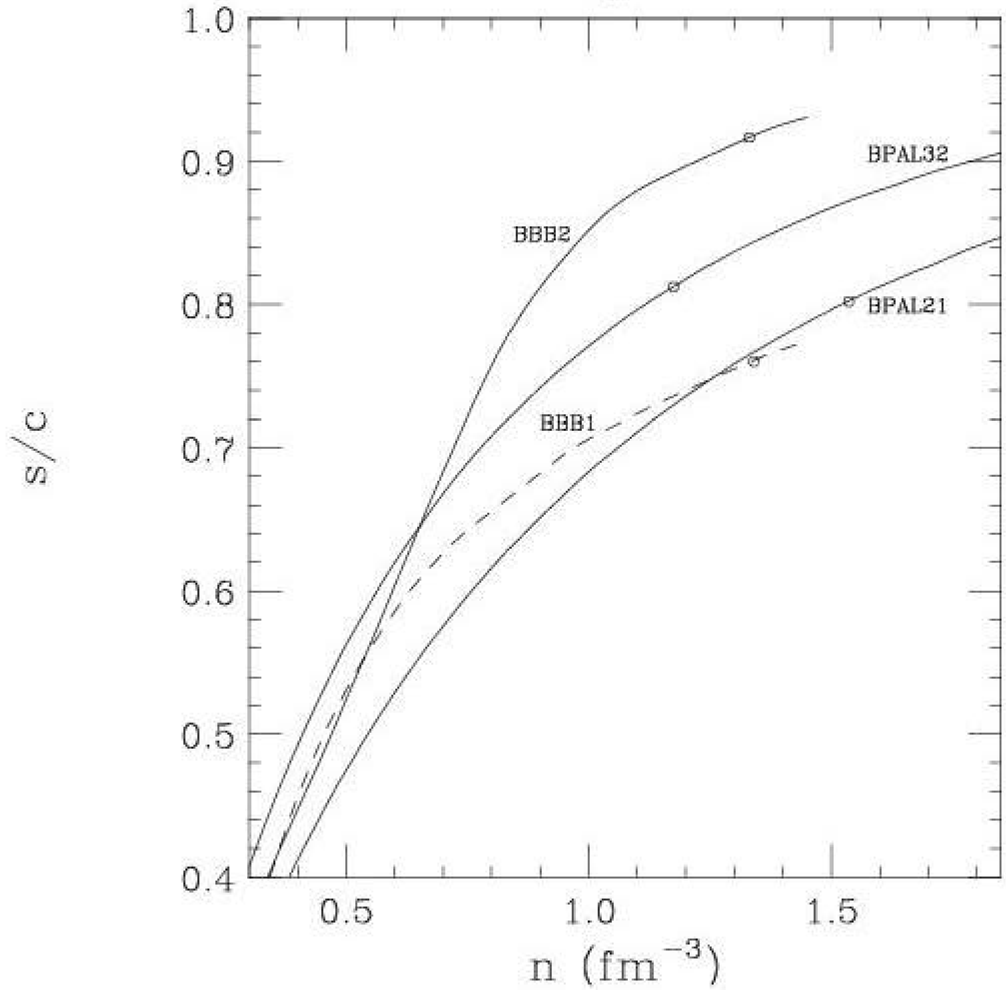
At high density extrapolation using

$$E/A = Q^{1}(n) \,/\, (1 + b\, n^{1})$$

$Q^{\lambda}(n)$ = polinomial of degree λ

Baldo, Bombaci, Burgio, A&A 328, (1997)

Speed of sound in beta-stable nuclear matter



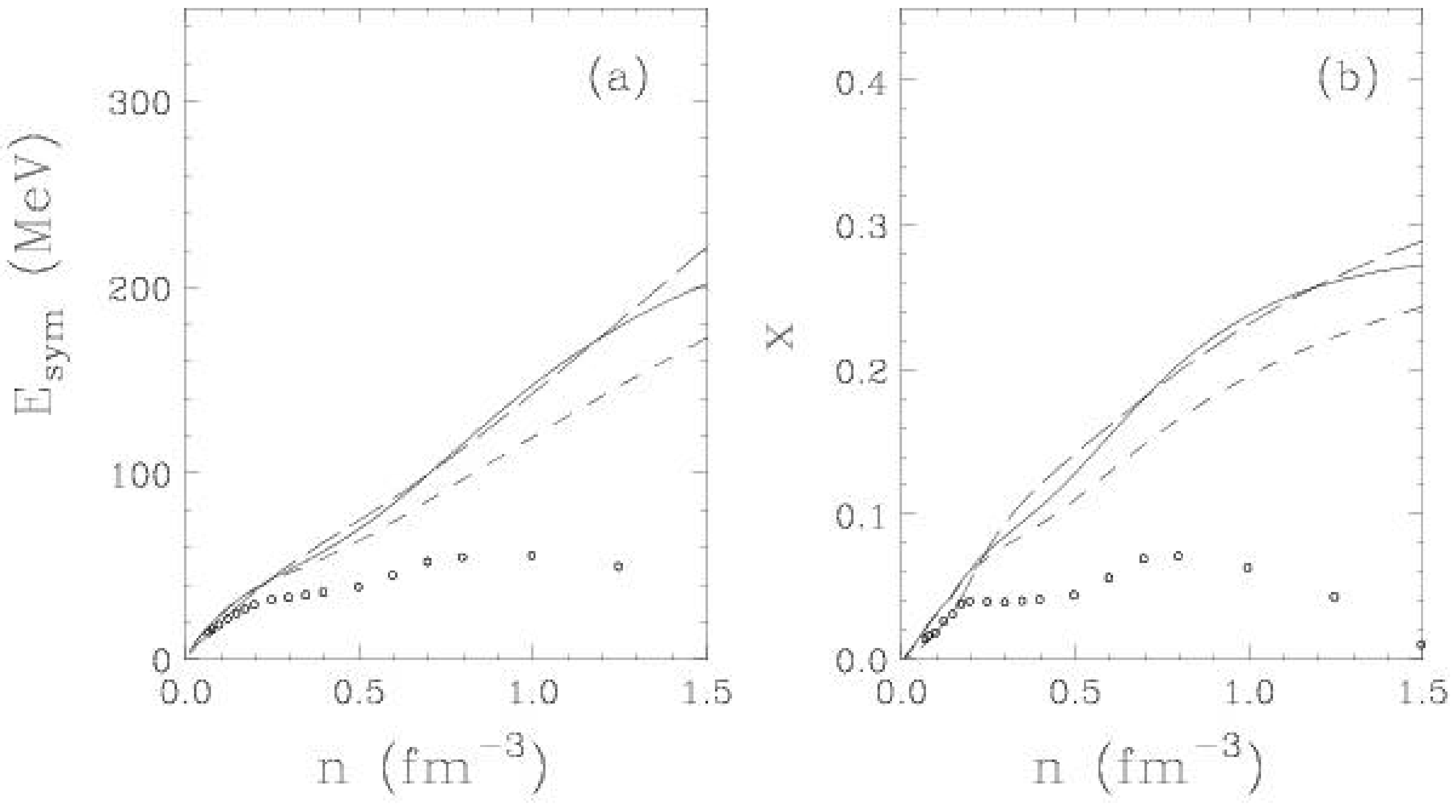
BBB1: BHF with A14+TBF

BBB2: BHF with Paris+TBF

BPAL: phenomen. EOS

Symmetry energy

Proton fraction in b-stable nucl. matter



BBB1: BHF with A14+TBF - - - - - - - -

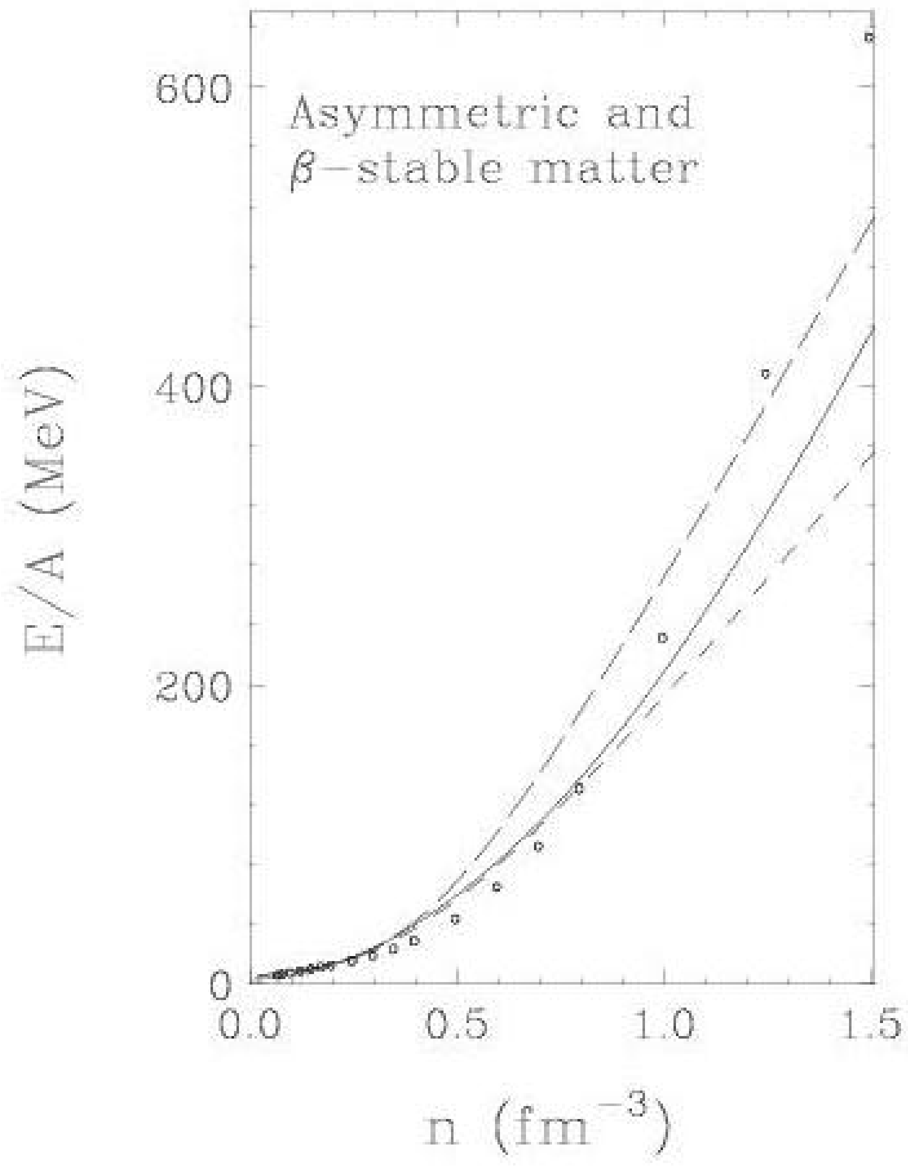
BBB2: BHF with Paris+TBF ————————

DBHF: Bonn A — — — —

WFF: CBF with A14+TBF o o o o o o o o o o

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

E/A in b-stable nuclear matter



BBB1: BHF with A14+TBF

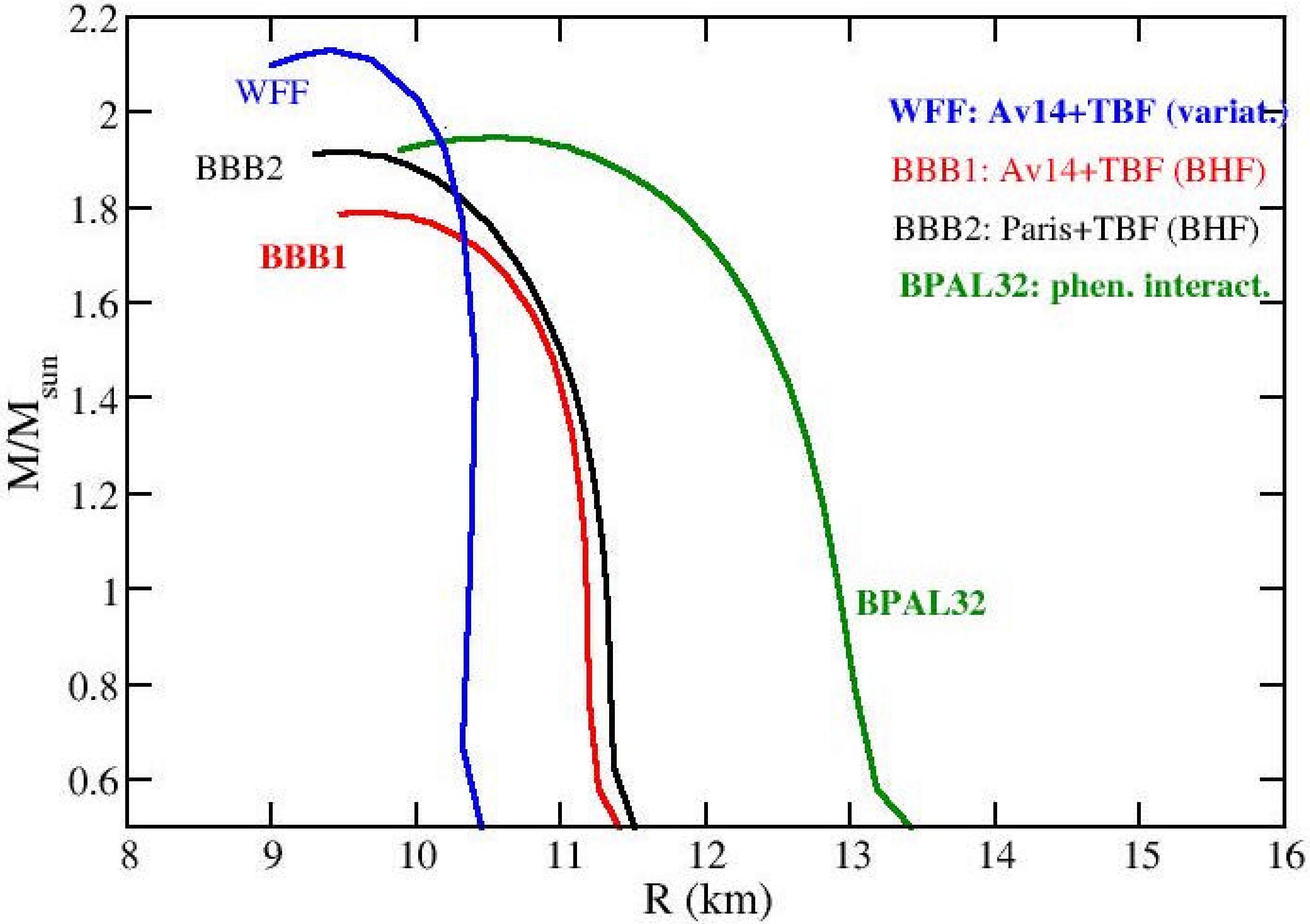
BBB2: BHF with Paris+TBF

DBHF: Bonn A

WFF: CBF with A14+TBF

Baldo, Bombaci, Burgio, Astr. & Astrophys. 328, (1997)

Mass-Radius relation for *nucleonic* Neutron Stars



WFF: Wiringa-Ficks-Fabrocini, 1988.
BPAL: Bombaci, 1995.
BBB: Baldo-Bombaci-Burgio, 1997.

