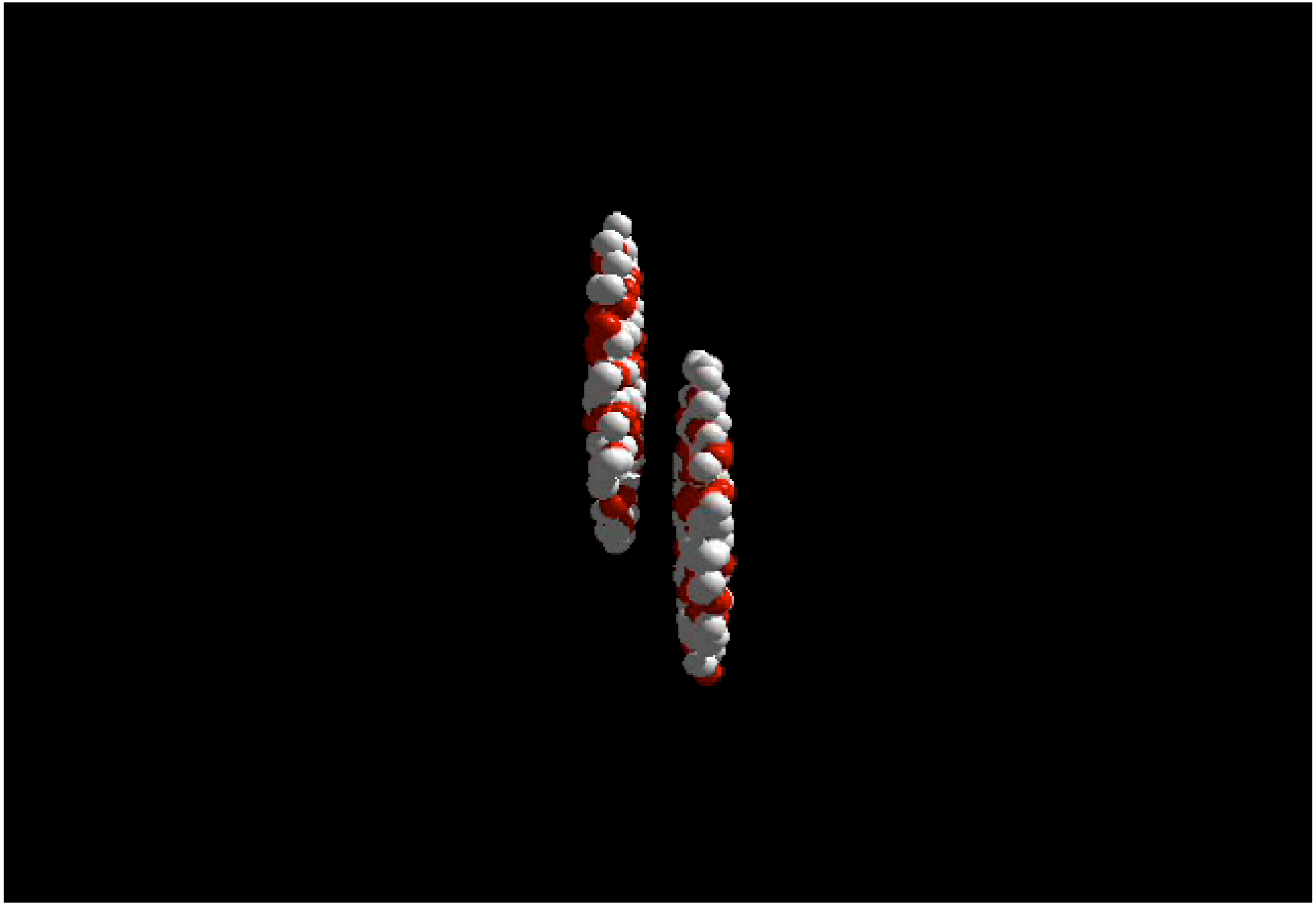
Kinetic approach to relativistic heavy ion collisions

-
- Introduction
- Molecular Dynamics with constraints for Pauli/Heisenberg principle (CoMD).
- Example: atoms and S-factor calculations, μCF.
- Infinite systems of ud quarks at zero temperature and finite baryon densities, preliminary result at finite T.
- Signals of phase transitions: order parameter, J/Ψ lifetime and large fluctuations of momentum distributions of D-mesons.
- Relativistic Boltzmann equation (ReB).
- Two and three body collisions.
- EOS.
- Conclusions and outlook



The exact one body classical distribution function satisfies the equation:

$$\partial_t f(\vec{r},\vec{p},t)+\frac{\vec{p}}{E}.\nabla_{\vec{r}} f(\vec{r},\vec{p},t)-\nabla_{\vec{r}} U.\nabla_{\vec{p}} f(\vec{r},\vec{p},t)=0$$

where: $E=\sqrt{p^2+m_q^2}$ is the energy, m_q is the (u,d) quark mass

$U=\sum_{ij} V(r_{ij})$ is the exact potential

with $V(r_{ij})$ Richardson's potential:

$$V(r_{ij})=3\sum_{a=1}^{8}\frac{\lambda_i^a}{2}\frac{\lambda_j^a}{2}\left[\frac{8\pi}{33-2n_f}\Lambda\left(\Lambda r_{ij}-\frac{f(\Lambda r_{ij})}{\Lambda r_{ij}}\right)+\frac{8\pi}{9}\alpha_s\frac{\langle\sigma_{q_i}\sigma_{q_j}\rangle}{m_{q_i}m_{q_j}}\delta(r_{ij})\right]$$

and

$$f(t)=1-4\int\frac{dq}{q}\frac{e^{-qt}}{\left[\ln\left(q^2-1\right)\right]^2+\pi^2}$$

n_f (number of flavours)=2

Λ _=0.250 Gev

λ _^a_i (Gell-Mann matrices)

The Balescu Lennard Vlasov equation

Define average (over ensembles) distribution function and mean field.

$$f_1 = \bar{f}_1 + \delta f_1 \qquad U = \overline{U} + \delta U$$

Inserting into the exact equation gives:

$$\partial_t \bar{f}_1 + \frac{p}{E}\nabla_r f_1 - \nabla_r \overline{U} \nabla_p f_1 = \left\langle \nabla_r \delta U \nabla_p \delta f_1 \right\rangle$$

Where we recognize the Vlasov equation on the LHS and The BL collision term on the RHS.

Numerically we solve CMD and average over ensembles to get the Mean field and fluctuations. Important to have particles creation and Annihilation.

Numerically the exact equation is solved by writing the one body distribution function as:

$$f(\vec{r},\vec{p},t)=\sum^{Q}\delta\left(\vec{r}-\vec{r}_i(t)\right)\delta\left(\vec{p}-\vec{p}_i(t)\right)$$

Q= $q+\bar{q}$ is the total number of quarks and antiquarks $(\bar{q}=0)$

Inserting this expression we get the Hamilton equations of motion for our system of quarks:

$$\frac{d\vec{r}_i}{dt}=\frac{\vec{p}_i}{E_i}$$

$$\frac{d\vec{p}_i}{dt}=-\vec{\nabla}_{r_i}U$$

S.Terranova and.A.B. Phys.Rev.**C70,** 024906 (2004).

- Initially we distribute randomly the quarks in a box of side L in coordinate space and in a sphere of radius P_f in momentum space.

For a Fermi gas model:

$$P_f^3 = \frac{6\pi^2 \rho_q}{g_q}$$

$g_q = n_c . n_f . n_s$ (*Degeneracy Number*)

$n_c \to 3$, $n_f \to 2$, $n_s \to 2$

- We impose periodic boundary conditions

- We consider many events and we impose that $\bar{f}_i(r,p,t) \le 1 \quad \forall i$

Constraint:

At each time step we control the value of $\bar{f}_i(\mathbf{r},\mathbf{p},t)$ *and consequently we change the momenta of the particles:*

$$\vec{p}_i = \vec{p}_i \,.\det$$

- If $f(\mathbf{r},\mathbf{p},t) > 1$

  → det > 1

- If $f(\mathbf{r},\mathbf{p},t) < 1$

  → det < 1

Test for atomic ground states where masses and forces (Coulomb) are exactly known

Convergence of Atomic G.S.

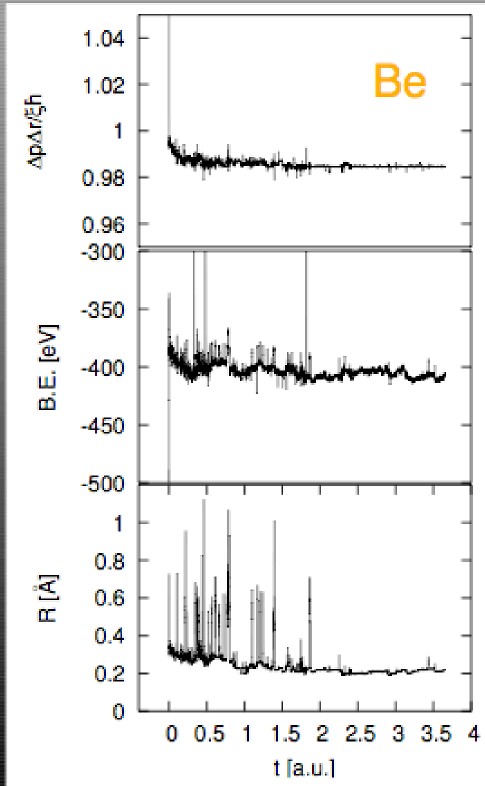
Constraint changes

Phase Space Occupation

$f(r,p,t) \leq 1$

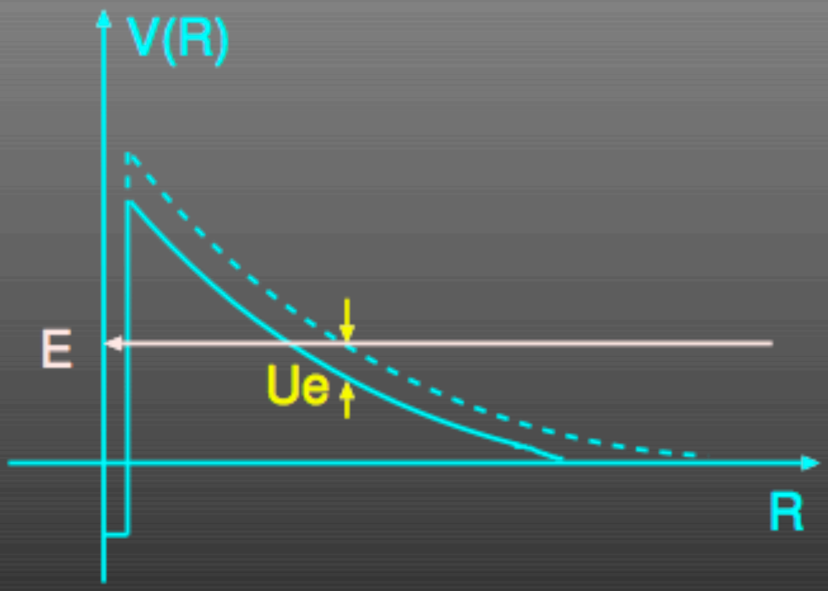
Binding energies of Atoms(in eV)