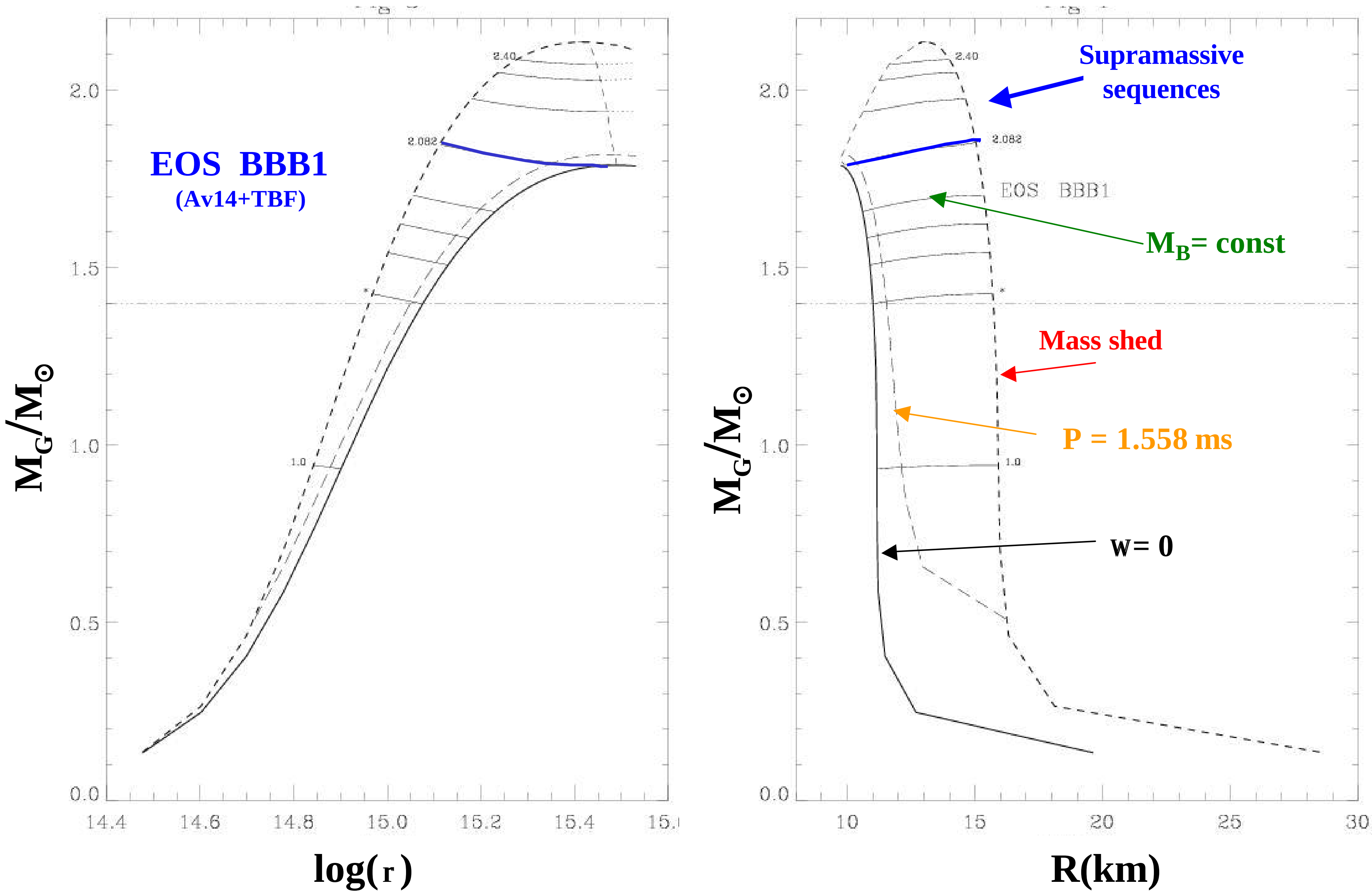
Maximum mass configuration of pure nucleonic Neutron Stars for different EOS

EOS	$M_G/M_\odot$	R(km)	n_c / n_0
BBB1	1.79	9.66	8.53
BBB2	1.92	9.49	8.45
WFF	2.13	9.40	7.81
BPAL12	1.46	9.04	10.99
BPAL22	1.74	9.83	9.00
BPAL32	1.95	10.54	7.58

Properties of neutron stars with $M_G = 1.4\ M_\odot$			
EOS	R(km)	n_c / n_0	x_c
BBB1	11.0	4.06	0.139
BBB2	11.1	4.00	0.165
WFF	10.41	4.13	0.066

Crustal properties of neutron stars with $M_G = 1.4\ M_\odot$						
EOS	r_c (10^{15} g/cm^3)	R(km)	R_{core}	$D R_{inner}$	$D R_{outer}$	$D R_{crust}$
BPAL12	2.5	9.98	8.56	1.15	0.27	1.42
BPAL22	1.2	11.81	9.63	1.75	0.43	2.18
BPAL32	0.9	12.60	10.06	2.05	0.49	2.54

Rotating Neutron Stars



Datta, Thampan, Bombaci, Astron. and Astrophys. 334 (1998)

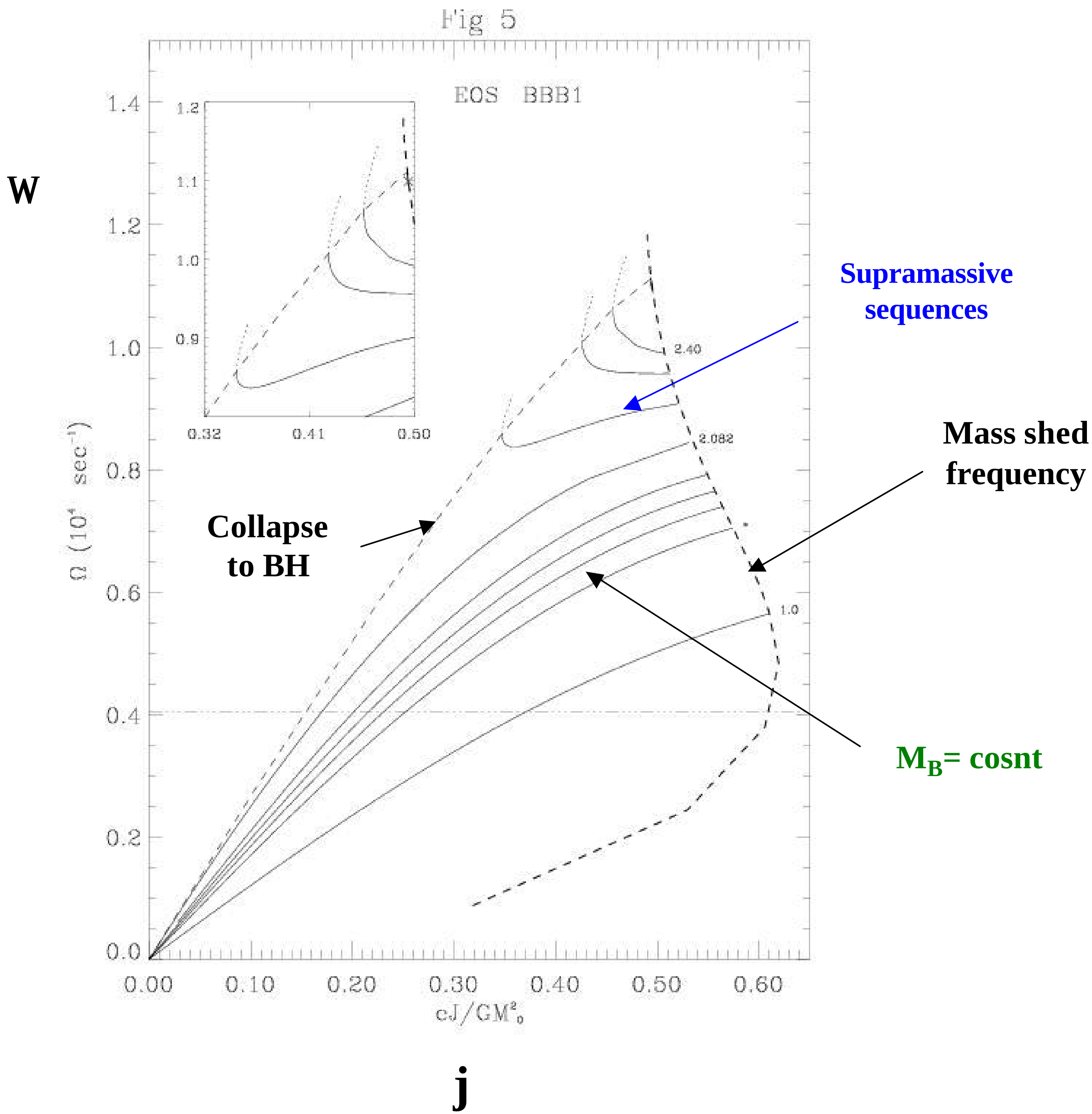


Fig 5
EOS BBB1
W
Ω (10⁴ sec⁻¹)
cJ/GM²₀
j
Supramassive sequences
Mass shed frequency
Collapse to BH
M_B= cosnt
2.40
2.082
1.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.9
1.0
1.1
1.2
0.32
0.41
0.50

Neutron Stars or Hyperon Stars

Why is it very likely to have hyperons in the core of a Neutron Star?

(1) The central density of a Neutron Star is “high”

$$\mathbf{r_c \text{ » } (4 - 10)\ r_0} \qquad\qquad (\rho_0 = 0.17\ \text{fm}^{-3})$$

(2) The nucleon chemical potentials increase very rapidly as function of density.

Above a threshold density ($\mathbf{r_c \text{ » } (2 - 3)\ r_0}$) hyperons are created in the stellar interior.

A. Ambarsumyan, G.S. Saakyan, (1960)
V.R. Pandharipande (1971)

Threshold density for hyperons in neutron matter

- **Non-relativistic free Fermi neutron gas**

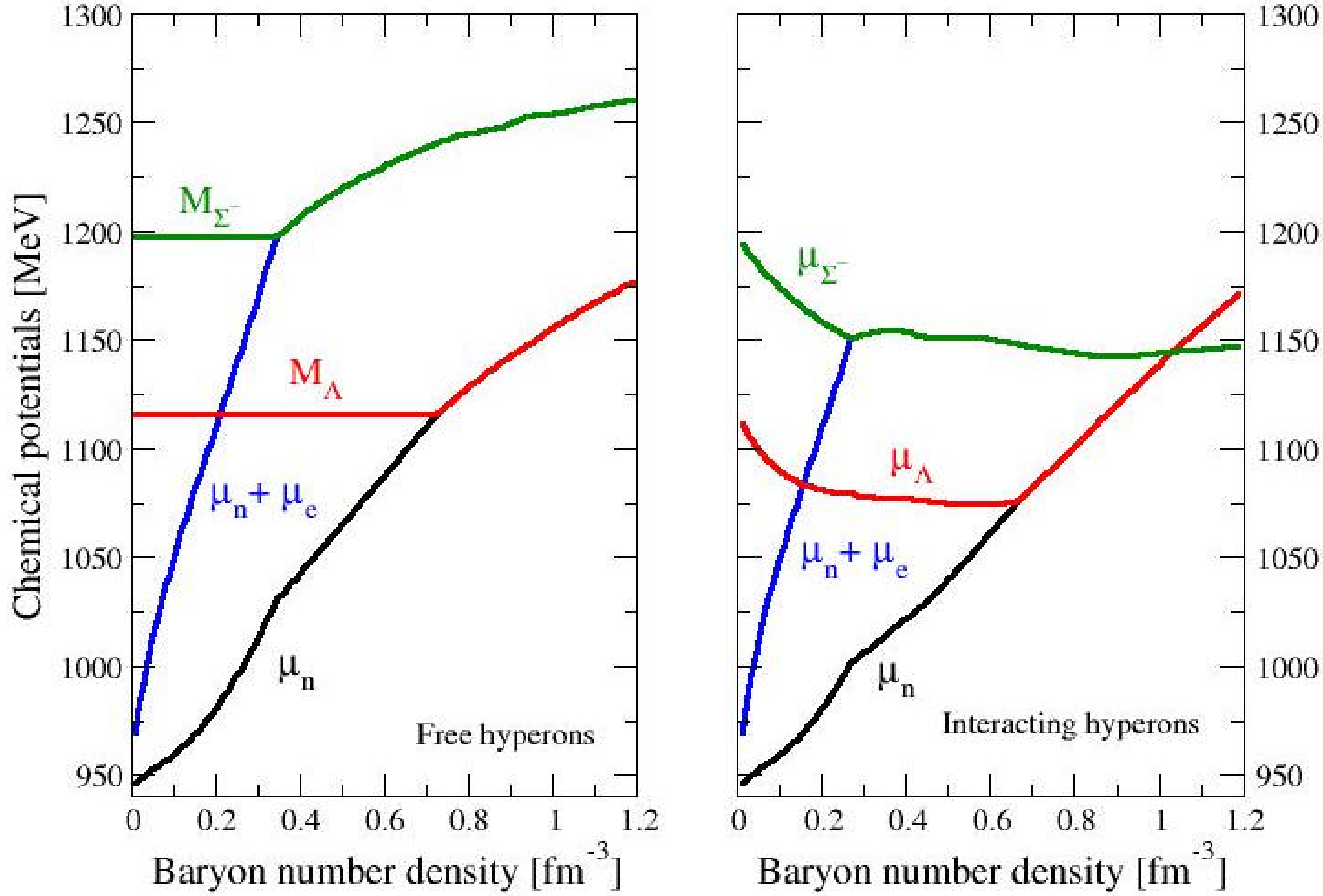
$$\frac{\hbar^2 k_{F_n}^2}{2m_n} + m_n c^2 \geq m_\Lambda c^2 \qquad\qquad n_n = \frac{k_{F_n}^3}{3\pi^2}$$

$$\boxed{n_{cr} = \frac{1}{3\pi^2}\left\{\frac{\left[2m_n c^2 (m_\Lambda - m_n)c^2\right]^{1/2}}{\hbar c}\right\}^3}$$

$m_L = 1115.68$ MeV/c^2 **$m_n = 939.56$ MeV/c^2**

$\boxed{\mathbf{n_{cr} = 0.837\ fm^{-3}}}$ $\boxed{\mathbf{n_{cr}/n_0 = 5.23}}$ **$n_0 = 0.16$ fm^{-3}**

Baryon chemical potentials in dense hyperonic matter



$$\mu_n = \mu_\Lambda$$

n + e$^-$ ® S$^-$ + n$_e$ $\quad \mu_n + \mu_e = \mu_{\Sigma^-}$

I. Vidaña, Ph.D. thesis (2001)

Microscopic EOS for hyperonic matter: extended Brueckner theory

$$G(\omega)_{B_1B_2B_3B_4} = V_{B_1B_2B_3B_4} + \sum_{B_5B_6} V_{B_1B_2B_5B_6} \frac{Q_{B_5B_6}}{\omega - e_{B_5} - e_{B_6}} G(\omega)_{B_5B_6B_3B_4}$$

$$e_{B_i}(k) = M_{B_i}c^2 + \frac{\hbar^2k^2}{2M_{B_i}} + U_{B_i}(k)$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k' \le k_{F\,B_j}} \langle \vec{k}\vec{k}' | G_{B_iB_jB_iB_j}(\omega = e_{B_i} + e_{B_j}) | \vec{k}\vec{k}' \rangle$$

V is the **baryon--baryon interaction for the baryon octet (n, p, L, s$^-$, s^0, s$^+$, x$^-$, x^0) (e.g. the Nijmegen potential).**

- **Energy per baryon in the BHF approximation**

$$E/N_B = 2\sum_{B_i} \int_0^{k_F[B_i]} \frac{d^3k}{(2\pi)^3} \left\{ M_{B_i}c^2 + \frac{\hbar^2k^2}{2M_{B_i}} + \frac{1}{2}U^N_{B_i}(k) + \frac{1}{2}U^Y_{B_i}(k) \right\}$$

Baldo, Burgio, Schulze, Phys.Rev. C61 (2000) 055801;
Vidaña, Polls, Ramos, Engvik, Hjorth-Jensen, Phys.Rev. C62 (2000) 035801;
Vidaña, Bombaci, Polls, Ramos, Astron. Astrophys. 399, (2003) 687.

b-stable hadronic matter

Equilibrium with respect to the weak interaction processes

$$\mu_p = \mu_n - \mu_e = \mu_{\Sigma^+}$$

$$\mu_n = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda$$

$$\mu_n + \mu_e = \mu_{\Sigma^-} = \mu_{\Xi^-}$$

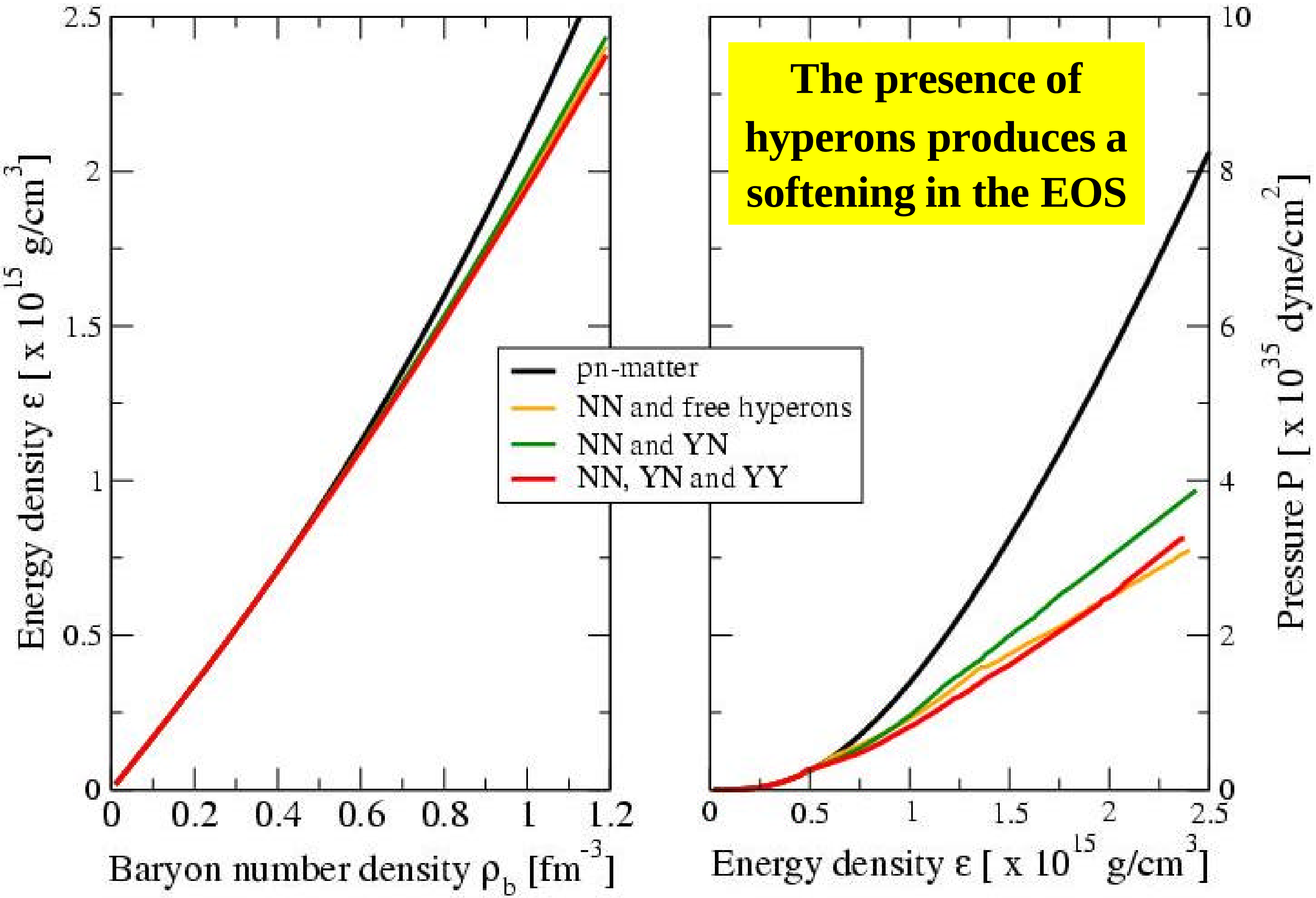
$$\mu_\mu = \mu_e$$

Charge neutrality

$$n_p + n_{\Sigma^+} = n_e + n_\mu + n_{\Sigma^-} + n_{\Xi^-}$$

For any given value of the total baryon number density n_B

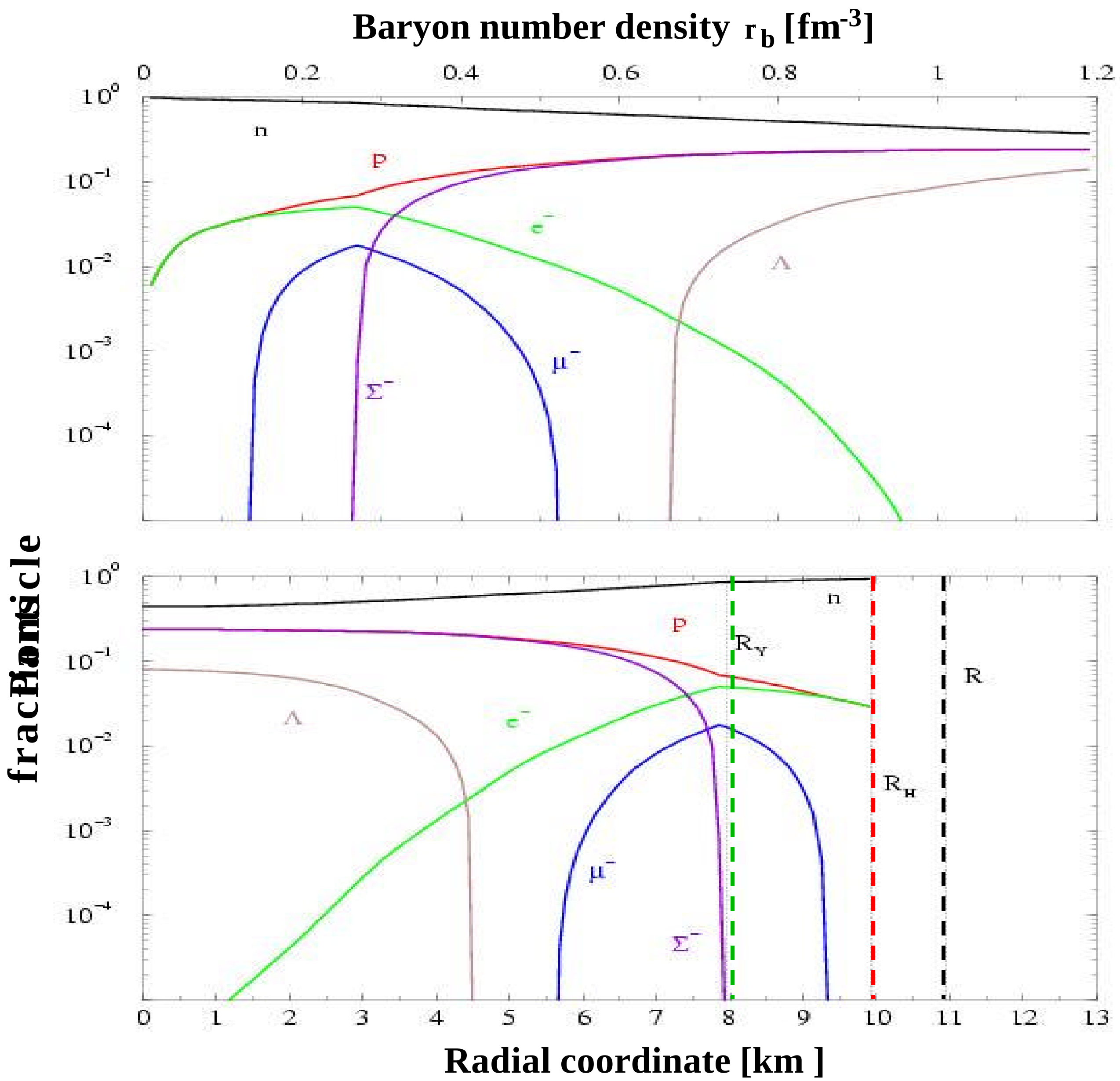
The Equation of State of Hyperonic Matter



NSC97e

I. Vidaña et al., Phys. Rev: C62 (2000) 035801

Composition of hyperonic beta-stable matter

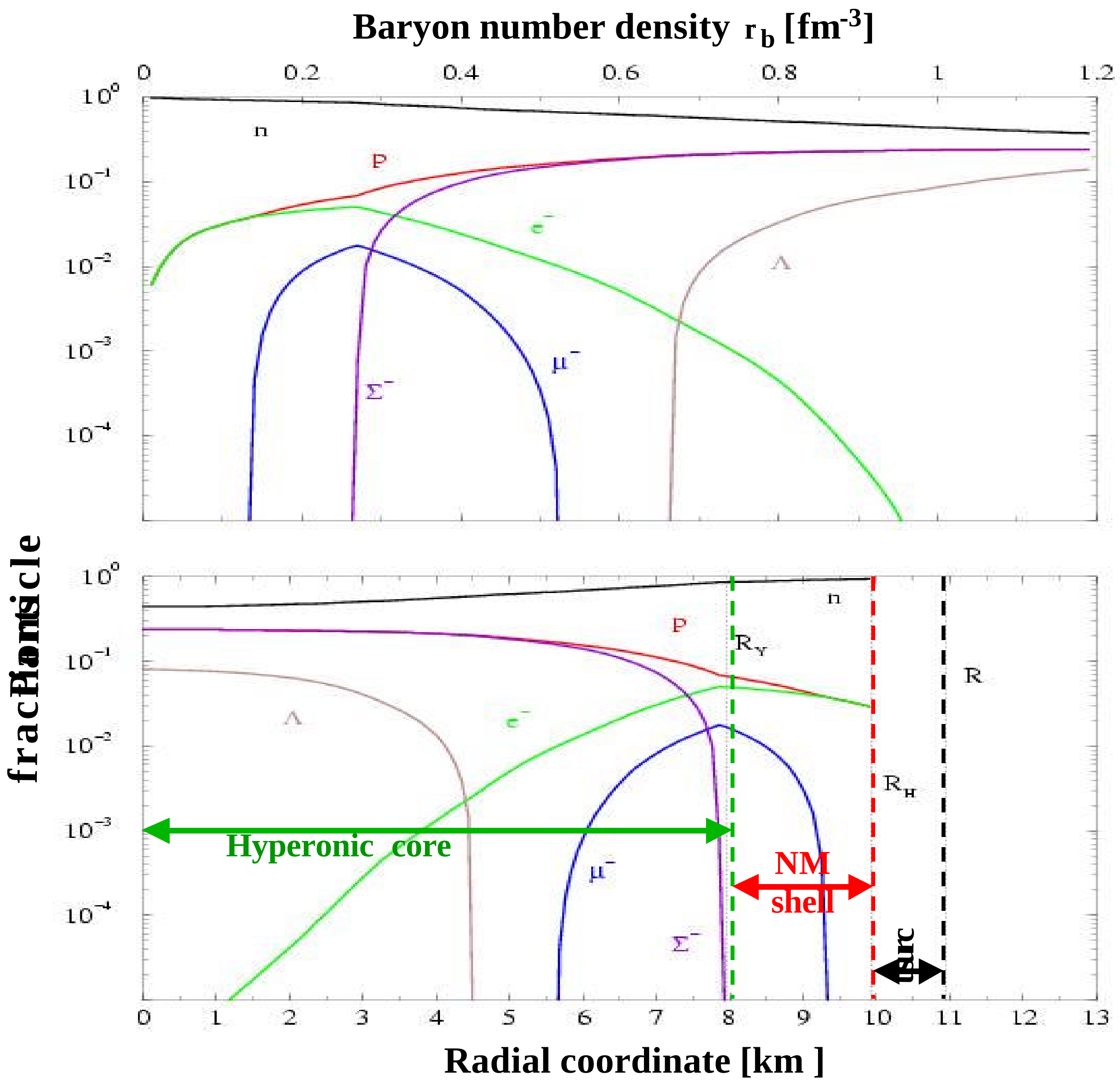


Hyperonic Star

$M_B = 1.34\ M_\odot$

I. Vidaña, I. Bombaci,
A. Polls, A. Ramos,
Astron. and Astrophys.
399 (2003) 687

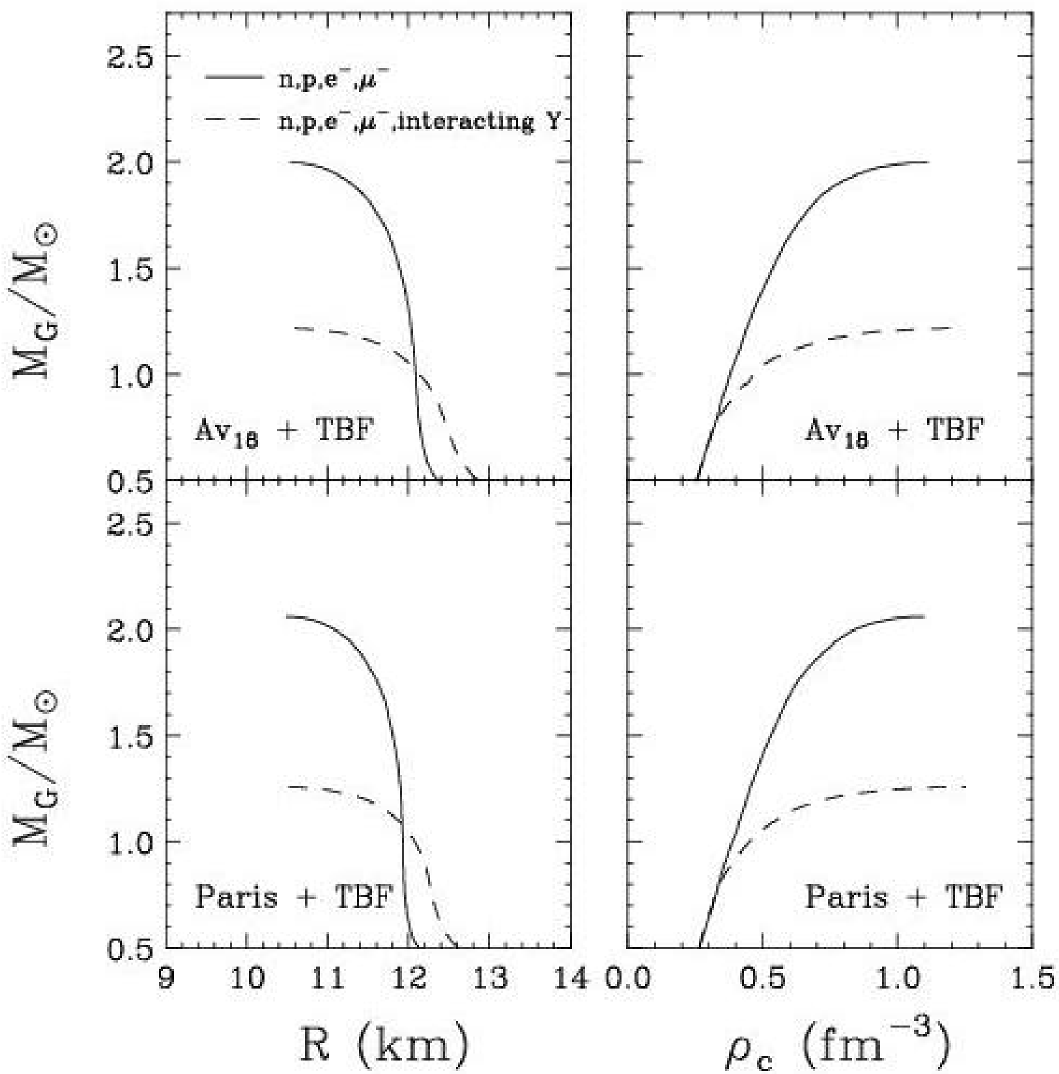
Composition of hyperonic beta-stable matter



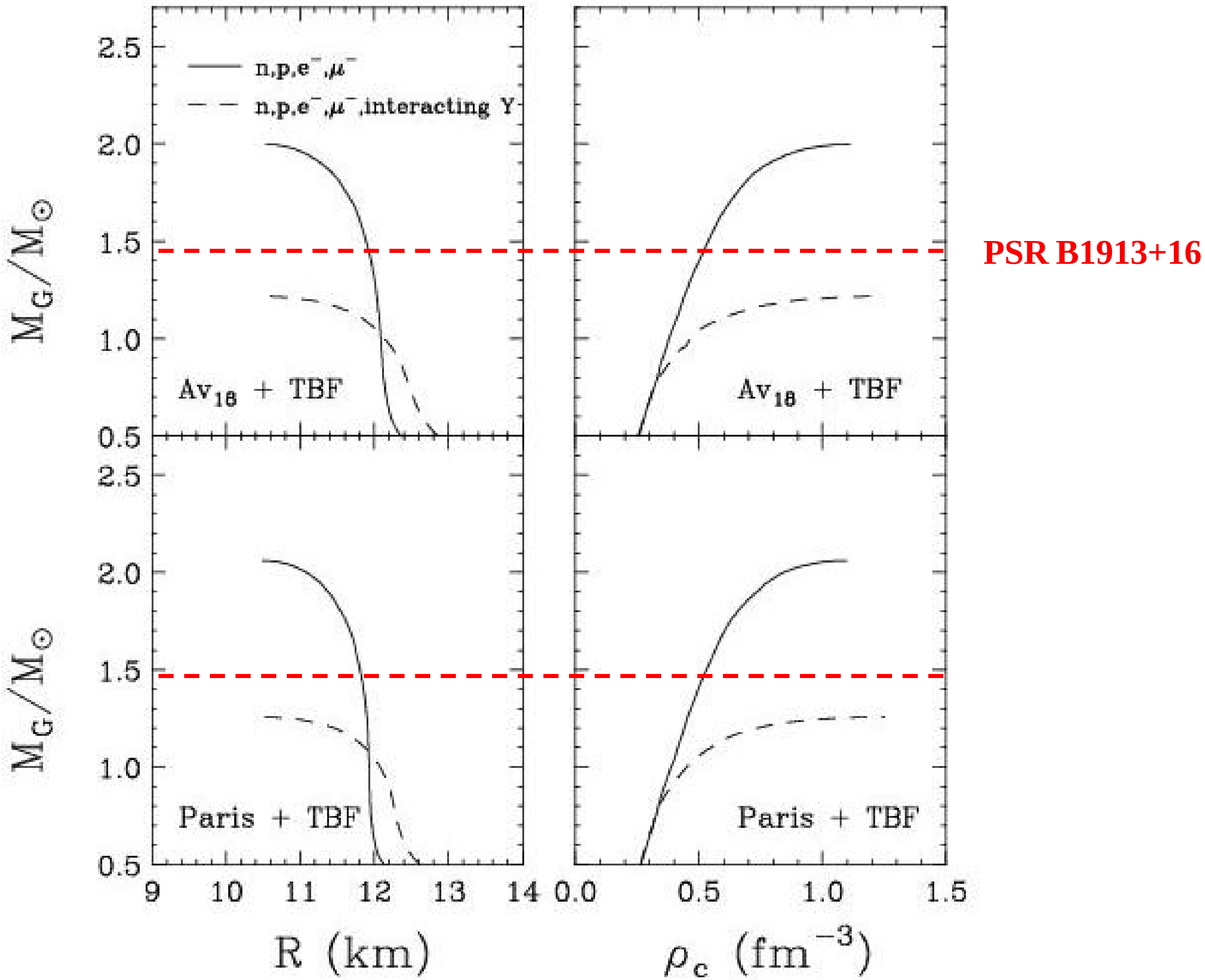
Hyperonic Star

$M_B = 1.34\ M_\odot$

I. Vidaña, I. Bombaci, A. Polls, A. Ramos, Astron. and Astrophys. 399 (2003) 687



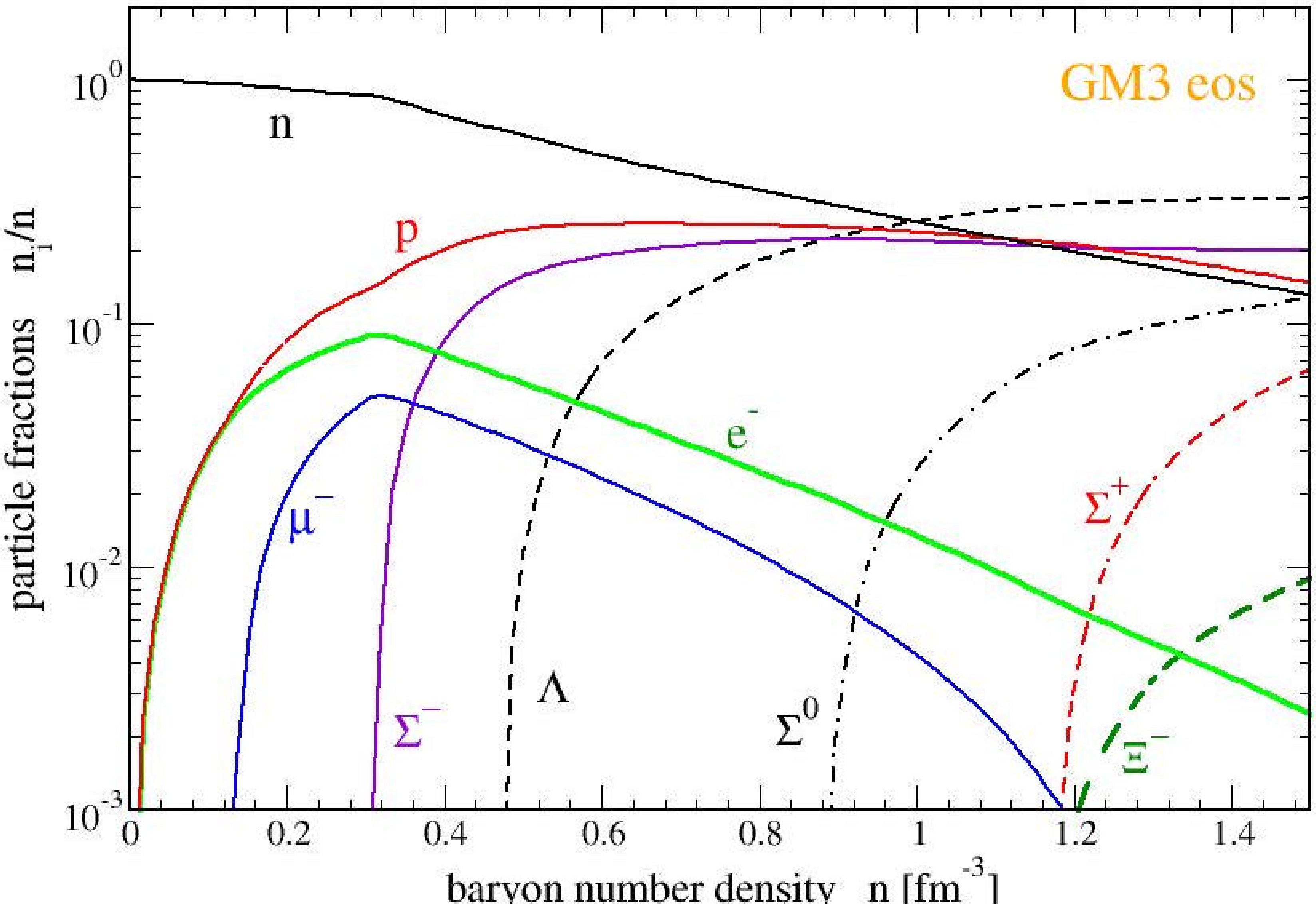
M. Baldo, G.F. Burgio, H.-J. Schulze, Phys.Rev. C61 (2000)