	CoMD	exper.
H	−13.56	−13.61
He	−77.70	−78.88
Li	−203.78	−203.43
Be	−404.91	−399.03
F	−2644.4	−2713.45



- S.Kimura and A.B. physics/0409008, Phys.ReV.A(2005)

Screening Energy



U_e : Screening Energy

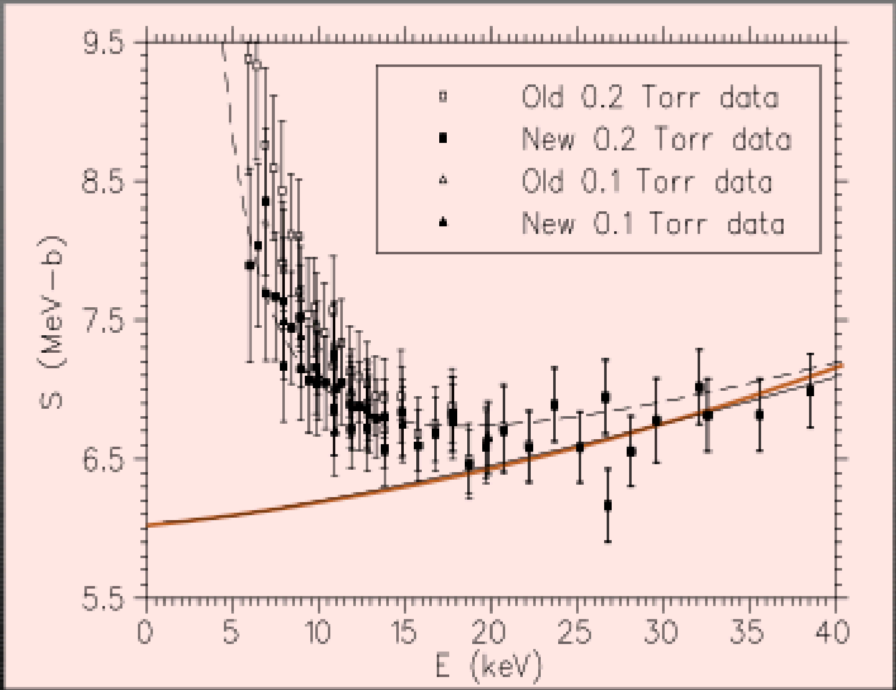
$$f_e = \frac{\sigma(E)}{\sigma_0(E)} = \frac{\sigma_0(E+U_e)}{\sigma_0(E)}$$

$$\sim \exp\{\pi\eta(E)\frac{U_e}{E}\}$$

$$U_e \sim \frac{E}{\pi\eta(E)}\log f$$

Large Enhancement

^{3}He(d,p)^{4}He



Ref. K. Langanke *et al.*, Phys. Lett. B 369,211(1996)

Energy loss, electron screening and the astrophysical ^{3}He(d,p)^{4}He cross section

Screening potential

Reaction	U_e [eV]		
	AD. app.	Ref.[1]	THM
D(d.p)T	20	8.7(Quadratic)	
		7.3(Cubic)	
^{3}He(d,p)^{4}He	119	34(Quadratic)	180±40
		60(R-matrix)	
		200(R-matrix)	
^{6}Li(d,α)^{4}He	175	259(Cubic)	320±50
		248($R-$matrix)	
^{7}Li(p,α)^{4}He	175	134(Cubic)	330±40
		204(Cubic)	
		155(R-matrix)	
		242(R-matrix)	

[1] F. C. Barker, Nucl. Phys. A 707,277(2002)

Problem has not been settled yet

Tunneling process

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i}{m_i}; \qquad \frac{d\mathbf{p}_i}{dt} = -\nabla_{\mathbf{r}} U(\mathbf{r}_i)$$

Collective coordinates and momenta

$$\mathbf{R}^{coll} \equiv \mathbf{r}_P - \mathbf{r}_T; \quad \mathbf{P}^{coll} \equiv \mathbf{p}_P - \mathbf{p}_T; \quad \mathbf{F}_P^{coll} \equiv \dot{\mathbf{P}}^{coll}$$

$$\frac{d\mathbf{r}^{\Im}_{T(P)}}{d\tau} = \frac{\mathbf{p}^{\Im}_{T(P)}}{m_{T(P)}}; \qquad \frac{d\mathbf{p}^{\Im}_{T(P)}}{d\tau} = -\nabla_{\mathbf{r}} U(\mathbf{r}^{\Im}_{T(P)}) - 2\mathbf{F}^{coll}_{T(P)}$$

Tunneling penetrability: $\Pi(E) = \left(1 + \exp\left(2\mathcal{A}(E)/\hbar\right)\right)^{-1}$

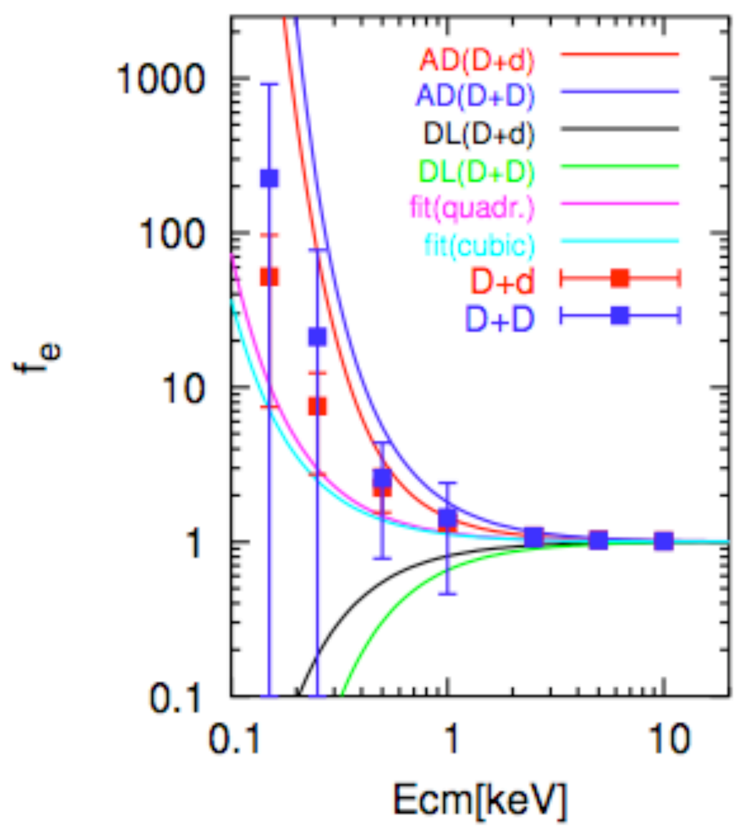
$$\mathcal{A}(E) = \int_{r_b}^{r_a} \mathbf{P}^{coll}\, d\mathbf{R}^{coll}$$

without electron $\Rightarrow \Pi_0(E)$

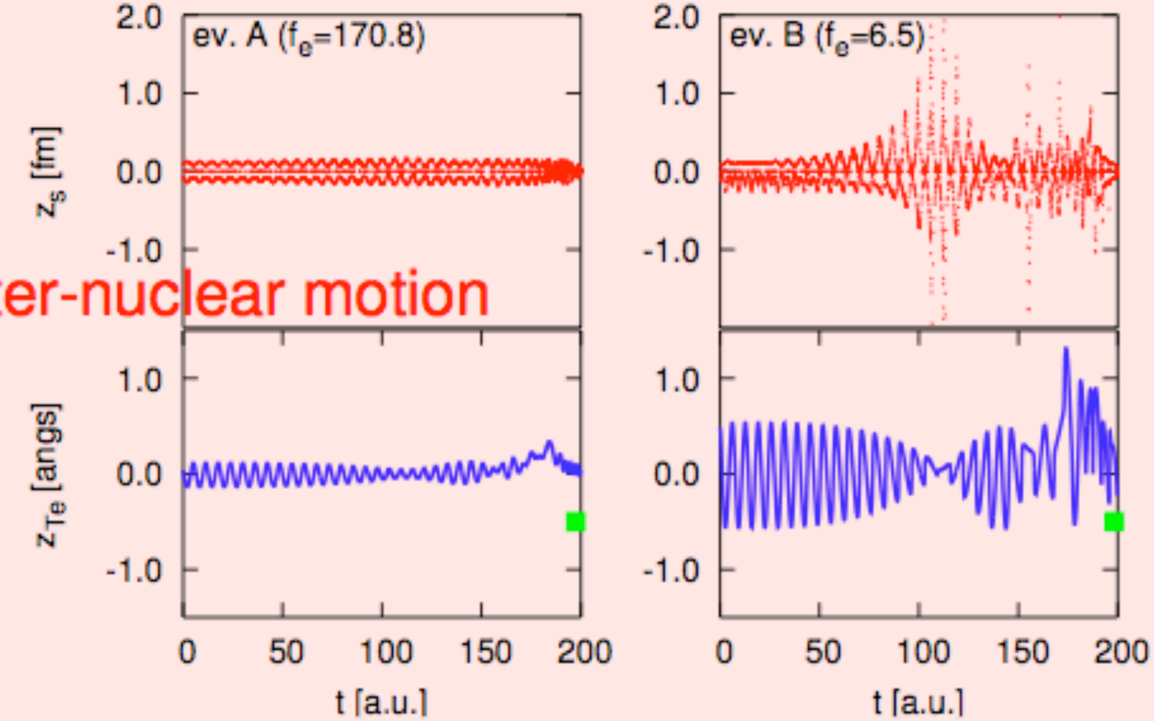
Enhancement factor: $f_e = \Pi(E)/\Pi_0(E)$

Dissipative Limit

Bound electron emission



Electronic Motion



Oscillational motion of
Electron around the target

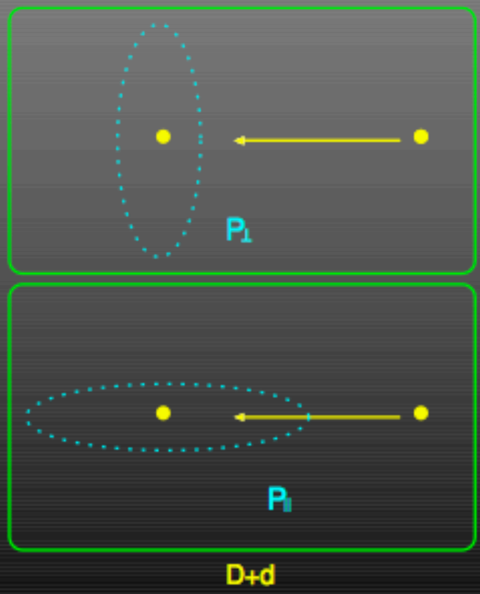
S.Kimura, and A.Bonasera
Phys.Rev.Lett.93,
262502(2004)