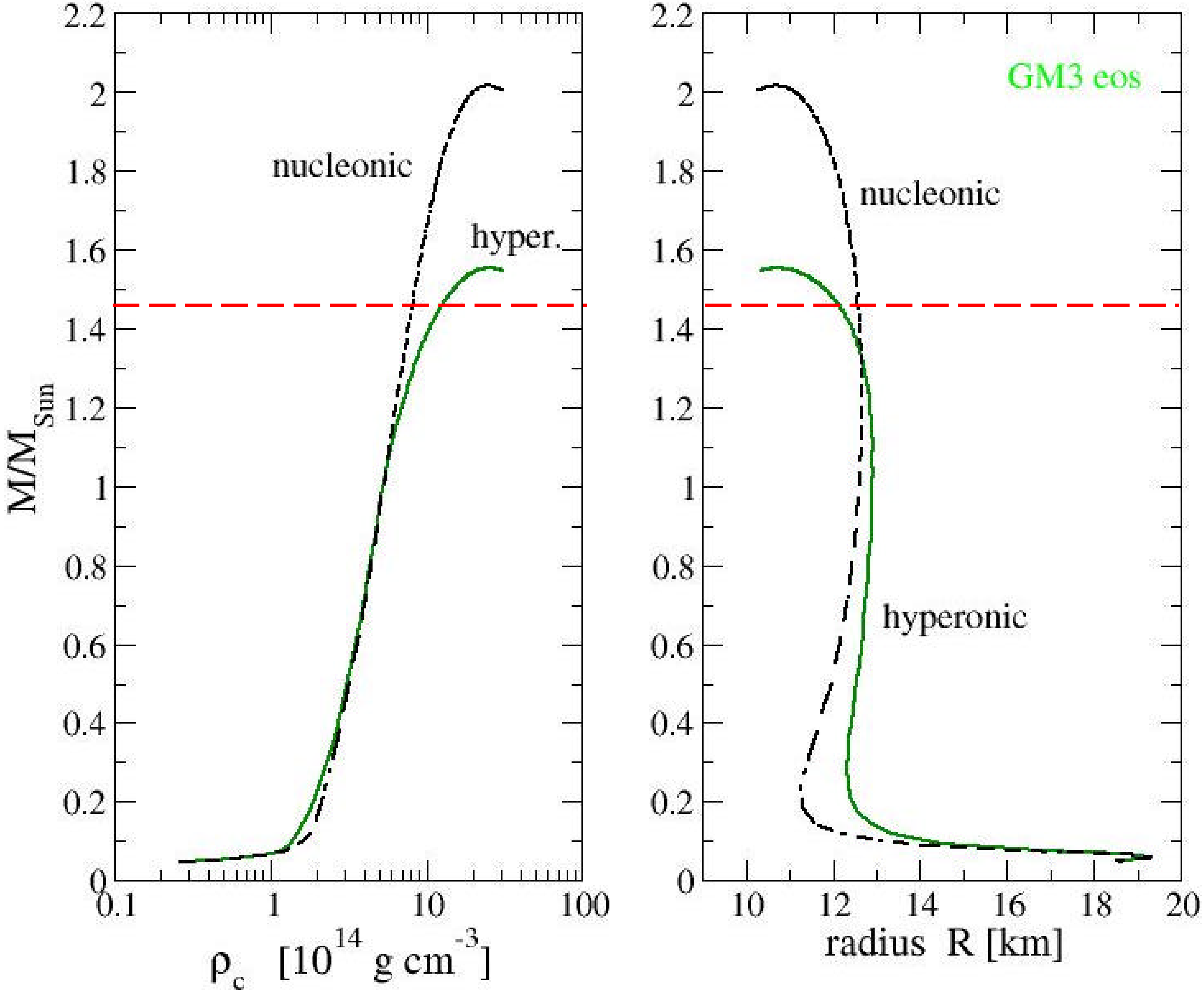
M. Baldo, G.F. Burgio, H.-J. Schulze, Phys.Rev. C61 (2000)

Relativistic Quantum Field Theory in the mean field approximation for Hyperonic Matter and Hyperon Stars

- Walecka, 1972
- Glendenning, Astrophys. Jour. 293 (1985)
- Glendenning and Moszkowski, Phys. Rev. Lett. 67, (1991)

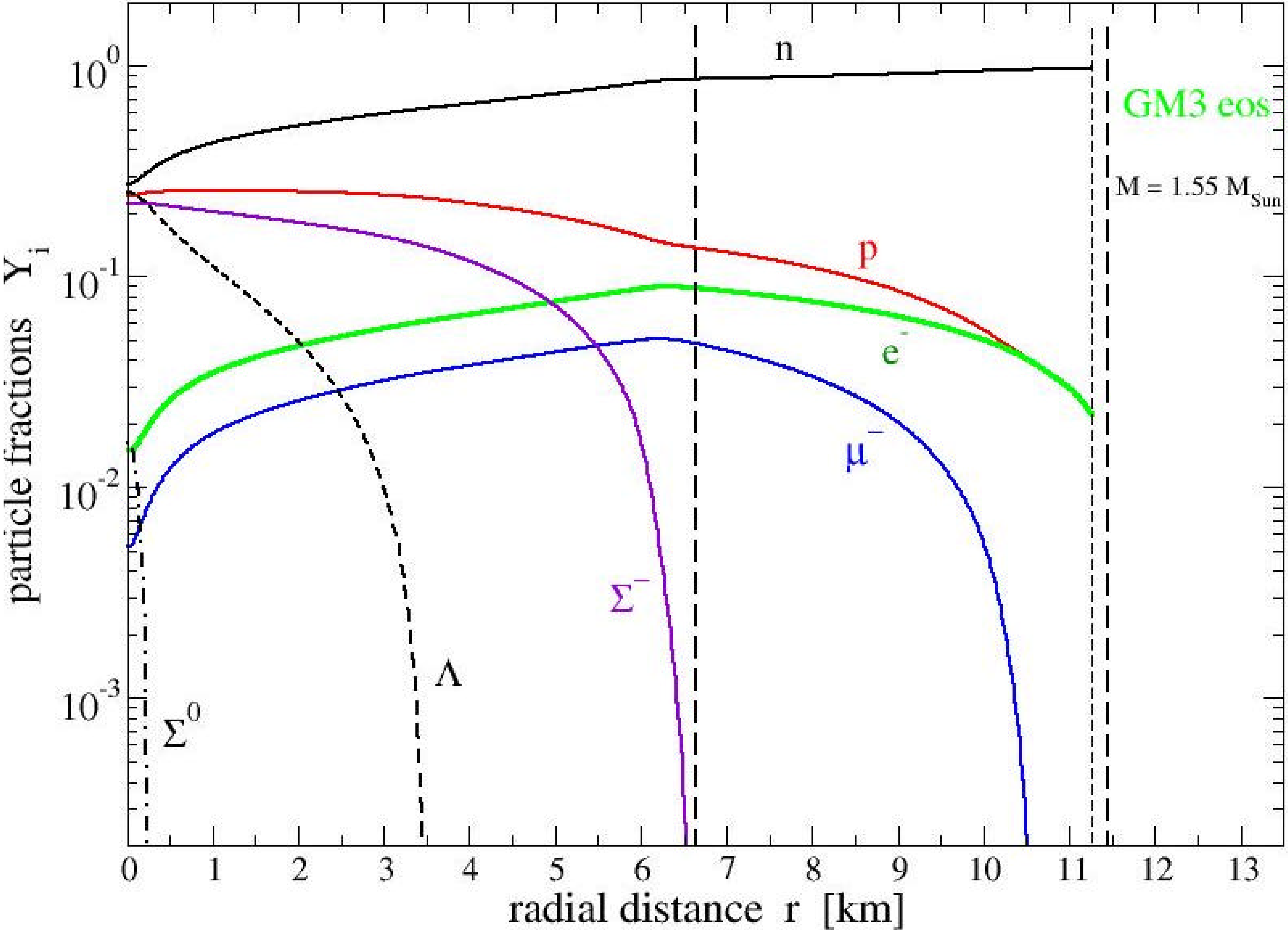


GM3 eos
n
p
e⁻
μ⁻
Σ⁻
Λ
Σ⁰
Σ⁺
Ξ⁻
particle fractions n_i/n
baryon number density n [fm⁻³]
10⁰
10⁻¹
10⁻²
10⁻³
0
0.2
0.4
0.6
0.8
1
1.2
1.4



GM3 EOS: **Glendenning, Moszkowsky, PRL 67(1991)**

Relativistic Mean Field Theory of hadrons interacting via meson exchange



n
GM3 eos
M = 1.55 M$_{Sun}$
p
e^-
μ^-
Σ^-
Λ
Σ^0
10^0
10^{-1}
10^{-2}
10^{-3}
particle fractions Y_i
0 1 2 3 4 5 6 7 8 9 10 11 12 13
radial distance r [km]

Hyperons in Neutron Stars: implications for the stellar structure

The presence of hyperons **reduces the maximum mass of neutron stars:** $\mathrm{D}M_{max}$ **»** $(0.5 – 0.8)\ M_{\odot}$

Therefore, **to neglect hyperons always leads to an overestimate of the maximum mass of neutron stars**

Microscopic EOS for hyperonic matter:

“very soft” EOS **non compatible with measured NS masses.**

→ **Need for extra pressure at high density**

- **Improved NY, YY two-body interaction**
- **Three-body forces: NNY, NYY, YYY**

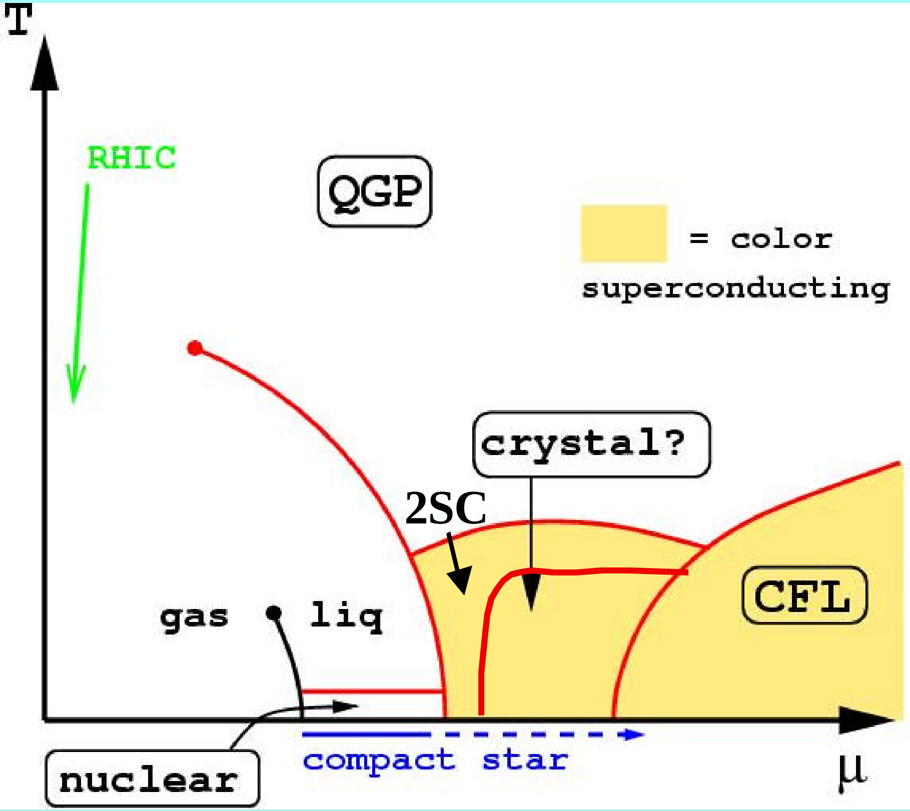
Quark Matter in Neutron Stars

QCD

Ultra-Relativistic Heavy Ion Collisions

→ **Quark-deconfinement phase transition** expected at

$\rho_c \approx (3-5)\ \rho_0$



The core of the most massive Neutron Stars is one of the best candidates in the Universe where such a deconfined phase of quark matter can be found

What quark flavors are expected in a Neutron Star?

What quark flavors are expected in a Neutron Star?

flavor	Mass	Q/|e|
u	5 ±3 MeV	2/3
d	10 ±5 MeV	-1/3
s	200 ±100 MeV	-1/3
c	1.3 ± 0.3 GeV	2/3
b	4.3 ±0.2 GeV	-1/3
t	175 ± 6 GeV	2/3

m_u , $m_{d,}$ m_s << m_c ,m_b ,m_t

Suppose: $m_u = m_d = m_s = 0$ (✱)

u,d,s non-interacting
(ideal ultrarelativ. Fermi gas)

- **Threshold density for the *c* quark**

$$\boxed{s \rightarrow c + e^{-} + \overline{\nu}_e}$$

$\left.\begin{array}{l} \textit{u,d,s} \text{ in â-equil.} \\ Q_{tot} = 0 \end{array}\right\} \xrightarrow{(\ast)} n_B = n_u = n_d = n_s$

$E_{Fq} = \; c\, k_{Fq} = \; c\, (p^2 n_q)^{1/3} = \; c\, (p^2 n_B)^{1/3} \; ³ \; m_c = 1.3 \text{ GeV} \; \Longrightarrow \; n_B \sim 29 \text{ fm}^{-3}$

$\sim 180\, n_0$

Only *u, d,s* quark flavors are expected in Neutron Stars.

A simple phenomenological model for the

EOS of Strange Quark Matter

Strange Quark Matter: u,d,s deconfined quark matter.

Strangeness quantum number ***S* :** **S = 0 (*u, d* quarks), *S* = –1 (*s* quark)**

We consider a simple phenomenological model which incorporates from the begining two foundamental features of **QCD**

- **Asymptotic freedom**
- **Confinement**

Ω = grand canonical thermodynamical potential <u>per unit volume</u> (Ω = – P)

$$\Omega = \Omega^{(0)} + \Omega_{int}$$

$\Omega^{(0)}$: **contribution for a *u,d,s* non–interacting realtivistic Fermi gas**

Ω_{int} : **contr. from quark interactions (in principle solving QCD eq.s)**

$$\Omega_{int} = \Omega_{short} + \Omega_{long}$$

SQM: high density limit

In the high density regime encountered in the cores of massive Neutron Stars quarks behave essentially as free particle (due to asymptotic freedom).