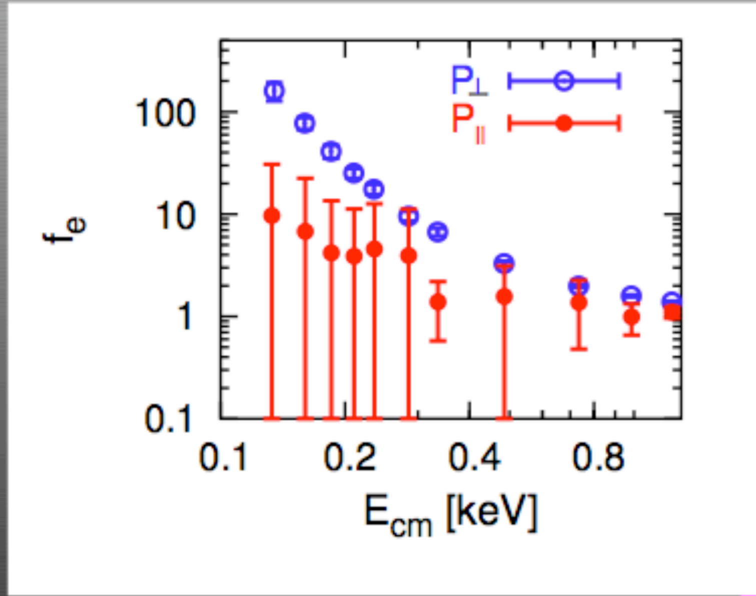
Sensitive Initial Phase

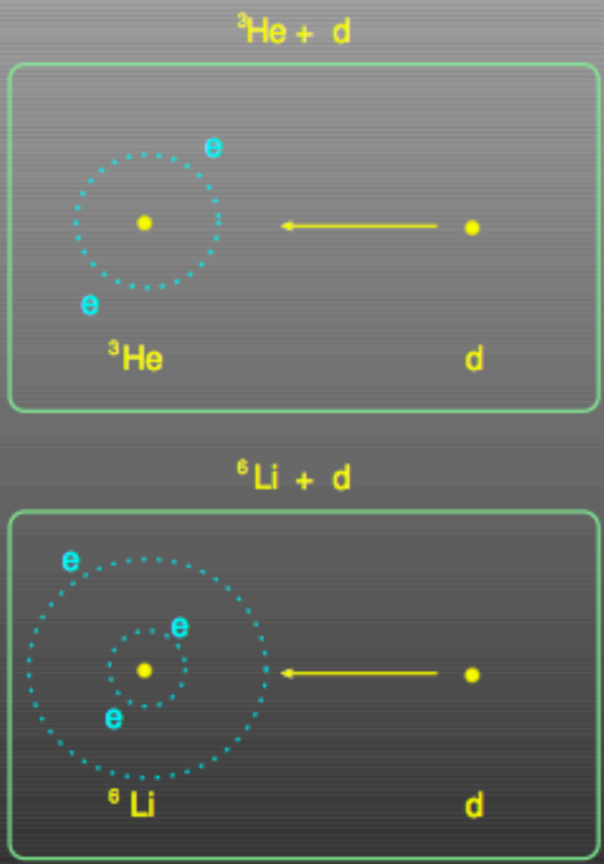
Space Configuration Dependence ⇒ Chaos

Polarized Target

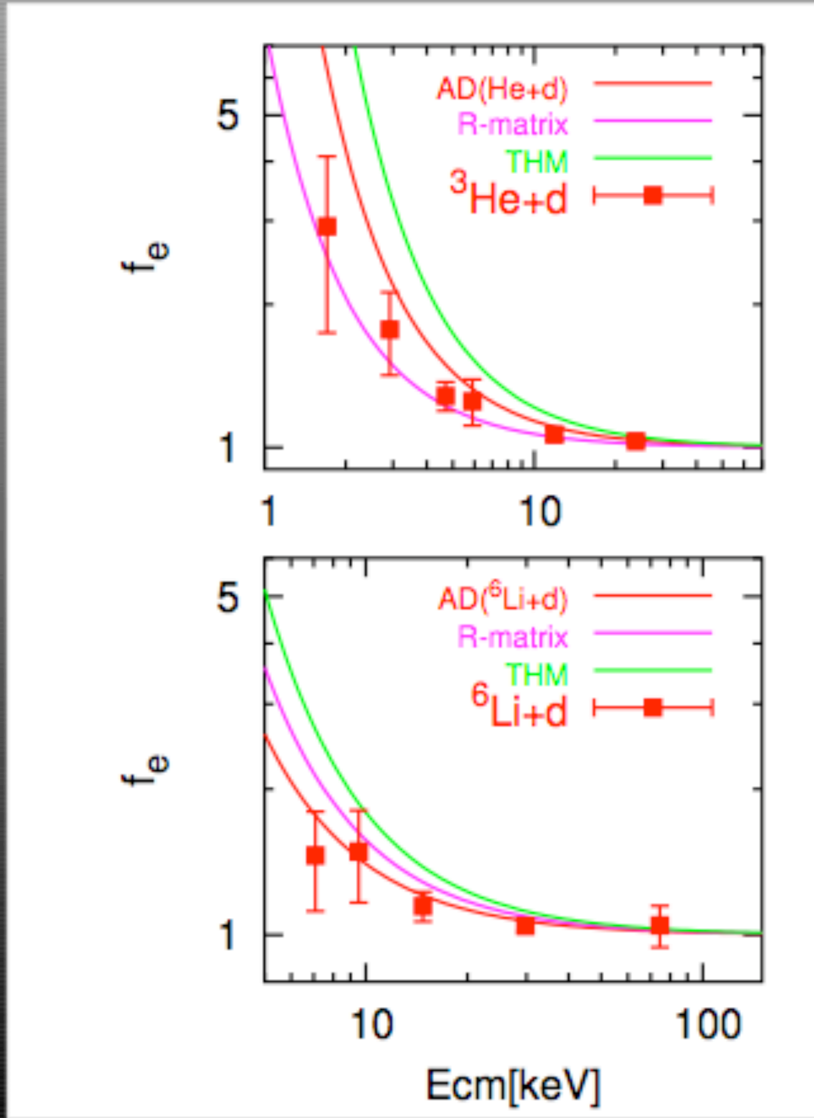
Enhancement factor with Polarized target



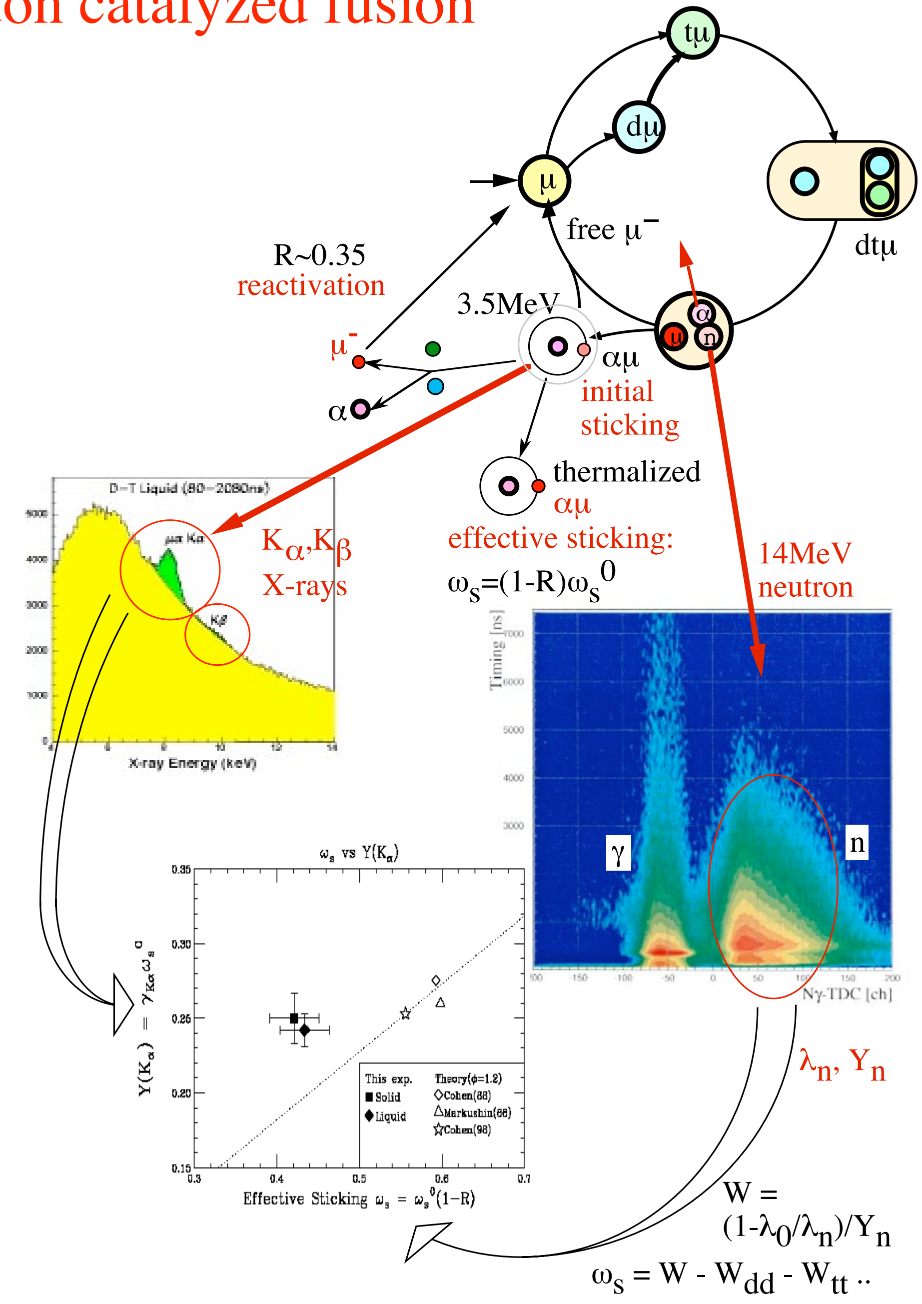




S.Kimura, and A. Bonasera
Nucl.Phys.A in press
nucl-th/0504005



Muon catalyzed fusion



arXiv:physics/0605206 v1 24 May 2006

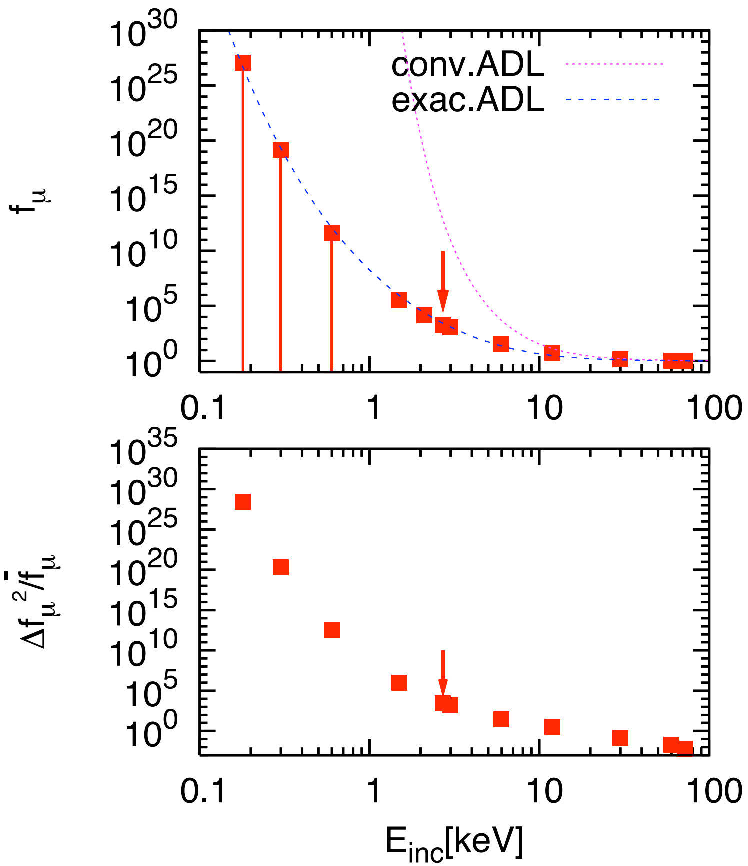


FIG. 1: Enhancement factor by the bound muon (top panel) and $\Delta f_\mu^2/\bar{f}_\mu$ (bottom panel) as functions of the incident center-of-mass energy. The arrows in the figure indicate the point where total energy is zero.

S.Kimura and A.B.

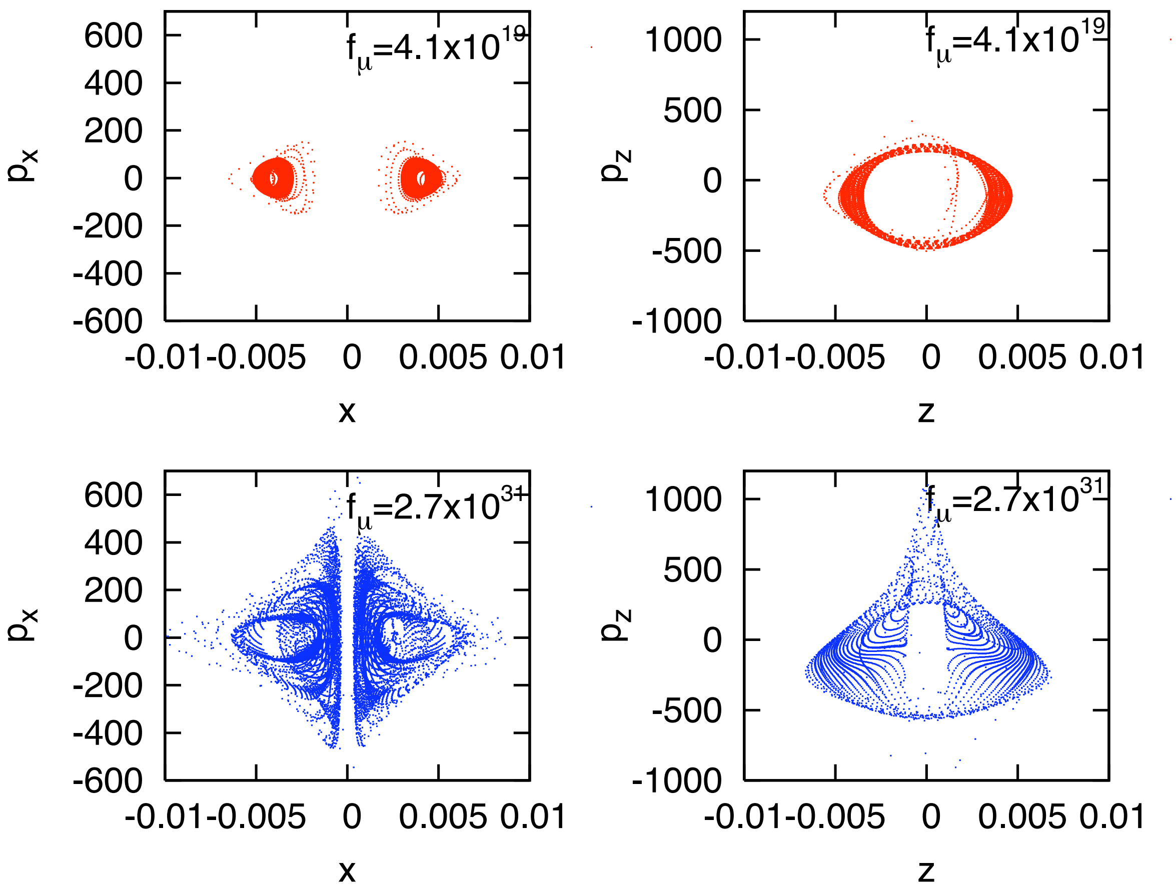


FIG. 2: Surface of section for 2 events, one has small f_μ (top panels) and the other has large f_μ (bottom panels), on the x-p_x(left panels) and the z-p_z(right panels) planes at the incident c.o.m energy 0.18keV, in the atomic unit

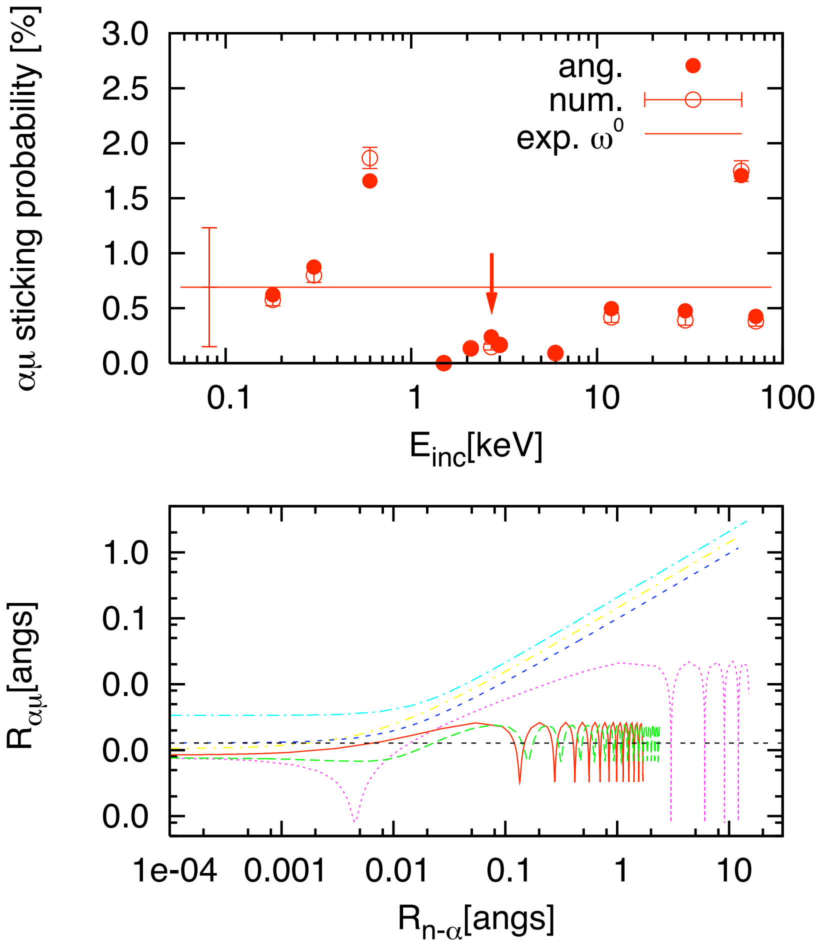
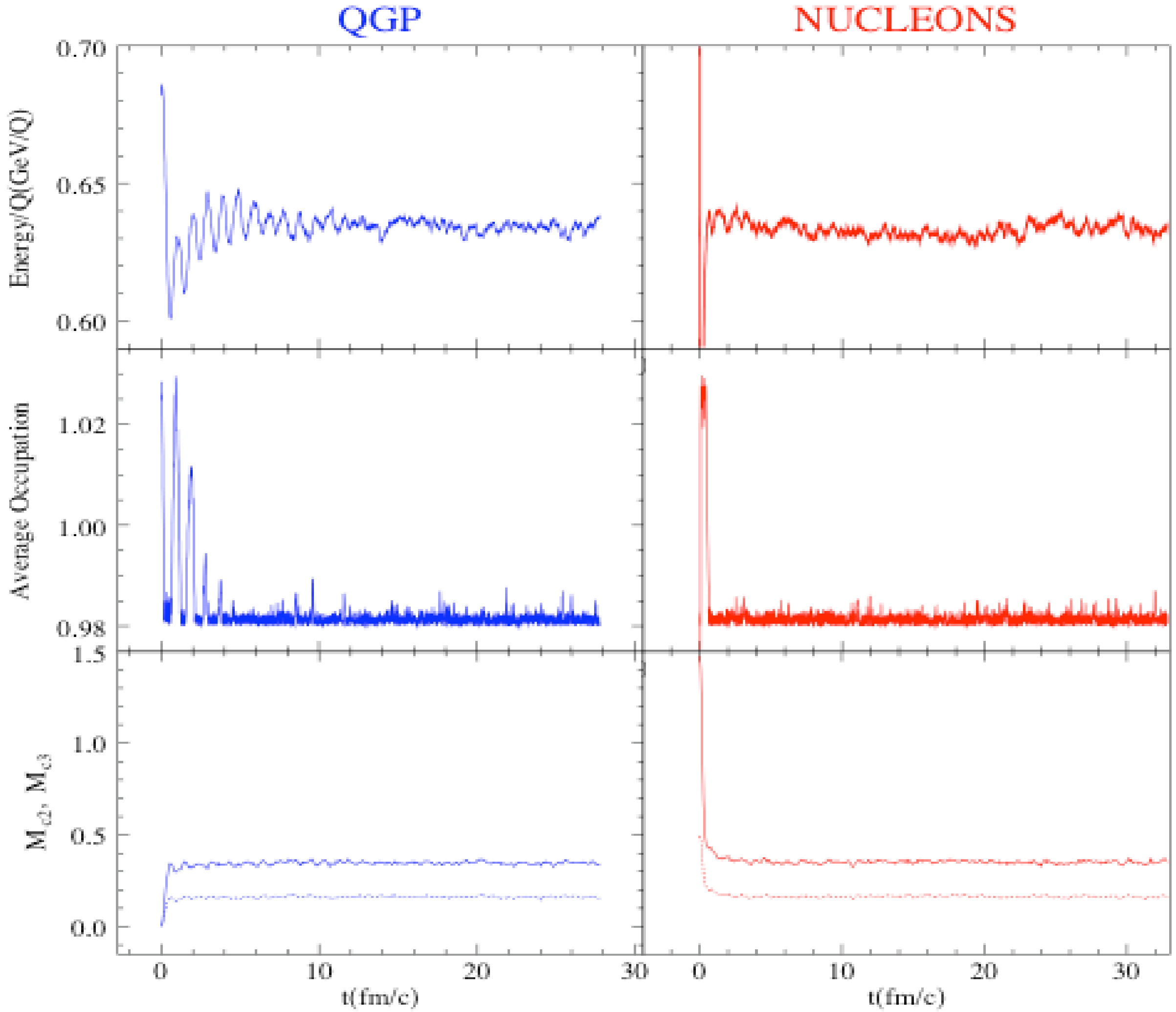