In this regime the short range quark interaction can be calcualted using perturbative QCD (perturbative expansion in powers of the QCD structure constant $\alpha_c = g^2/(4\pi)$ **)**

$$\Omega_{short} \leftarrow \text{perturbative QCD}$$

The long range part of the quark interaction is roughly approssimated by the so called bag constant B. In the **MIT bag model** for hadrons the bag pressure $P = -B$, is responsible for the **confinement** of quarks in hadrons

$$\Omega_{long} \sim B$$

Grand canonical potential (per unit volume)

In the following we assume**:** $m_u = m_d = 0\,, \qquad m_s \neq 0$

$$\Omega^{(0)} = \Omega_u^{(0)} + \Omega_d^{(0)} + \Omega_s^{(0)}$$

$$\Omega_q^{(0)} = -\frac{1}{(\hbar c)^3}\frac{1}{4\pi^2}\mu_q^4 \qquad (q = u, d)$$

$$\Omega_s^{(0)} = -\frac{1}{(\hbar c)^3}\frac{1}{4\pi^2}\left\{\mu_s\mu_s^*\left(\mu_s^2 - \frac{5}{2}m_s^2\right) + \frac{3}{2}m_s^4 \ln\left(\frac{\mu_s + \mu_s^*}{m_s}\right)\right\}$$

$$\mu_s^* \equiv \left(\mu_s^2 - m_s^2\right)^{1/2} = \hbar c \; k_{Fs}$$

$\mu_u \quad \mu_d \quad \mu_s$: chemical potentials for quarks

The expression for the **linear (in α_c) perturbative contribution** $\Omega^{(1)}$ to the grand canonical potential con be found in **Farhi and Jaffe, Phys. Rev. D30 (1984) 2379**

Equation of State (T = 0)

$$\begin{cases} P(\mu_u, \mu_d, \mu_s) = -\Omega \;\cong\; -\Omega^{(0)} - \Omega^{(1)} - B \\[2ex] \rho(\mu_u, \mu_d, \mu_s) \cong \dfrac{1}{c^2}\left\{ \Omega^{(0)} + \Omega^{(1)} + \sum\limits_{f=u,d,s} \mu_f n_f + B \right\} \end{cases}$$

$$n_f = -\left(\frac{\partial \Omega_f}{\partial \mu_f}\right)_{T,V}$$

$$n = \frac{1}{3}(n_u + n_d + n_s)$$ total baryon number density

b-stable Strange Quark Matter

$$u + e^- \leftrightarrow d + \nu_e$$

$$u + e^- \leftrightarrow s + \nu_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e$$

$$s \rightarrow u + e^- + \bar{\nu}_e$$

$$s + u \leftrightarrow d + u$$

$$e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

$$......, \quad etc.$$

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$

neutrino-free matter

b-stable Strange Quark Matter

- **Equilibrium with respect to the weak interaction processes**

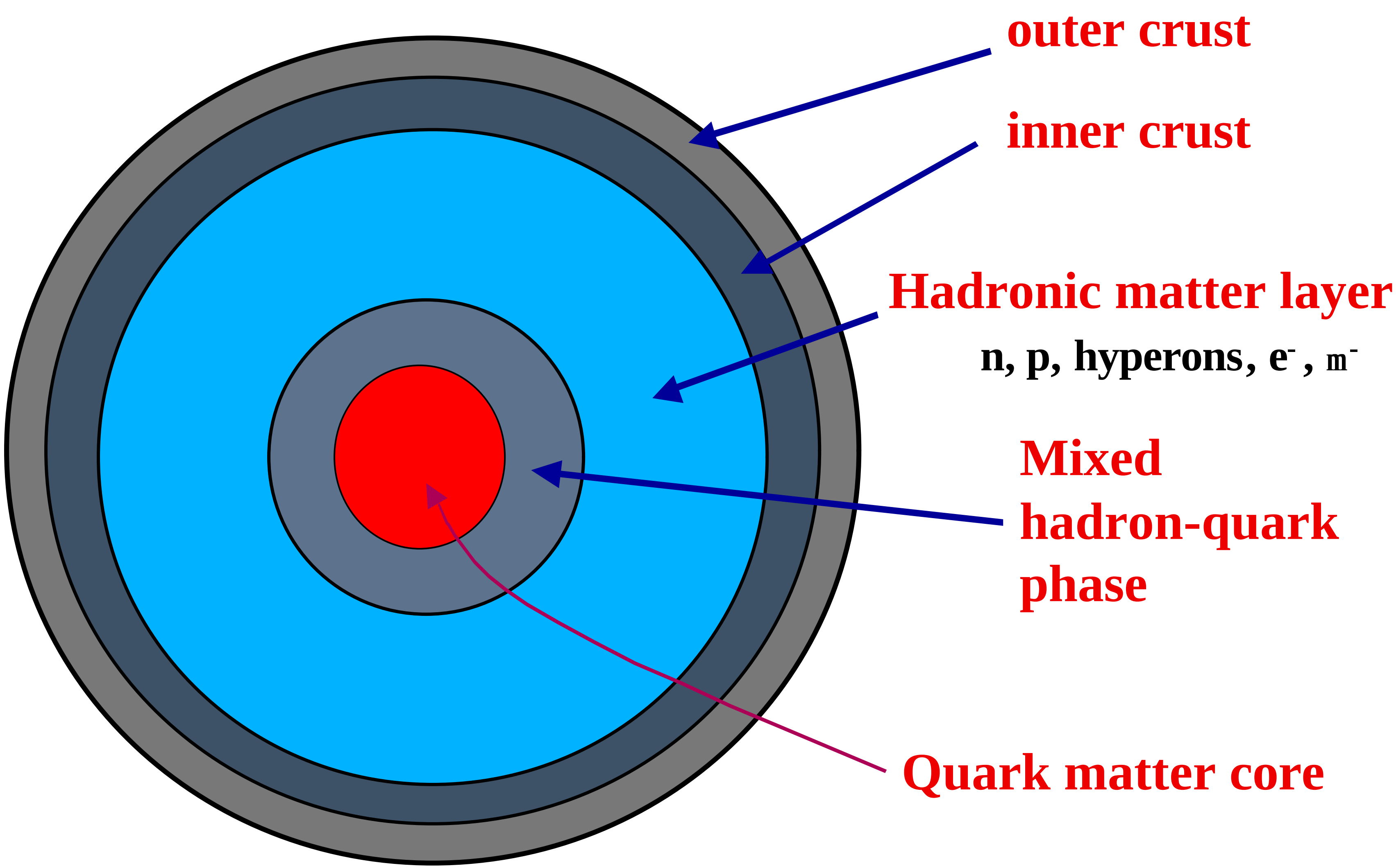
$$\mu_d = \mu_u + \mu_e$$
$$\mu_d = \mu_s$$
$$\mu_\mu = \mu_e$$

- **Charge neutrality**

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu = 0$$

To be solved for any given value of the total baryon number density n_B

Hybrid Stars



I. Bombaci, Formazione e struttura delle stelle compatte, Scuola “R. Anni”, Otranto 2006

The EOS for Hybrid Stars

⁕ **Hadronic phase :**

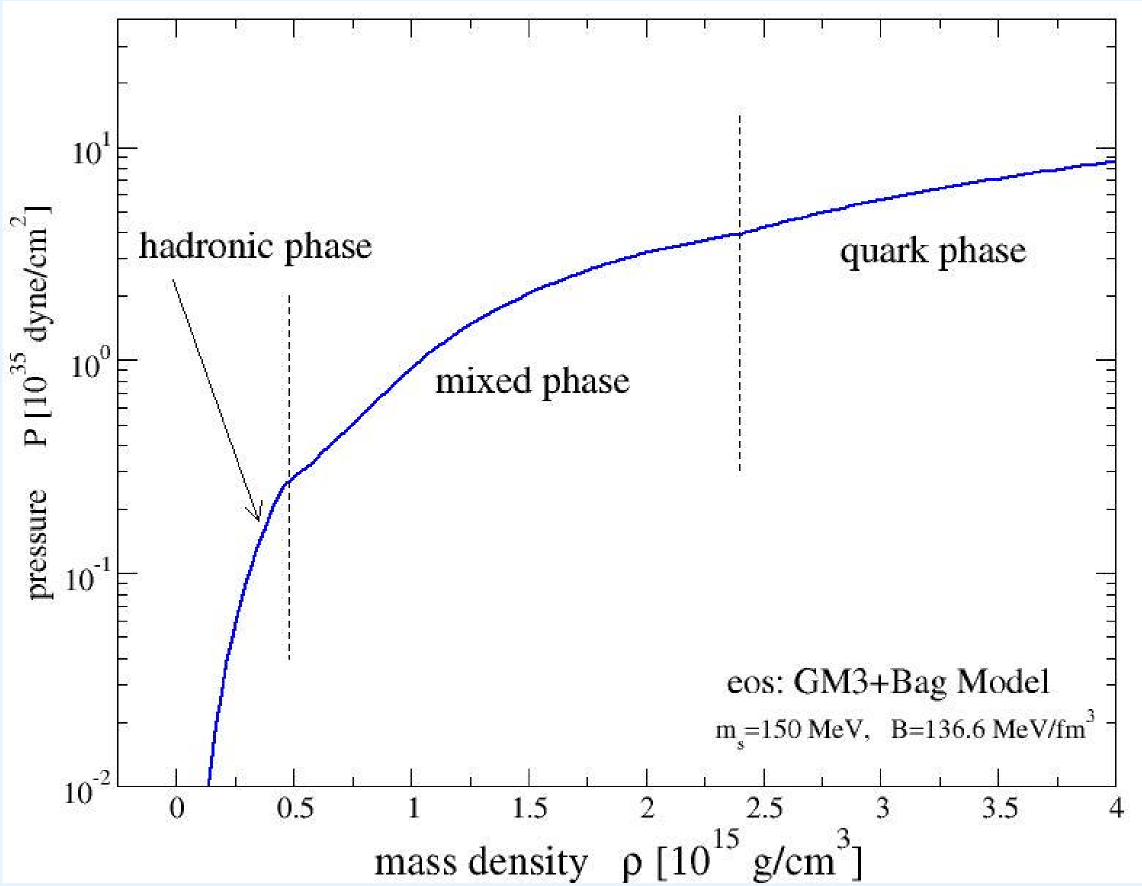
Relativistic Mean Field Theory of hadrons interacting via meson exch.
[e.g. Glendenning, Moszkowsky, PRL 67(1991)]

⁕ **Quark phase :**

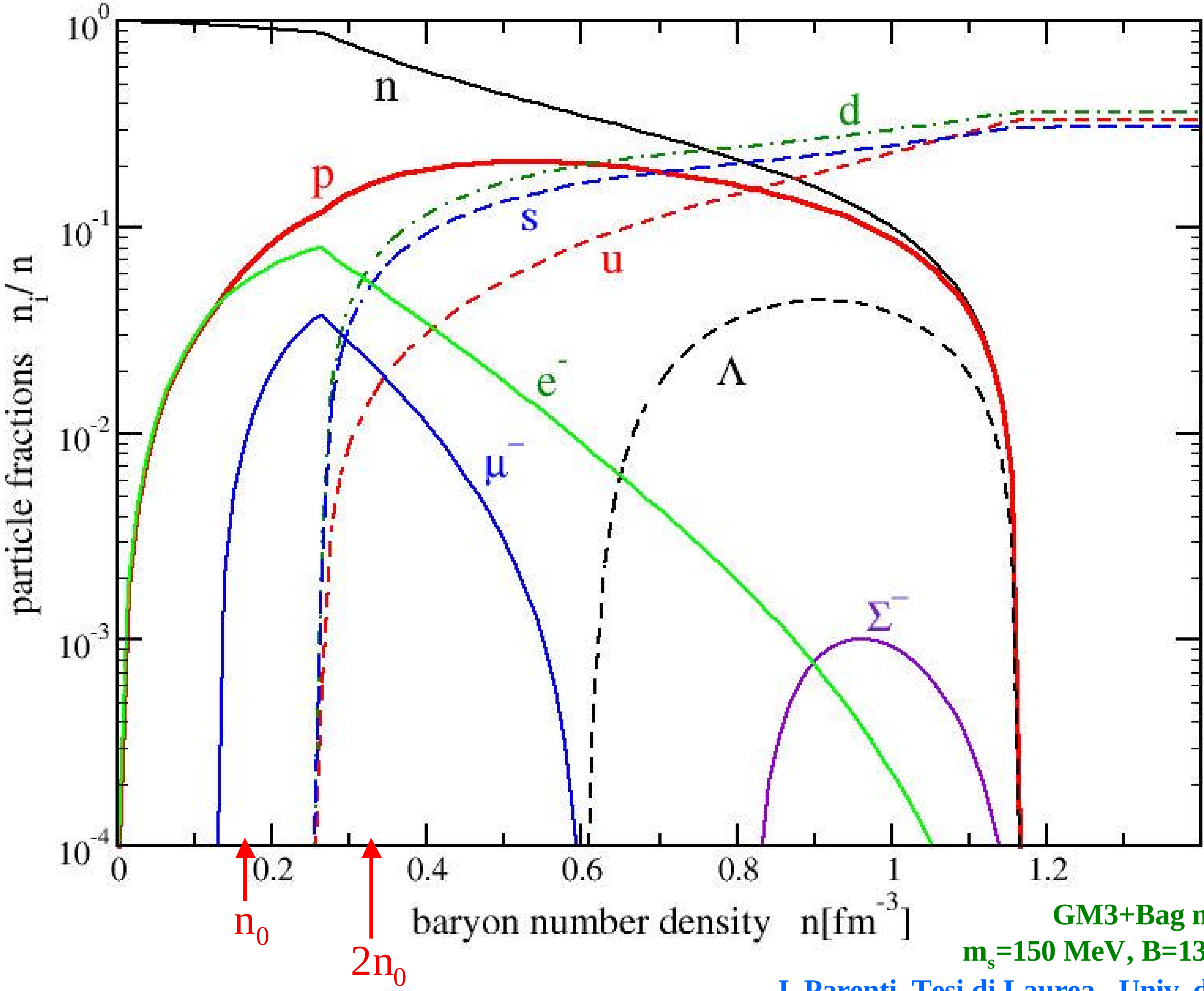
EOS based on the MIT bag model for hadrons. [Farhi, Jaffe, Phys. Rev. D46(1992)]

⁕ **Mixed phase :**

Gibbs construction for a multicomponent system with two conserved "charges".
[Glendenning, Phys. Rev. D46 (1992)]



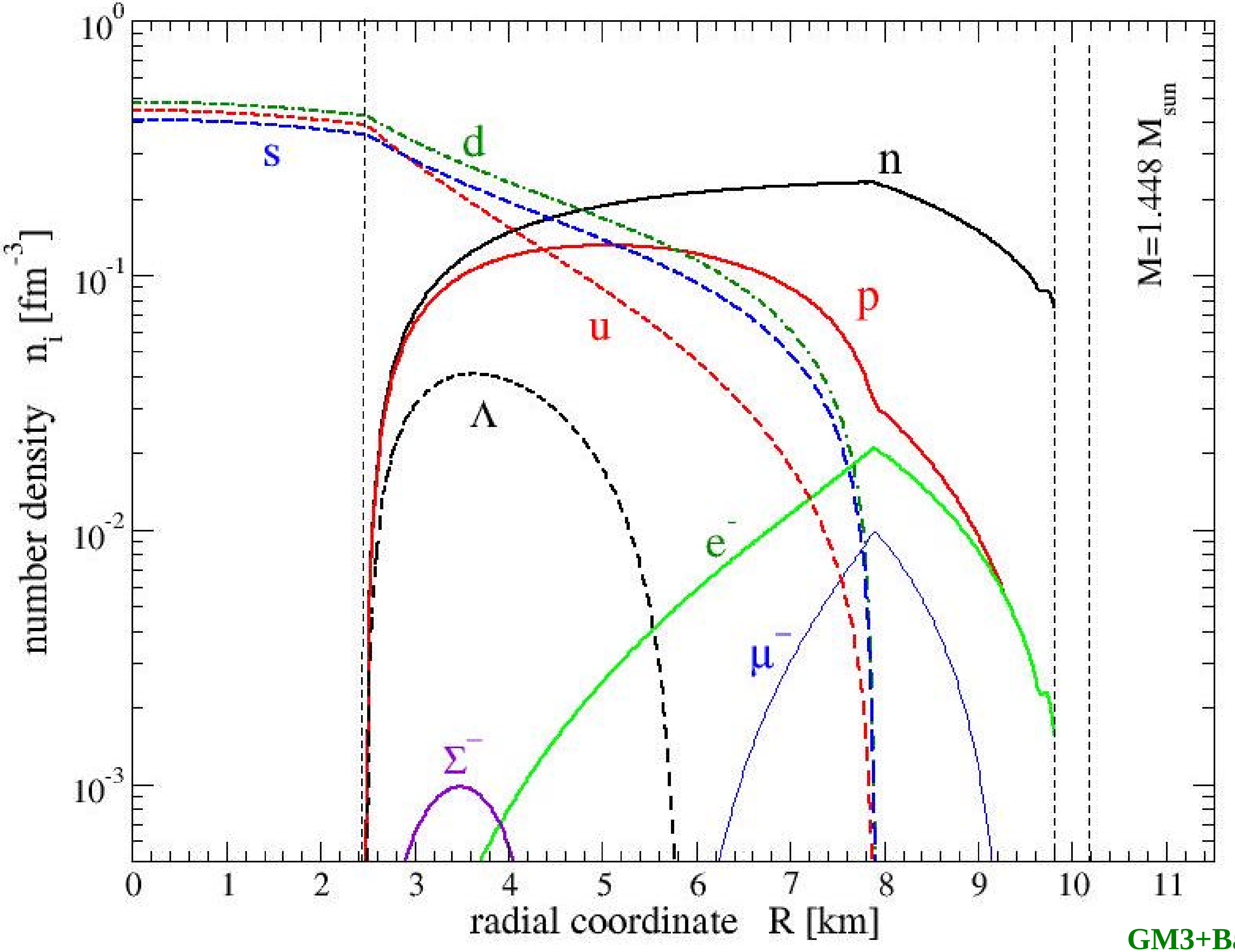
Composition of dense matter with a phase trans to Quark Matter