FIG. 3: Incident energy dependence of the sticking probability of the muon on the α particle. The statistical error is shown by error bars, otherwise it is within the size of the points in the figure. (top panel) Distance between the muon and the alpha particle as a function of the inter-nuclear separation (bottom panel)



QGP
NUCLEONS
Energy/Q(GeV/Q)
0.70
0.65
0.60
Average Occupation
1.02
1.00
0.98
M_{c2}, M_{c3}
1.5
1.0
0.5
0.0
0
10
20
30
t(fm/c)
t(fm/c)

We define an ***Order Parameter*** as:

$$M_c = \frac{1}{N}\sum_{i=1}^{N}\sum_{a=3,8}(\lambda_j^a\lambda_k^a + \lambda_i^a\lambda_j^a + \lambda_i^a\lambda_k^a)$$
$$= M_{cr} + \frac{1}{N}\sum_{a=3,8}(\lambda_j^a\lambda_k^a + \lambda_i^a\lambda_k^a)$$

j and k are the two quarks closest to quark i

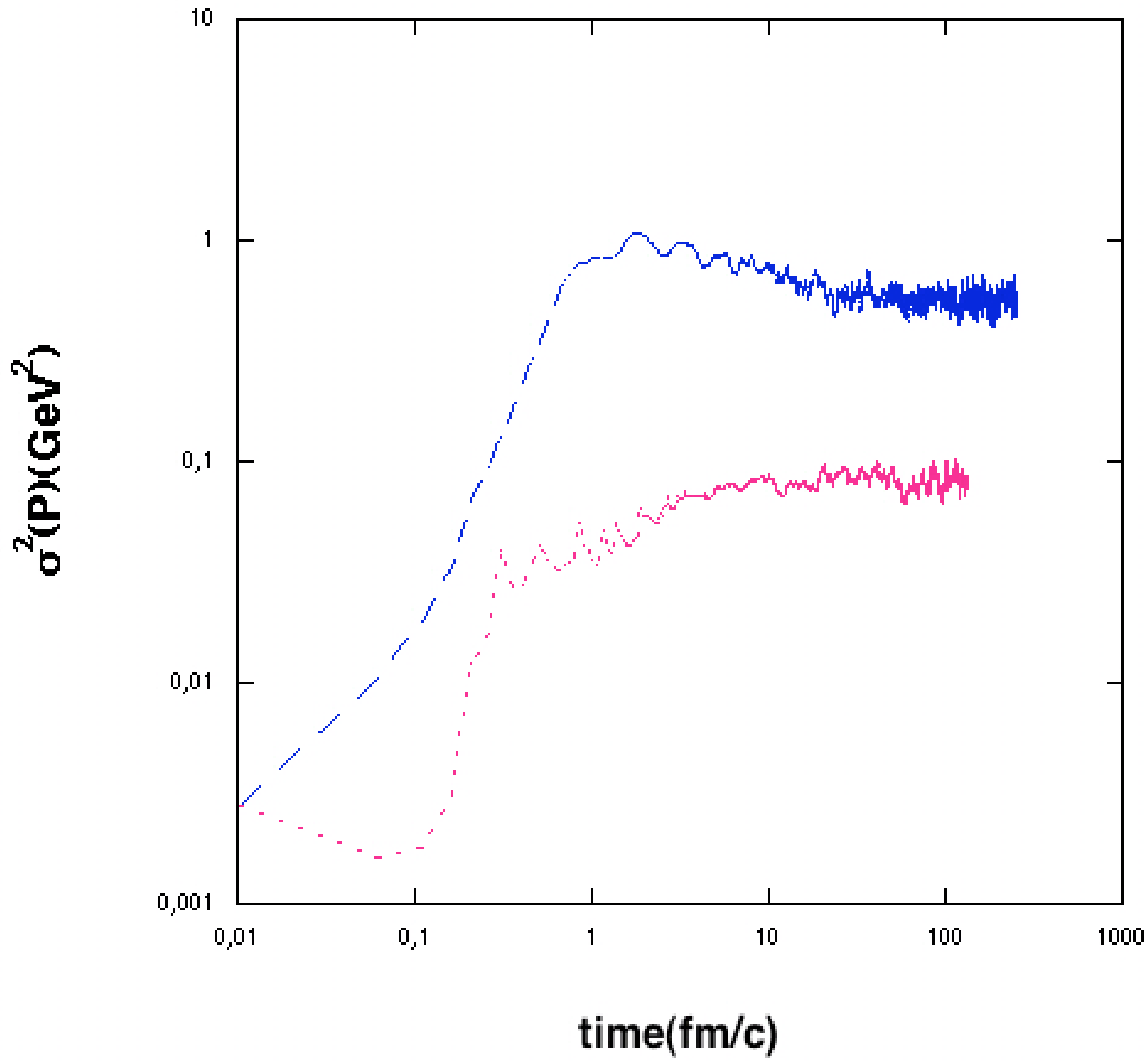
M_{cr} = Reduced order parameter.

We normalize the order parameter:

$$\tilde{M}_c = \frac{2}{9}[M_c + 3]$$

$$\tilde{M}_{cr} = \frac{2}{3}[M_{cr} + 1]$$

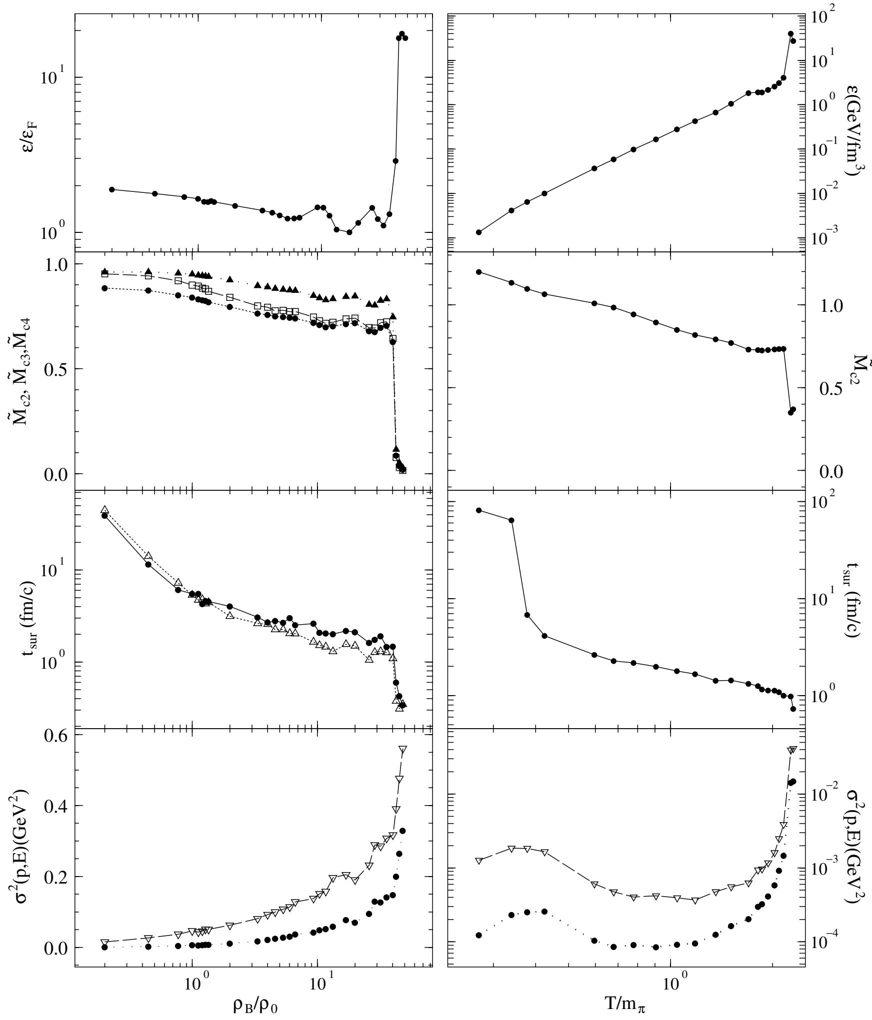
- if the three closest quarks have different colours
  $\tilde{M}_c = 1$; $\tilde{M}_{cr} = 1$ ***Isolated white nucleon***
- if the three closest quarks have two different colours $\tilde{M}_c = \frac{2}{3}$; $\tilde{M}_{cr} = \frac{2}{3}$ ***Quark Gluon Plasma***
- if the three closest quarks have the same colour
  $\tilde{M}_c = 0$; $\tilde{M}_{cr} = 0$ ***Exotic colour clustering***



10
1
0,1
0,01
0,001
0,01
0,1
1
10
100
1000
$\sigma^2(P)(GeV^2)$
time(fm/c)

First order phase transition

m_u=5MeV,
m_d=10MeV,
r_{cut}=2fm



S.Terranova, D.M.Zhou and A.B.,arXiv-nucl/th0412031 EPJA26(2005)333.

Kinetic approach

- Introduction
- JPCIAE model (old code)
- ReB model (new code)
- Preliminary results

both based on PYTHIA

Z.G.Tan, D.M.Zhou, S.Terranova and A.B., in preparation

JPCIAE model (old code)

JPCIAE is based on PYTHIA*,

PYTHIA is a well- known event generator for hadron- hadron collisions.

(Using particle list and collision list method)

(1) Radial position of a nucleon in colliding nucleus A
Woods-Saxon distribution.

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R_A)/\alpha)}$$

(2) Solid angle of the nucleon sampled uniformly in 4π

An Tai and Ben-Hao Sa, Comput. Phys. Commun. 116, 353 (1999)

* T. Sjostrand, Comput. Phys. Commun. 82, 74(1994)

The nucleon-nucleon collision with the least collision time is selected from the initial collision list to perform the first collisions.

The particle list and the collision list are then updated.

New collision list may consist of:

(1) nucleon-nucleon collisions,
(2) nucleon-produced particle collisions,
(3) produced particle-produced particle collisions

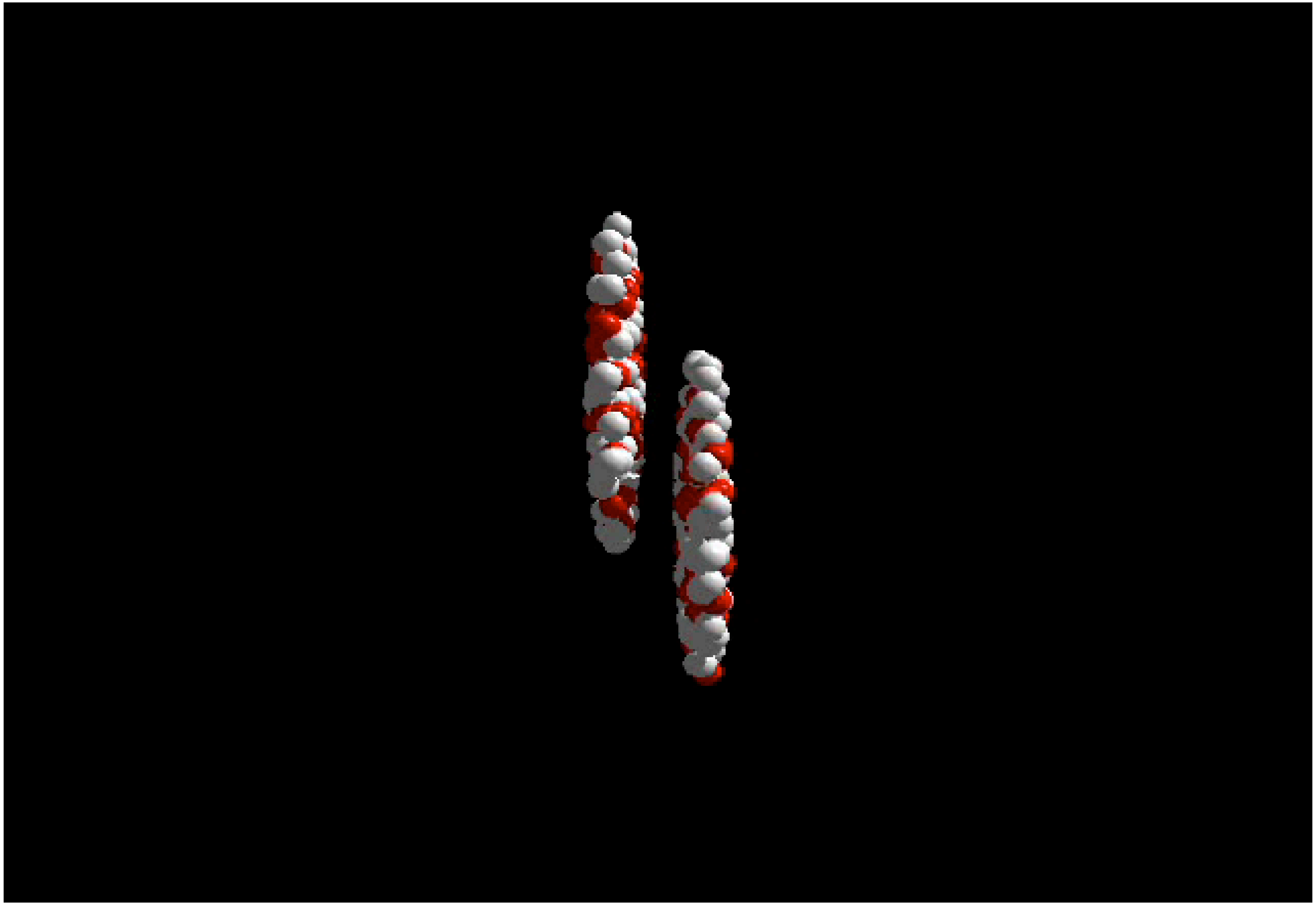
The next collision is selected from the new collision list and the processes above are repeated until the collision list is empty.

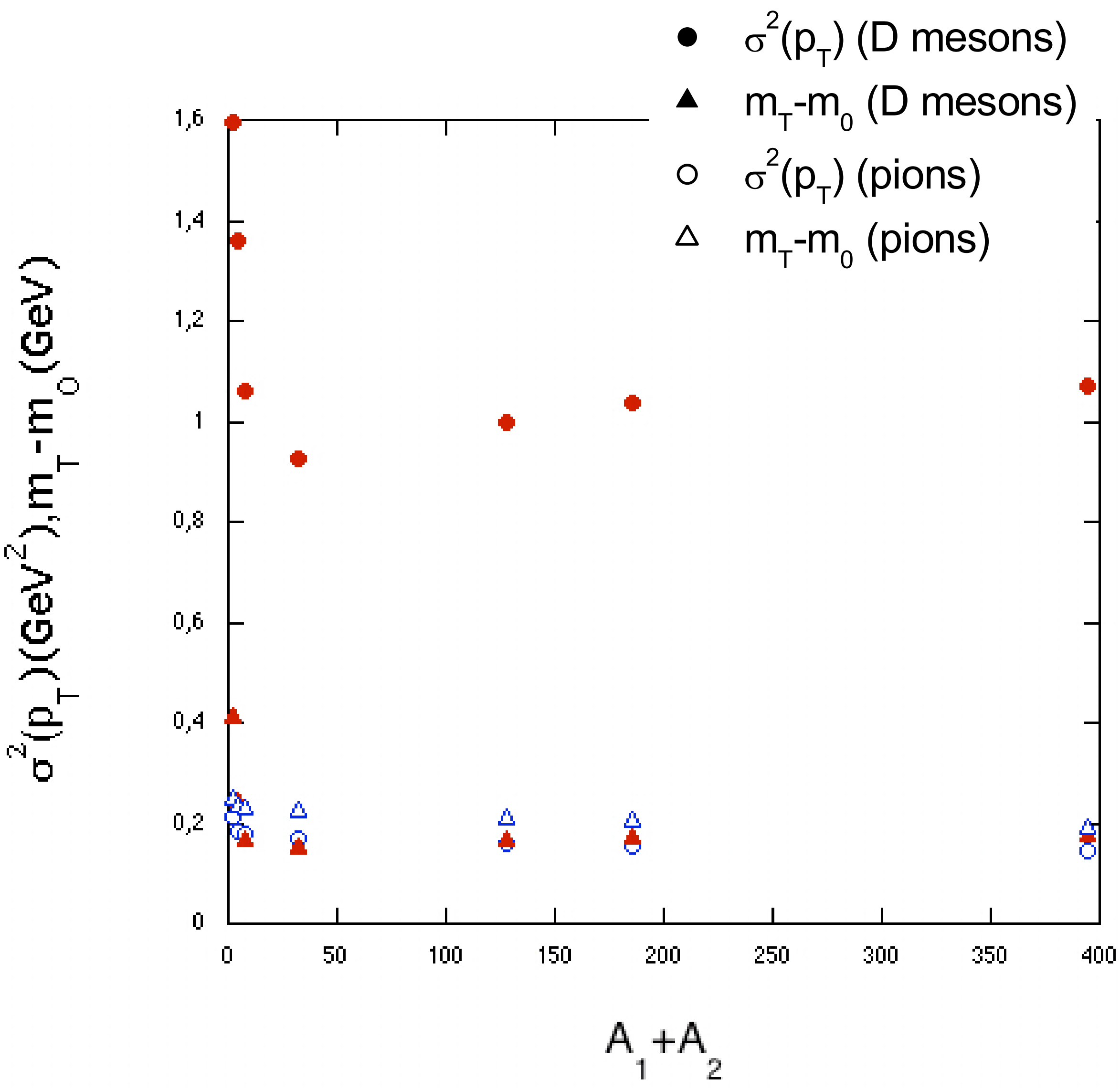
3) Beam momentum of each nucleon is given in *z* direction and zero initial momenta in x and y directions.

⟹ initial particle list

4)The collision time of each colliding pair is calculated under the requirement that the least approach distance of the colliding pair along their straight line trajectory should be smaller than $\sqrt{\sigma_{tot}/\pi}$.

⟹ initial collision list





$\sigma^2(p_T)$ (D mesons)
m_T-m_0 (D mesons)
$\sigma^2(p_T)$ (pions)
m_T-m_0 (pions)
$\sigma^2(p_T)(GeV^2), m_T - m_0$ (GeV)
$A_1 + A_2$
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
0
50
100
150
200
250
300
350
400

☆ ReB model (new code)

⟹ based on mean free path idea

(1) Radial position of a nucleon in colliding nucleus A sampled in Woods-Saxon distribution.

(2) Solid angle of the nucleon sampled uniformly in 4π

(3) Beam momentum of each nucleon is given in *z* direction and zero initial momentum in x and y direction

(4) The origin of the time is set at the moment when the projectile and target nuclei touch

⟹

Using time evolution method

At each time step a two(three)-body collision takes place in this way:

1) For each particle i, find the closest particle j in phase space

2) A mean free path is defined as:

$$\lambda = \frac{1}{\bar{\sigma}\rho(1 + \rho\sigma^{3/2} + ..)\Pi_i(1 \pm f_i)}$$

$\bar{\sigma}$ ⟶ **Energy dependent cross section**

ρ ⟶ **density**

f_i ⟶ occupation function (+ bosons,-Fermions)

A. Bonasera, F. Gulminelli, J. Molitoris, Phys. Rep., 243(1&2), (1994) 1-124

3) A collision probability is:

$$\Pi_{i,j} = \frac{\Delta t}{\Delta t_{coll}} = \frac{\Delta t \upsilon_{ij}}{\lambda} = \Delta t \upsilon_{ij} \overline{\sigma}\, \rho(1+..)$$

Δt ⟹ is the time step interval

υ_{ij} ⟹ the relative velocity of particle i and j

4) A random number χ, in the $(0,1)$ interval, is compared with Π_{ij} ,if $x < \Pi_{ij}$ the collision can occur .

After each collision the particle list is updated

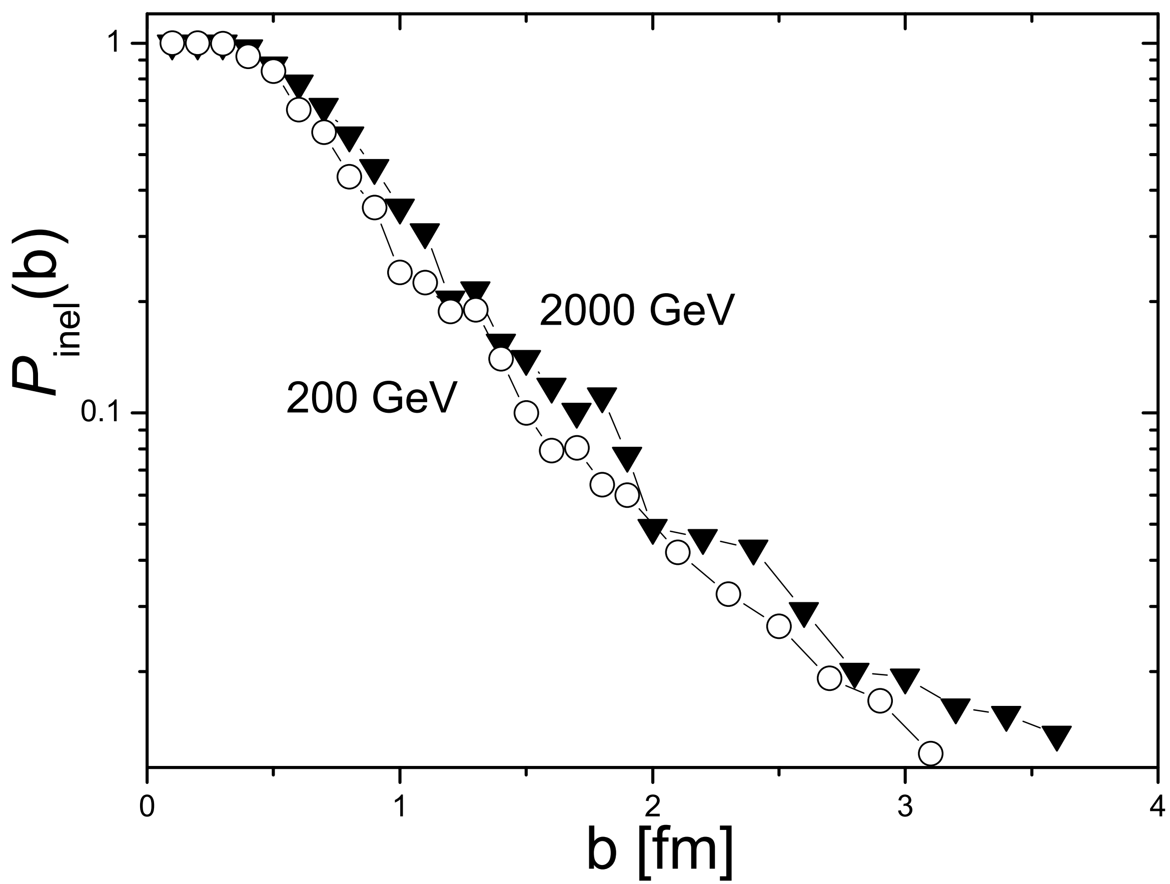
Both the old code and the new code:

For each collision pair:

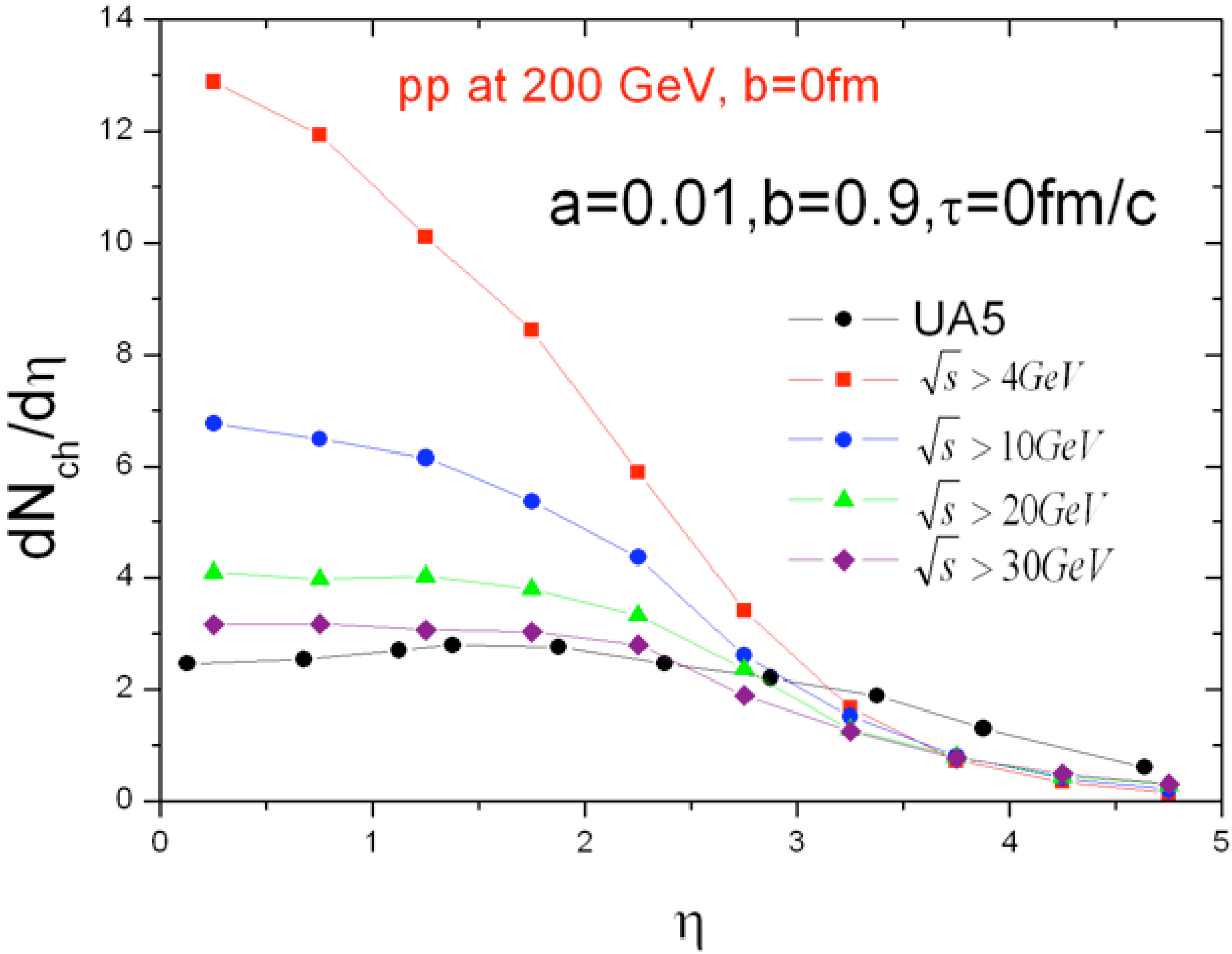
(a) If CMS energy > 4 GeV → strings are formed → PYTHIA is used to deal with particle production.

(b) Otherwise, the collision is treated as a two-body collision.

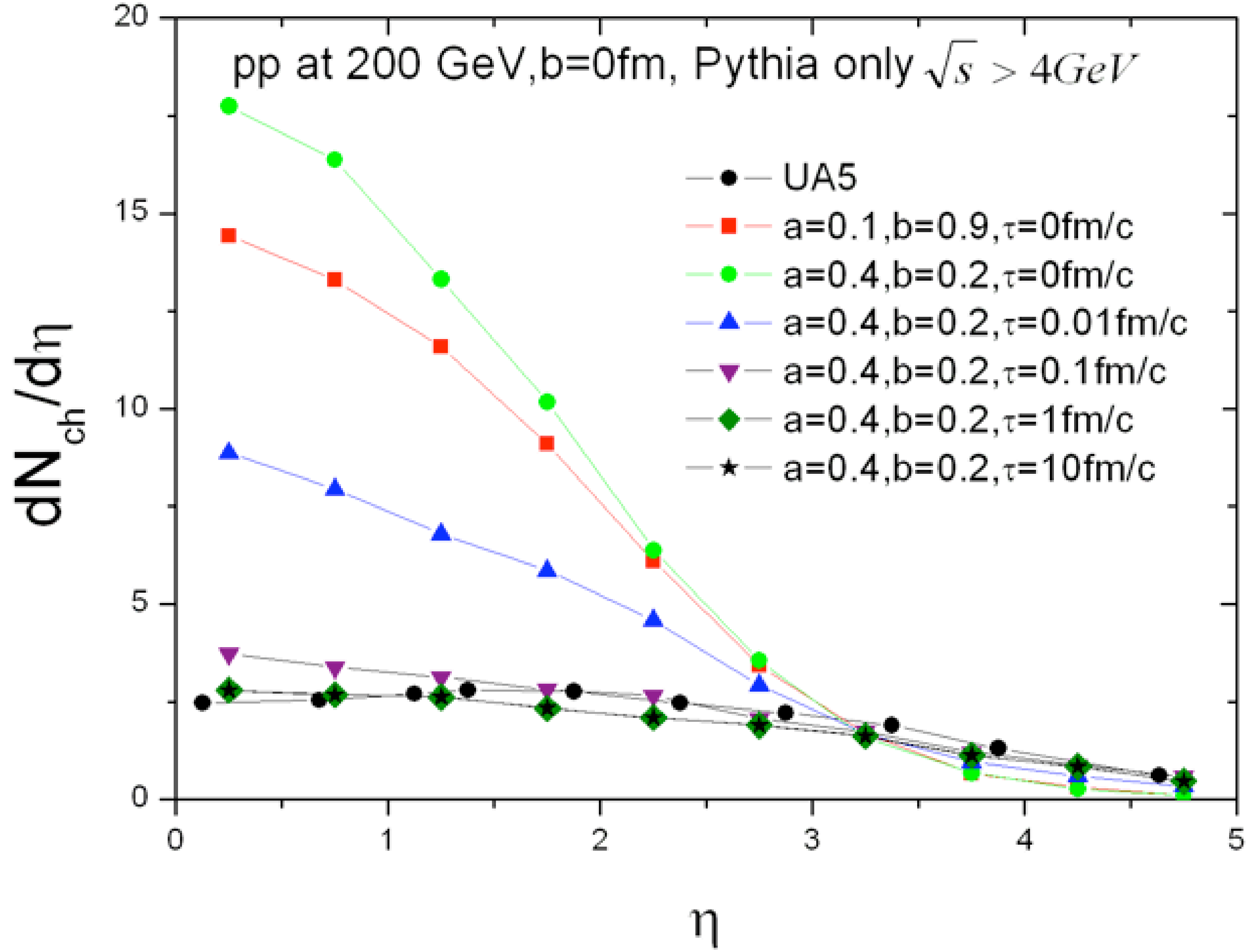
The threshold , 4 GeV, is chosen in such a way that JPCIAE correctly reproduces the charged multiplicity distributions in nucleon-nucleon collisions.