GM3+Bag model
m_s=150 MeV, B=13.6.6Mev/fm³

I. Parenti, Tesi di Laurea , Univ. di Pisa, 2003

Hybrid Star



GM3+Bag model
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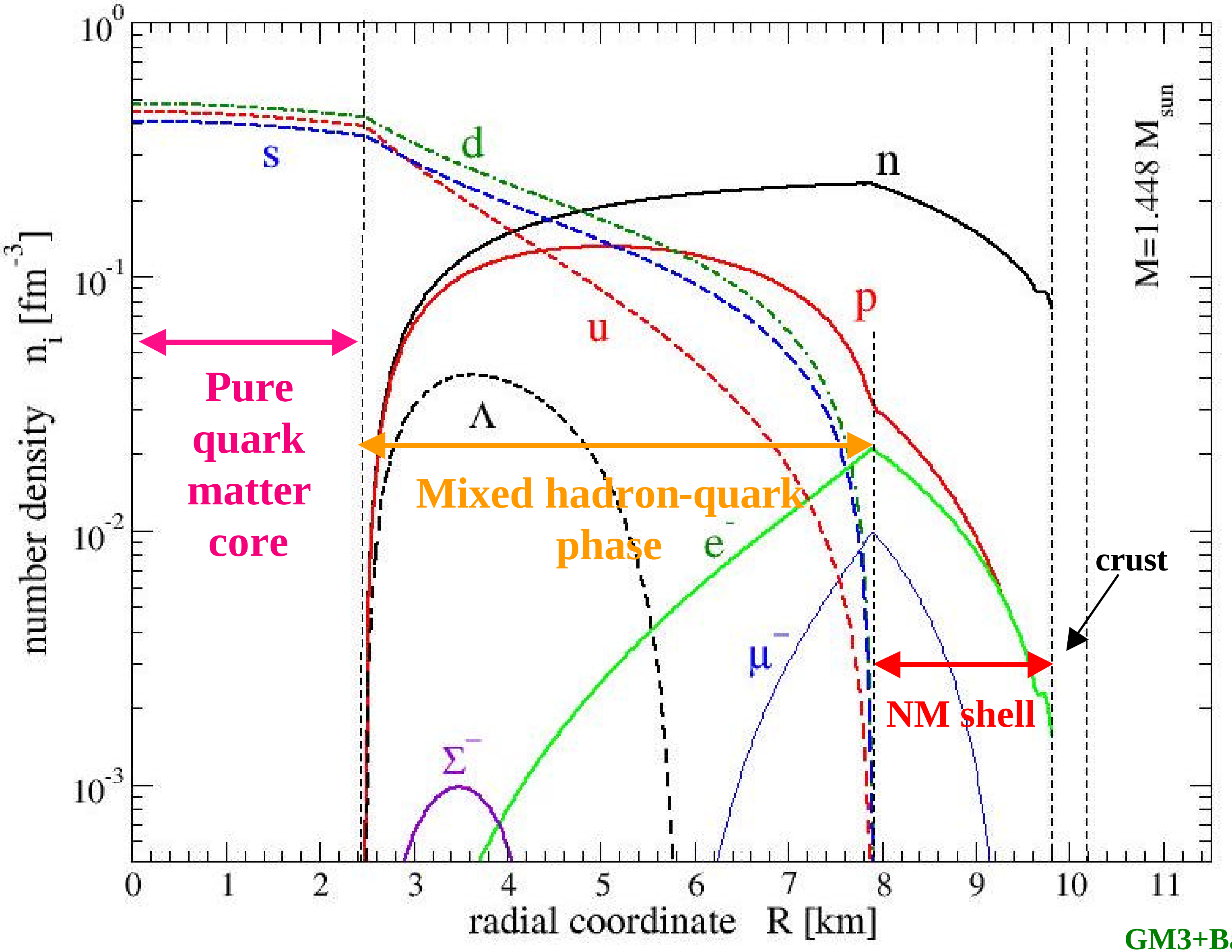
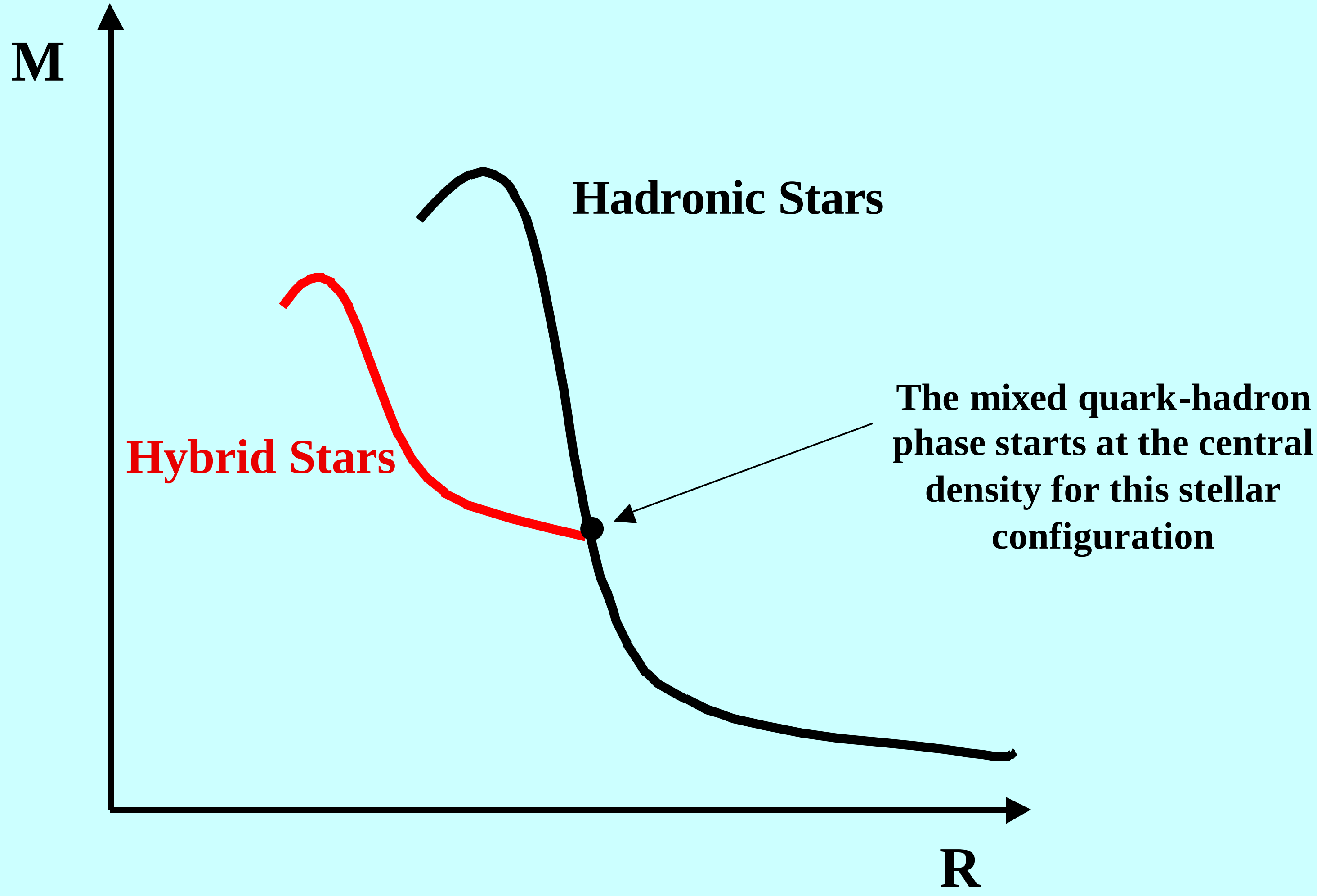
Hybrid Star



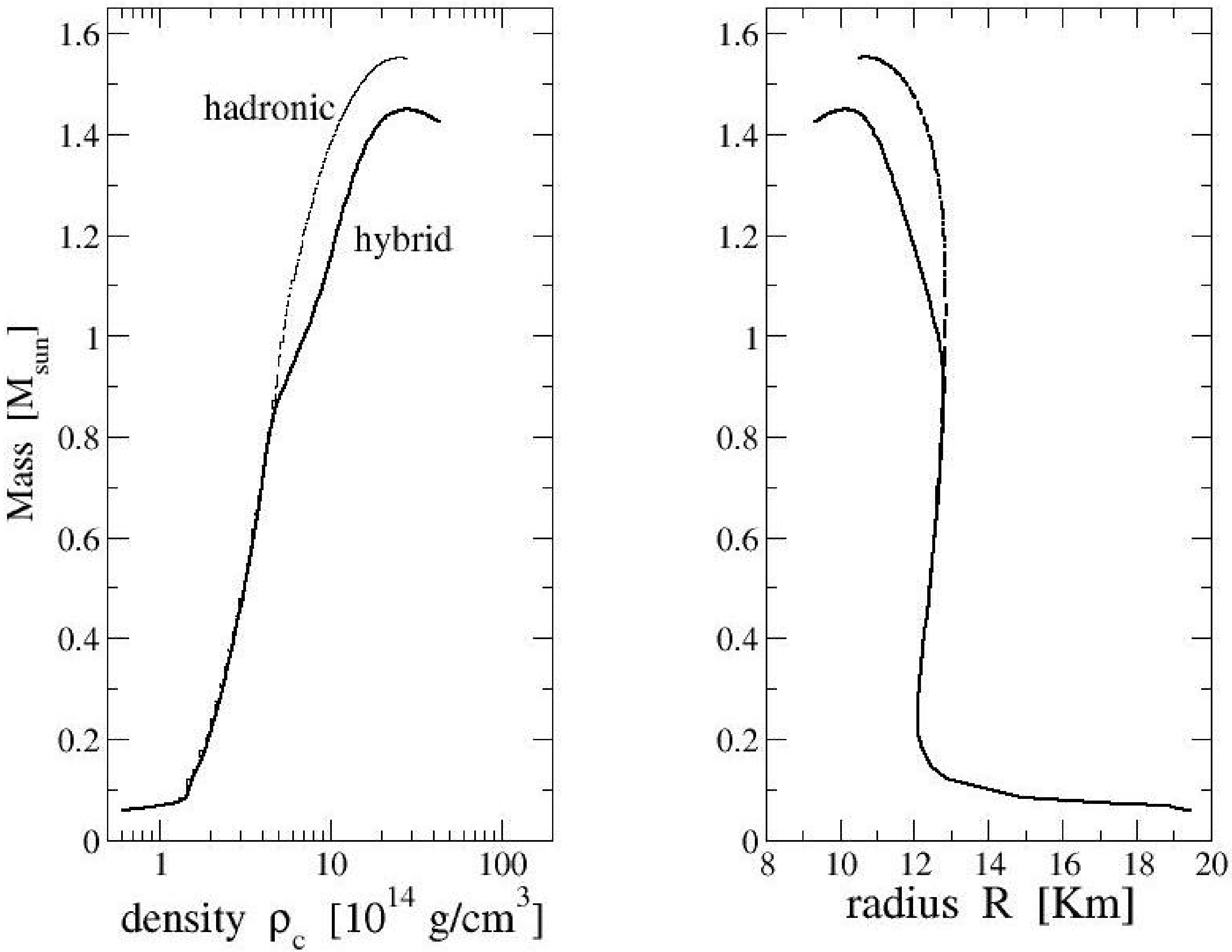
GM3+Bag model
m_s=150 MeV, B=13.6.6Mev/fm^3

I. Parenti, Tesi di Laurea , Univ. di Pisa, 2003

The mass-radius relation



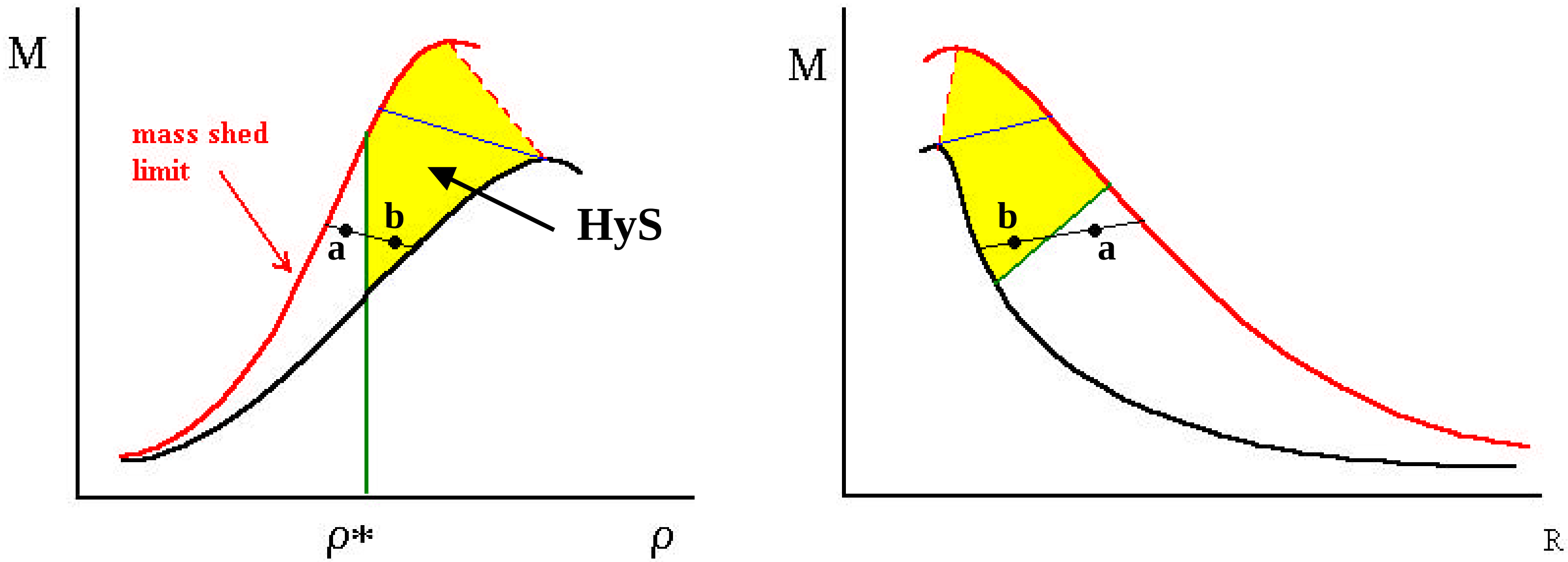
Hybrid Stars



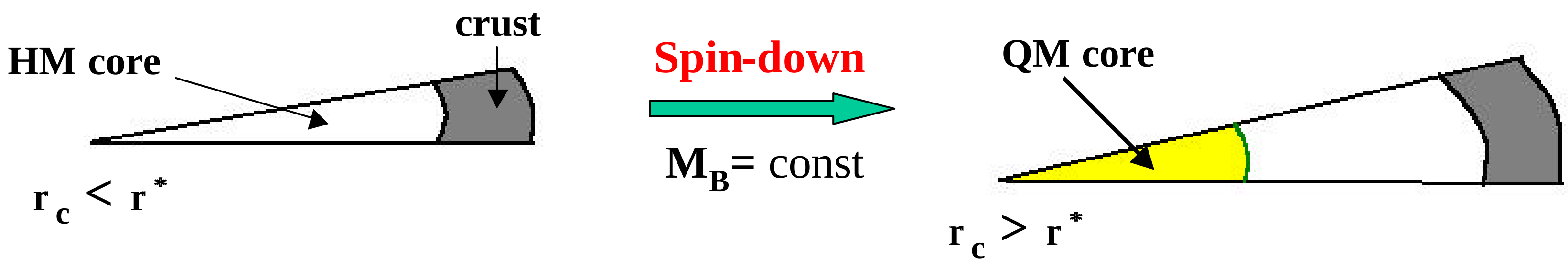
EOS: GM3 + Bag model (B=136 MeV/fm^3, m_s=150 MeV

I. Parenti, Tesi di Laurea , Univ. di Pisa, 2003

Deconfinement phase transition in isolated spinning-down neutron stars



r^* = critical density for quark deconfinement

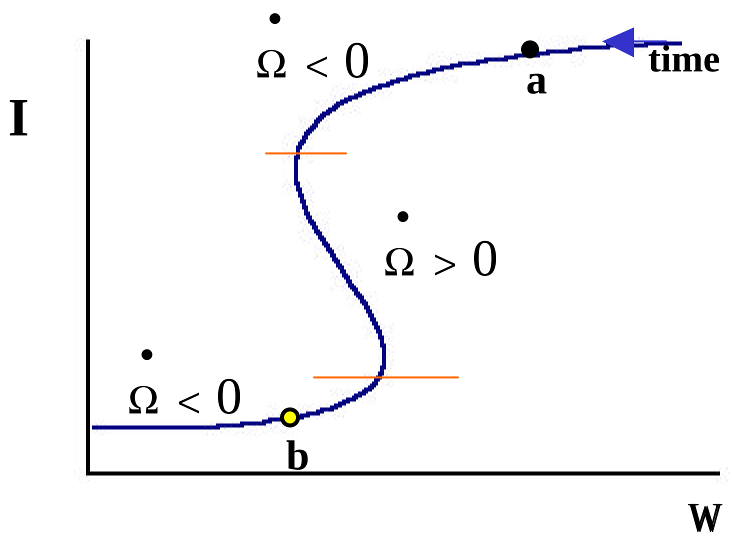


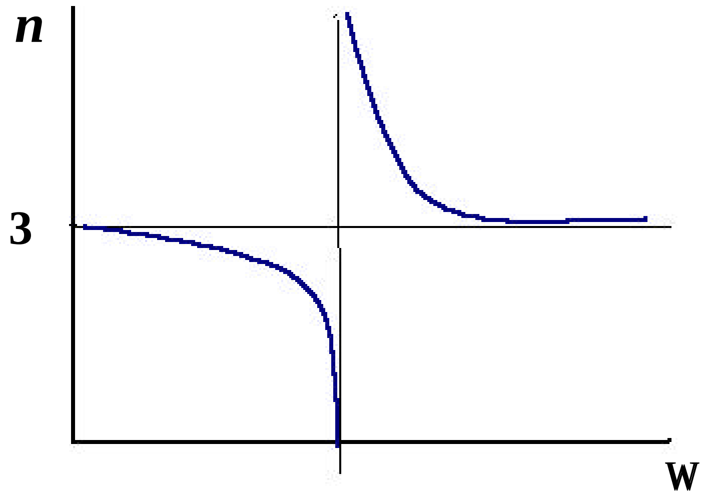
Spin-down: **J decreases**, Ω decreases
ρ_c increases, I decreases

I. Bombaci, Formazione e struttura delle stelle compatte, Scuola “R. Anni”, Otranto 2006

apparent braking index

$$\tilde{n}(\Omega) \equiv \Omega\,\ddot{\Omega}/\dot{\Omega}^{2} = n - \frac{3I'\Omega + I''\Omega^{2}}{2I + I'\Omega}$$





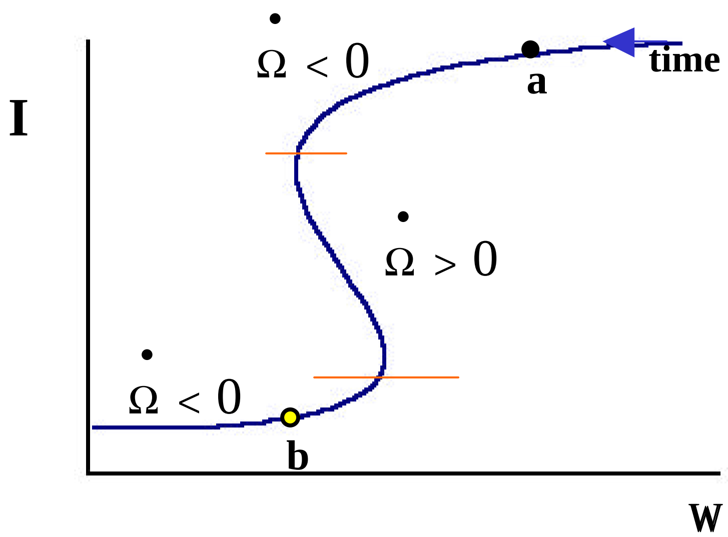
measured large value of the braking index $|n| >> 3$

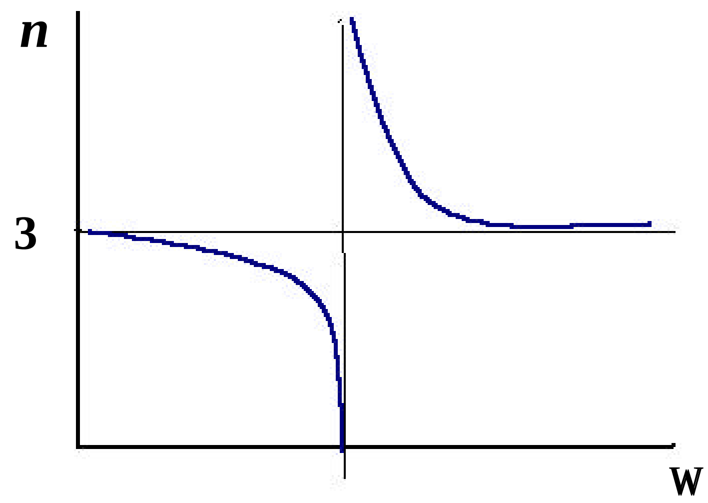
Observational signature for quark deconfinement phase transition in compact stars

Glendenning, Pei, Weber, 1997

apparent braking index

$$\tilde{n}(\Omega) \equiv \Omega\,\ddot{\Omega}/\dot{\Omega}^{2} = n - \frac{3I'\Omega + I''\Omega^{2}}{2I + I'\Omega}$$

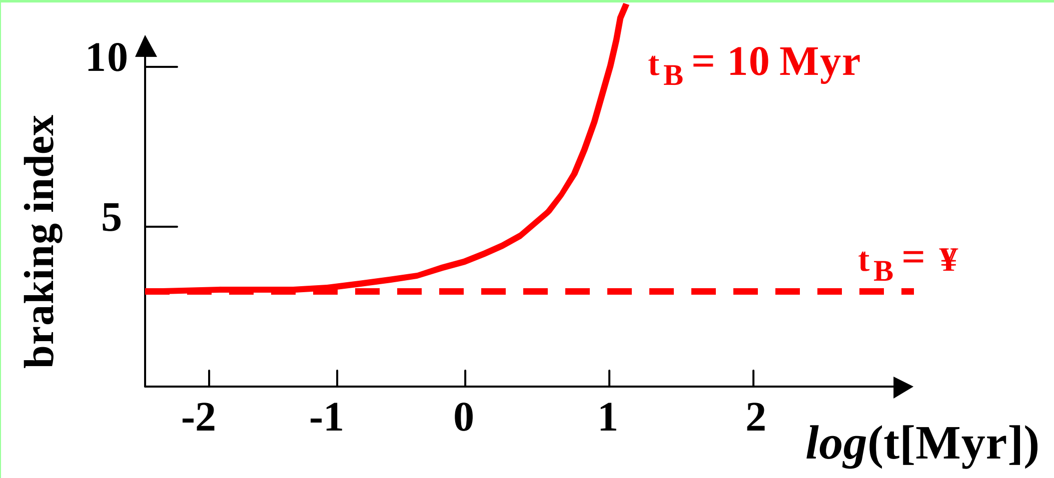




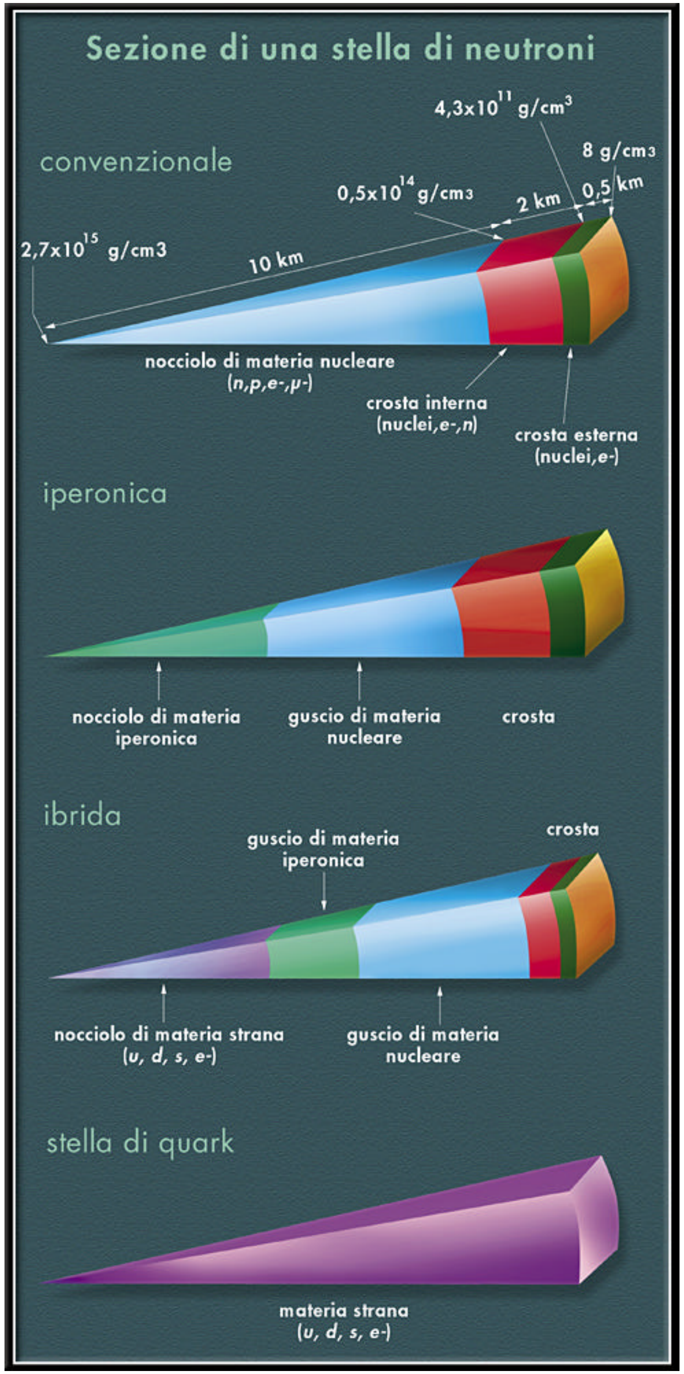
Remember the effects of magnetic field decay on the braking index
(see one of the previous lectures)

braking index

$$n(t) \equiv \Omega\,\ddot{\Omega}/\dot{\Omega}^{2} = 3 - \frac{3c^{3} I\,\dot{B}}{R^{6} B^{3} \sin^{2}\alpha\;\Omega^{2}}$$



Tauris and Konar,
Astron. and Astrophys. 376
(2001)



"Neutron Stars"

"traditional" Neutron Stars

Hyperon Stars

} **Hadronic Stars**

Hybrid Stars

Strange Stars

} **Quark Stars**

I. Bombaci, Formazione e struttura delle stelle compatte, Scuola "R. Anni", Otranto 2006

The Strange Matter hypothesis

Bodmer (1971), Terazawa (1979), Witten (1984)

Three-flavor *u,d,s* quark matter, in equilibrium with respect to the weak interactions, could be the true ground state of strongly interacting matter, rather than ^{56}Fe

$$E/A|_{SQM} \leq E(^{56}Fe)/56 \sim 930 \text{ MeV}$$

Stability of Nuclei with respect to u,d quark matter

The success of traditional nuclear physics provides a clear indication that **quarks in the atomic Nucleus are confined within protons and neutrons**

$$E/A|_{ud} \geq E(^{56}Fe)/56$$

EOS for SQM: massless quarks

(ultra-relativistic ideal gas +bag constant)

$$e = K\, n^{4/3} + B$$

$$P = (1/3) K\, n^{4/3} - B$$

$$P = (1/3)\,(e - 4B)$$

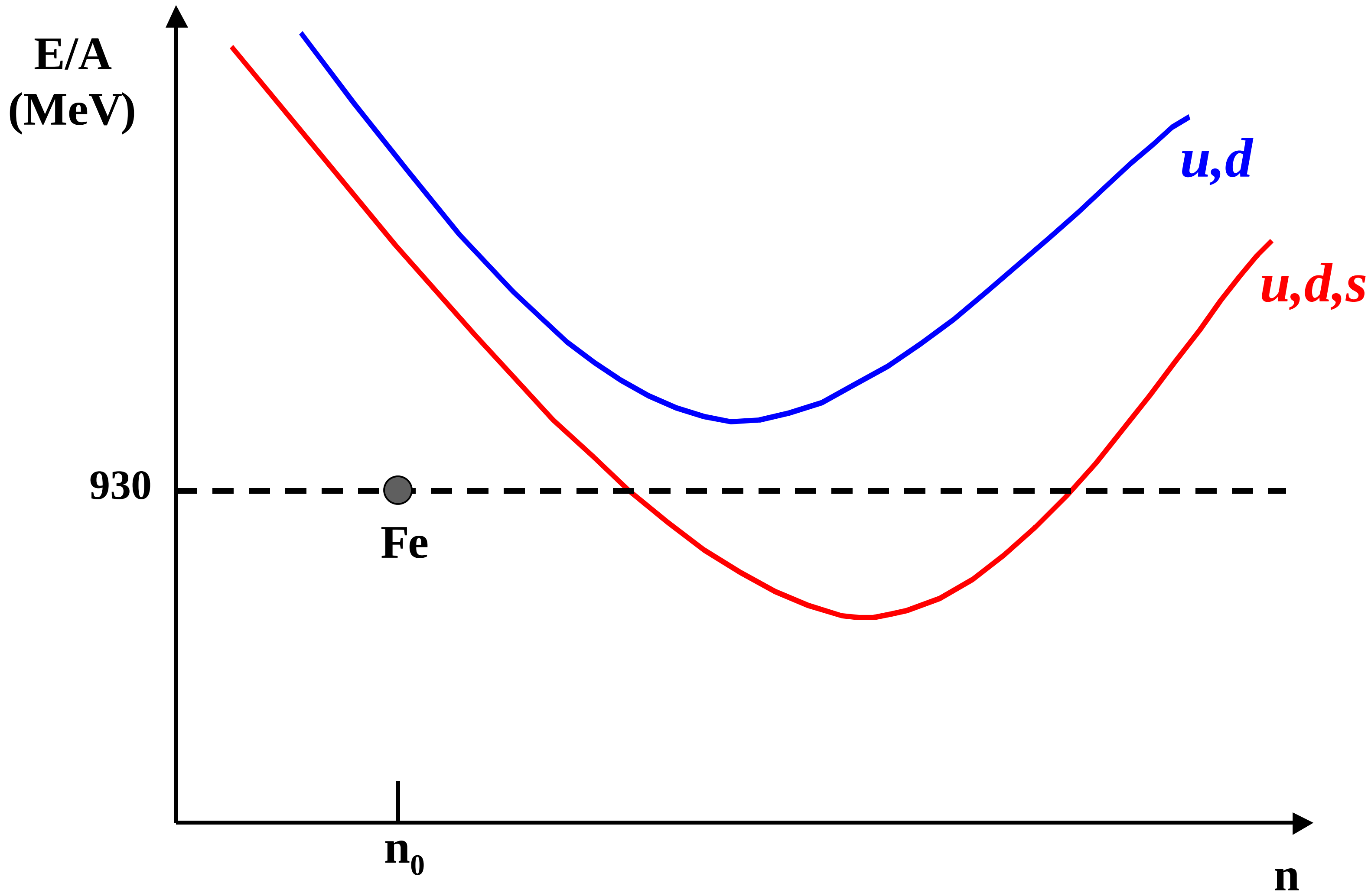
$$E/A = K\, n^{1/3} + B/n$$

$$K = \frac{9}{4}\hbar c\, \pi^{2/3}$$

u,d,s QM : deg.fact.= 2×3×3 ($n_u = n_d = n_s$)

$$K = \frac{9}{4}\hbar c \left(\frac{3}{2}\pi^{2}\right)^{1/3}$$

u,d (isospin-symm.)QM : deg.fact.= 2×2×3



E/A
(MeV)
u,d
u,d,s
930
Fe
n_0
n

Stability of atomic nuclei against decay to SQM droplets

- If the SQM hypothesis is true, why nuclei do not decay into SQM droplets (strangelets) ?

- One should explain the existence of atomic nuclei in Nature.

a) Direct decay to a SQM droplet

$$^{56}\mathrm{Fe} \rightarrow \; ^{56}(\mathrm{SQM}) \qquad (1)$$

weak process

$$u \rightarrow s + e^{+} + \nu_e$$
$$d + u \rightarrow s + u \qquad (2)$$

To have the direct decay to ^{56}(SQM) one needs ~ 56 simultaneous strangeness changing weak processes (2).

The probability for the direct decay (1) is : $P \sim (G_F^{\,2})^A \sim 0$

The *mean-life time* of ^{56}Fe with respect to the direct decay to a drop of SQM is

t >> age of the Universe

b) Step by step decay to a SQM droplet

$$^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{X}^{1L} \rightarrow {}^{56}\mathrm{Y}^{2L} \; .\,.\,.\,. \rightarrow {}^{56}(\mathrm{SQM})$$

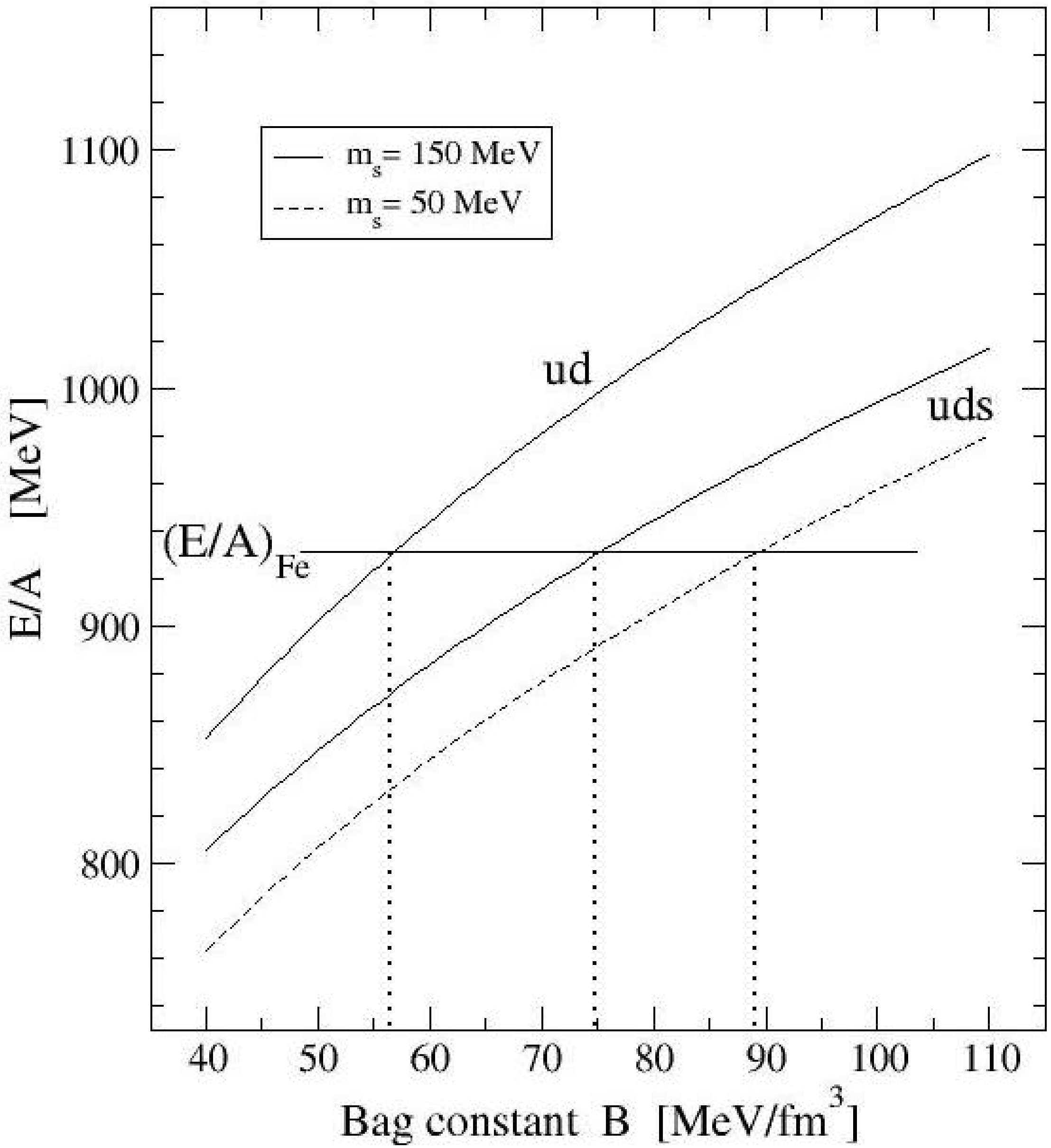
$$^{56}\mathrm{Fe} \rightarrow \begin{cases} {}^{56}\mathrm{Fe}^{L} \\ {}^{56}\mathrm{Mn}^{L} \end{cases}$$

These processes are not energetically possible since

$$Q = M(^{56}\mathrm{Fe}) - M(^{56}\mathrm{X}^{1L}) < 0$$

Thus, **according to the BTW hypothesis**, **nuclei are metastable states of strong interacting matter with a *mean-life time***

t >> **age of the Universe**



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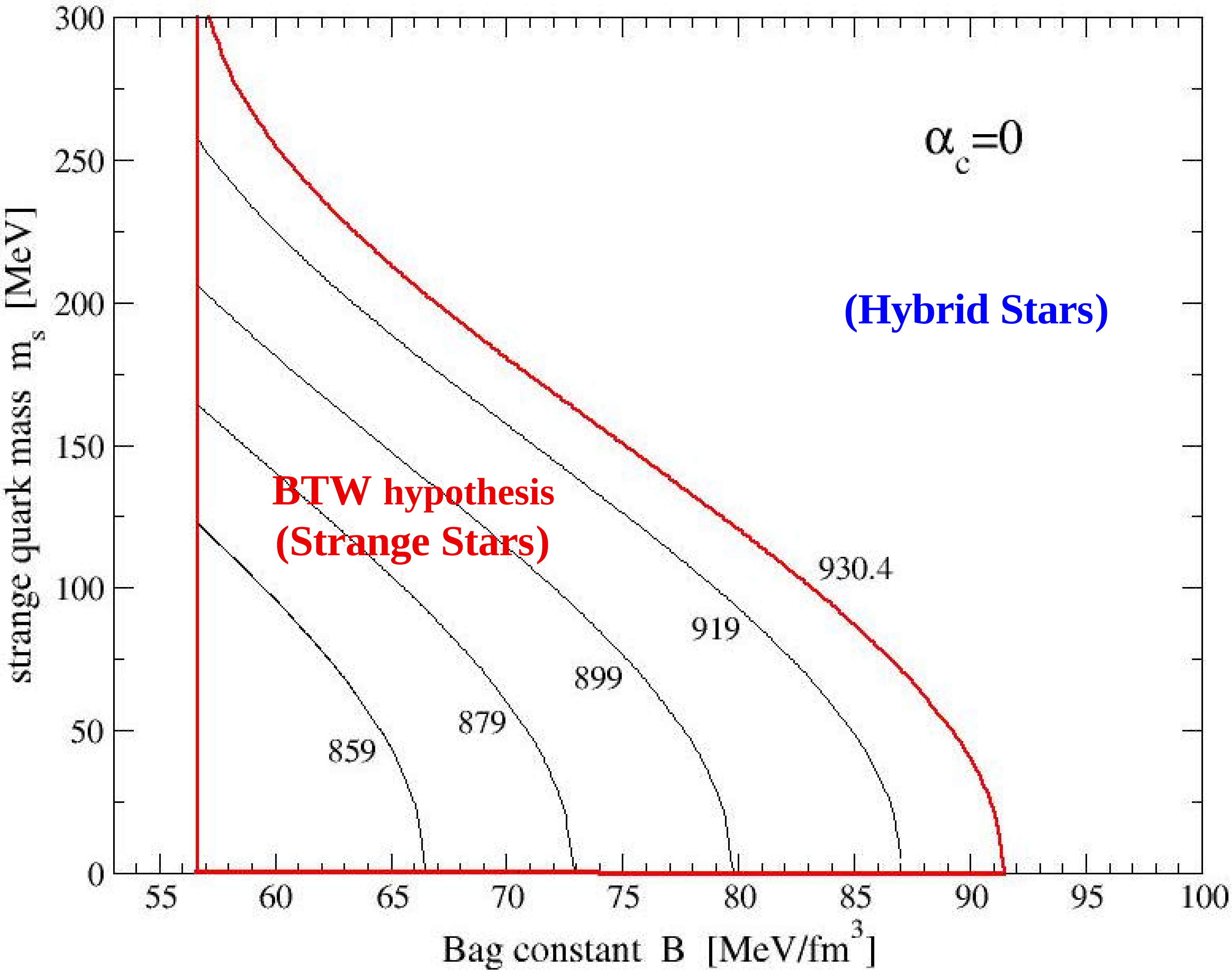
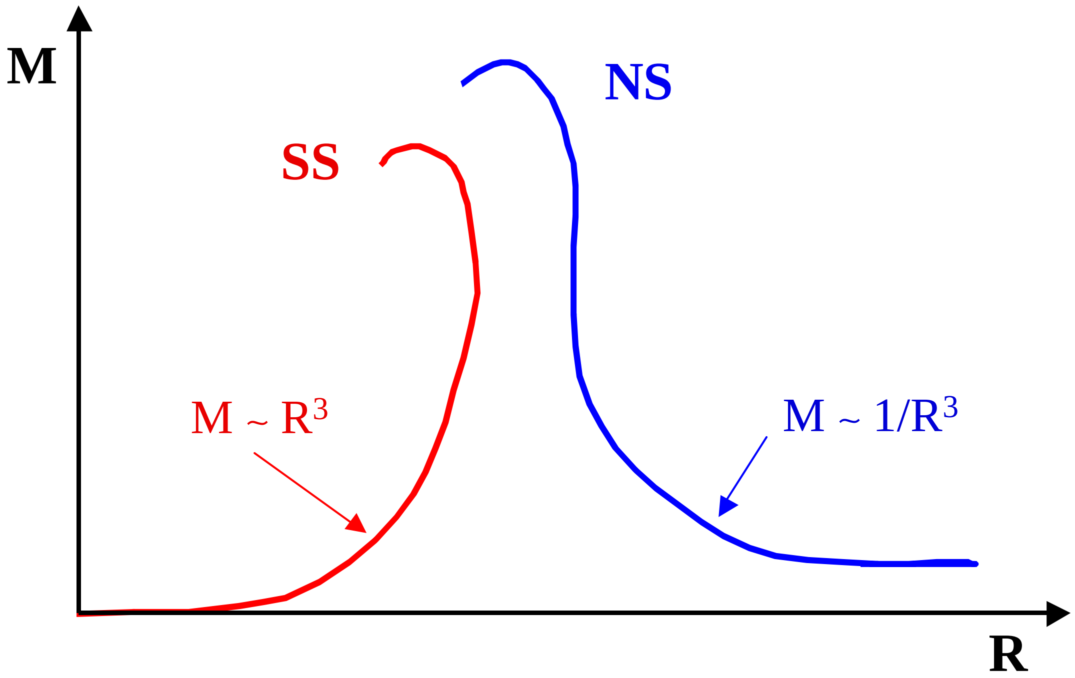


$\alpha_c=0$
(Hybrid Stars)
BTW hypothesis
(Strange Stars)
930.4
919
899
879
859
strange quark mass m_s [MeV]
Bag constant B [MeV/fm^3]
0
50
100
150
200
250
300
55
60
65
70
75
80
85
90
95
100

The Mass-Radius relation for Strange Stars



- "low" mass **Strange stars** are **self-bound** bodies
  i.e. they are bound by the strong interactions.

- Neutron Stars (Hadronic Stars) are **bound by gravity**.

The X-ray burster SAX J1808.4-3658

- **Discovered in Sept. 1996 by Beppo SAX**
- **Type-I X-ray burst source (DT < 30 sec.)**
- **Transient X-ray source (XTE J1808-369) detected with the proportional counter array on board of the Rossi X-ray Timing Explorer (RXTE) (1998)**
- **Millisecond PSR: Coerent pulsation with P = 2.49 ms**
- **Member of a LMXB: P_{orb} = 2.01 hours**

SAXJ1808.4-3658 is the first of the (so far) 3 discovered accreting X-ray millisecond PSRs.

XTE J1751-305 : P = 2.297 ms, P_{orb} = 42.4 min [2002]
XTE J0929-314 : P = 5.405 ms, P_{orb} = 43.6 min [2002]

▶ **millisecond X-ray PSRs were expected from theoretical models on the genesis of millisecond radio pulsars.**

The Mass-Radius relation for SAX J1808.4-3658

(i) In the course of RXTE observation in April – May 1998, the 3—150 keV X-ray luminosity of the source decreased by a factor of ~100.

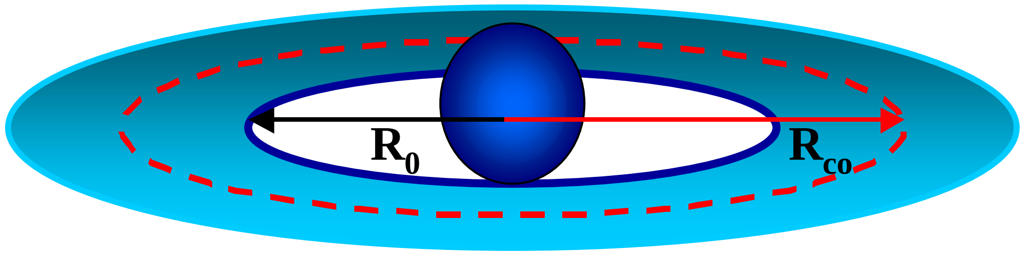
(ii) X-ray pulsation was observed over this range of X-ray luminosity.

The Mass-Radius relation for SAX J1808.4-3658

Detection of X-ray pulsation requires:

(1) $R < R_0$

(2) $R_0 < R_{co}$



Accretion disk

R = radius of the compact star

R_0 = radius of the inner edge of the accretion disk

R_{co} = corotation radius: $P_{orb}(R_{co}) = P$

$$R_{co} = \left[\frac{GM}{4\pi^2} P^2 \right]^{1/3}$$

The Mass-Radius relation for SAX J1808.4-3658

- **Spherical accretion (and dipolar magnetic field)**

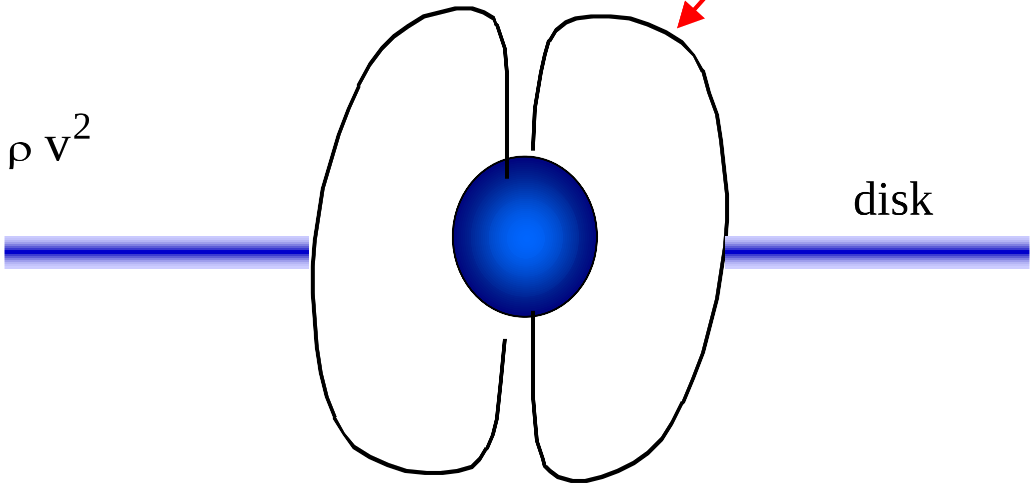
$$R_0 = R_A = \left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/7}\left(\frac{B_s^4 R^{12}}{M\dot{m}^2}\right)^{1/7}$$

Alfvén radius

radius of the stellar magnetosphere

$\dot{m}$ = **mass accretion rate**

$$p_{mag} = p_{ram}\ , \quad p_{mag} \approx B_s^2\left(\frac{R}{r}\right)^6, \quad p_{ram} = \rho\, v^2$$



- **Disk accretion**

$$R_0 = \xi\ R_A \qquad \xi \approx 1$$

x does not depend on the accretion rate

The Mass-Radius relation for SAX J1808.4-3658

F = **X-ray flux measured with the RXTE**

$$\mathbf{F = k\,\dot{m}} \qquad \mathbf{k = const}$$

$$R_0 = \xi \left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/7} \left(\frac{B_s^4 R^{12}}{M}\right)^{1/7} k^{-2/7} F^{-2/7} \equiv A\,F^{-2/7}$$

$\mathbf{F}_{\mathbf{min}}$ £ $\mathbf{F}$ £ $\mathbf{F}_{\mathbf{max}}$ X-ray flux variation observed during april-may 1998

Using (1) and (2)

$$R < \frac{A}{F_{\max}^{2/7}} < \frac{A}{F_{\min}^{2/7}} < R_{co} \Longrightarrow R < \left(\frac{F_{\min}}{F_{\max}}\right)^{2/7} R_{co}$$

The Mass-Radius relation for SAX J1808.4-3658

upper limit for the radius of the compact object in SAX J1808.4-3658

$$R < (F_{min} / F_{max})^{2/7} (GM_{\odot} / 4p^2)^{1/3} P^{2/3} (M / M_{\odot})^{1/3}$$

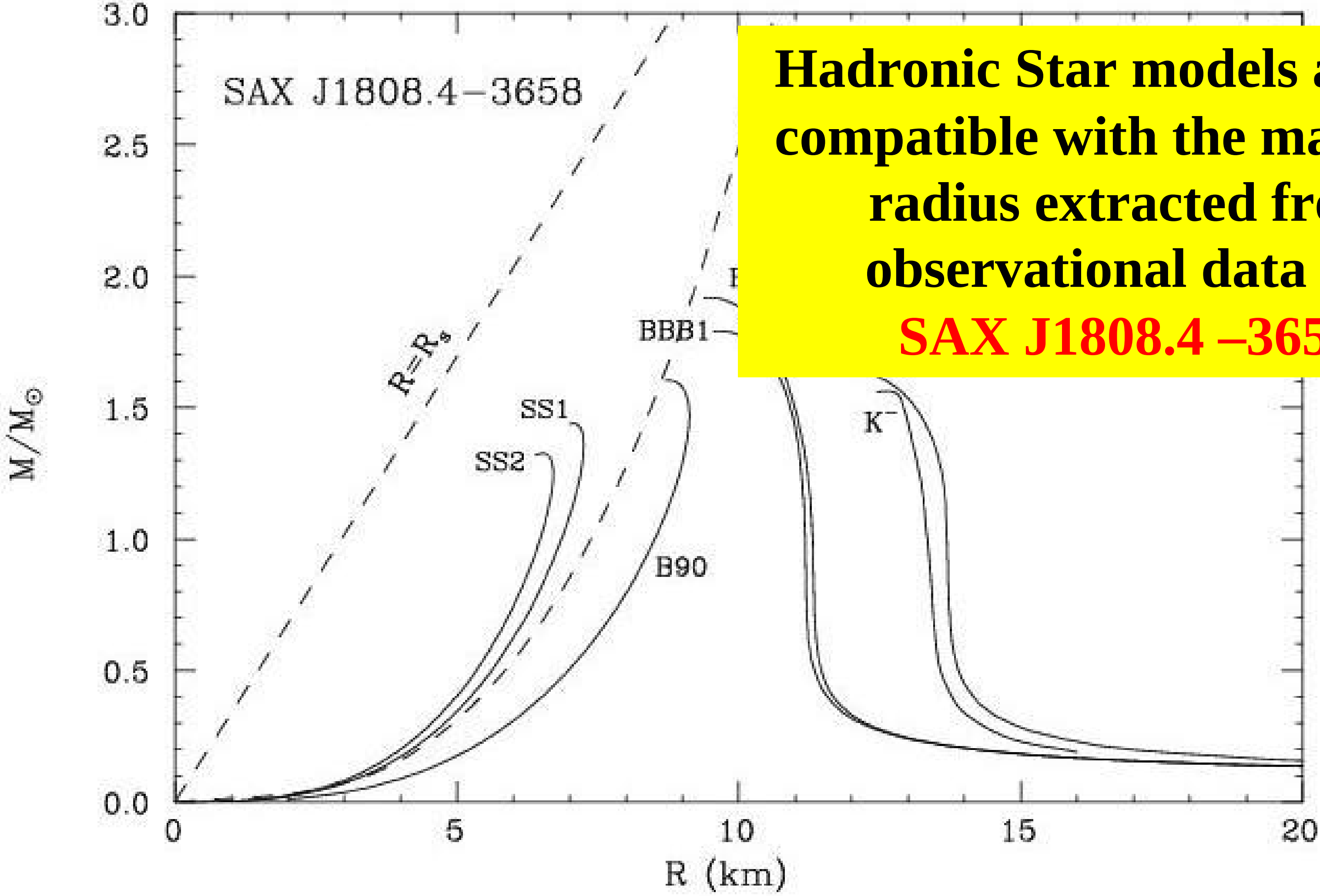
F_{min} = X-ray flux measured during the “low state” of the source
F_{max} = X-ray flux measured during the “high state” of the source

$F_{max} / F_{min} \sim 100$

P = period of the X-ray PSR

X.-D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel,
Phys. Rev. Lett. 83, (1999), 3776

A strange star candidate: SAX J1808.4 –3658



Hadronic Star models are not compatible with the mass and radius extracted from observational data for SAX J1808.4 –3658

X.-D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel, Phys. Rev. Lett. 83, (1999), 3776

Thank you for your attention.

Ignazio Bombaci