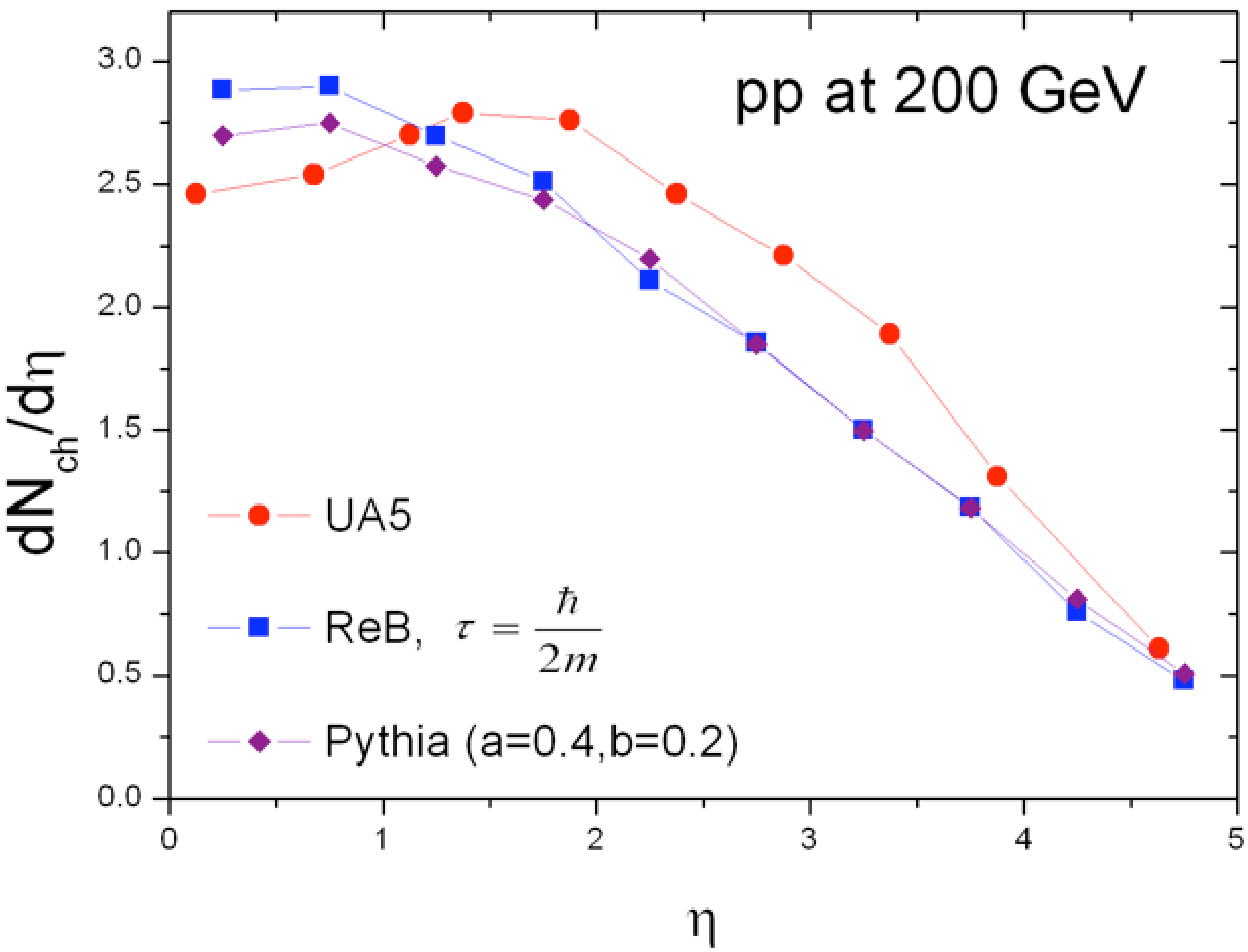
1
0.1
0
1
2
3
4
$P_{inel}(b)$
b [fm]
2000 GeV
200 GeV



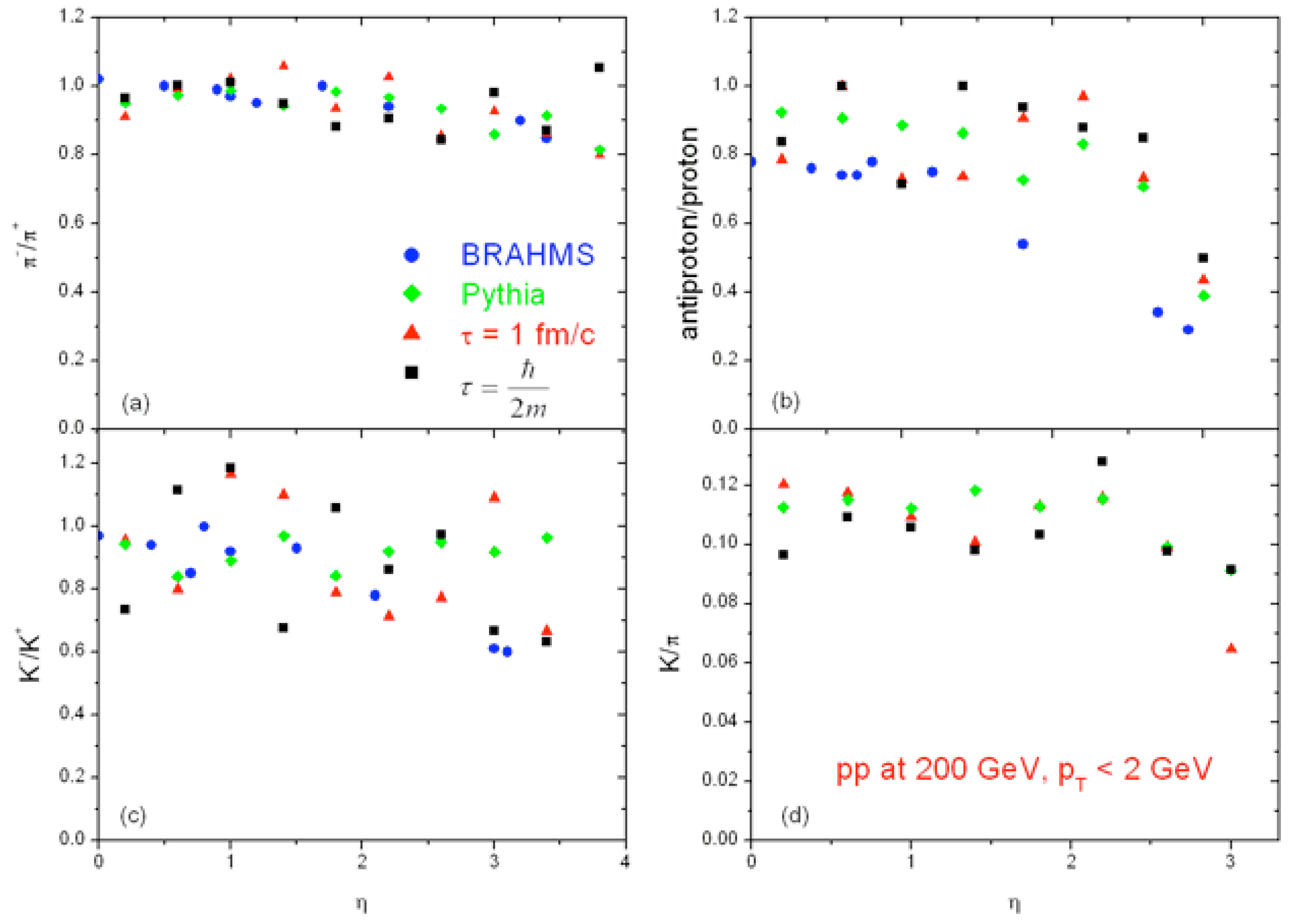
pp at 200 GeV, b=0fm
a=0.01,b=0.9,τ=0fm/c
UA5
$\sqrt{s} > 4GeV$
$\sqrt{s} > 10GeV$
$\sqrt{s} > 20GeV$
$\sqrt{s} > 30GeV$
$dN_{ch}/d\eta$
η
0
2
4
6
8
10
12
14
0
1
2
3
4
5



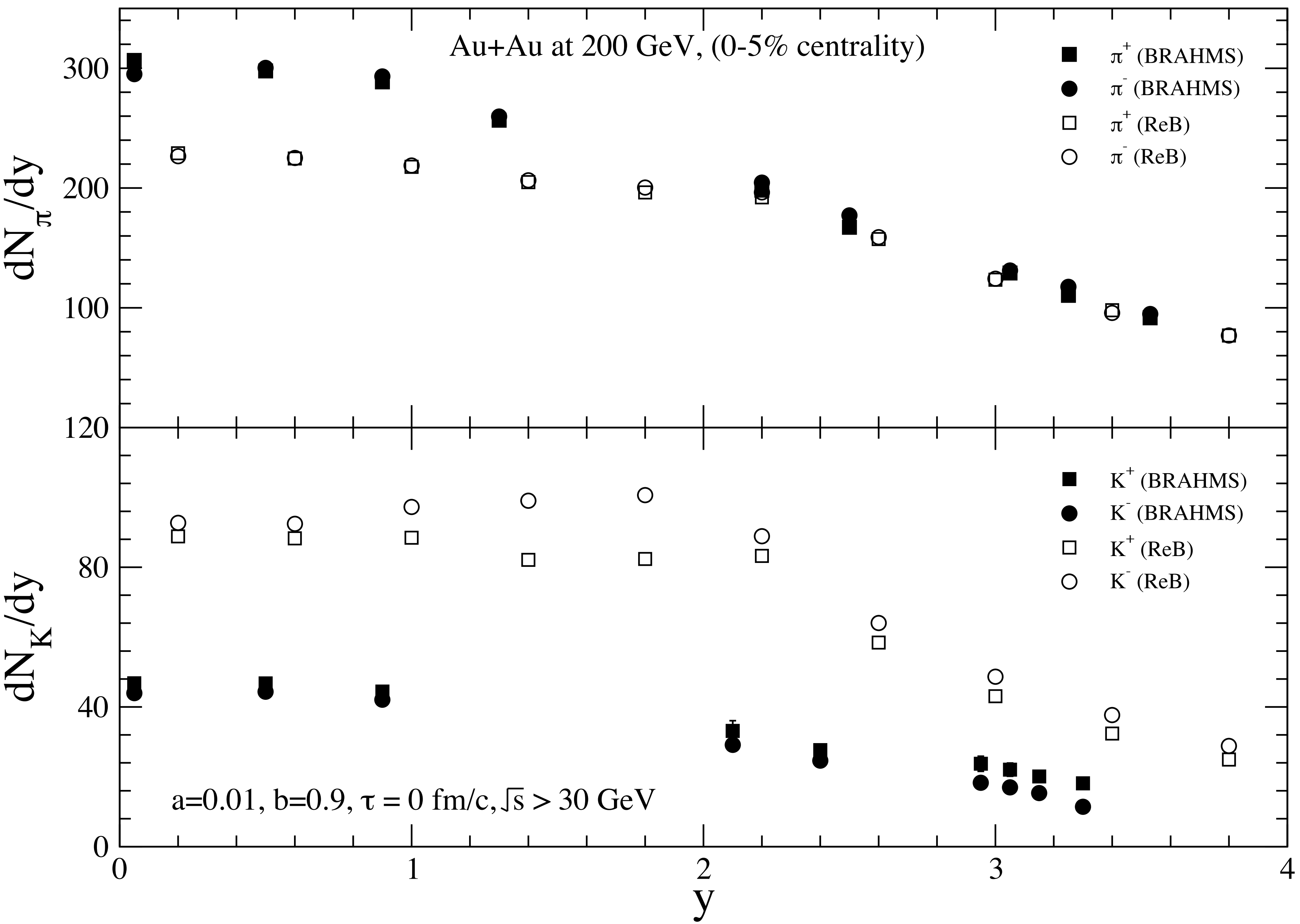
pp at 200 GeV,b=0fm, Pythia only $\sqrt{s} > 4GeV$
UA5
a=0.1,b=0.9,τ=0fm/c
a=0.4,b=0.2,τ=0fm/c
a=0.4,b=0.2,τ=0.01fm/c
a=0.4,b=0.2,τ=0.1fm/c
a=0.4,b=0.2,τ=1fm/c
a=0.4,b=0.2,τ=10fm/c
$dN_{ch}/d\eta$
η
0
5
10
15
20
0
1
2
3
4
5



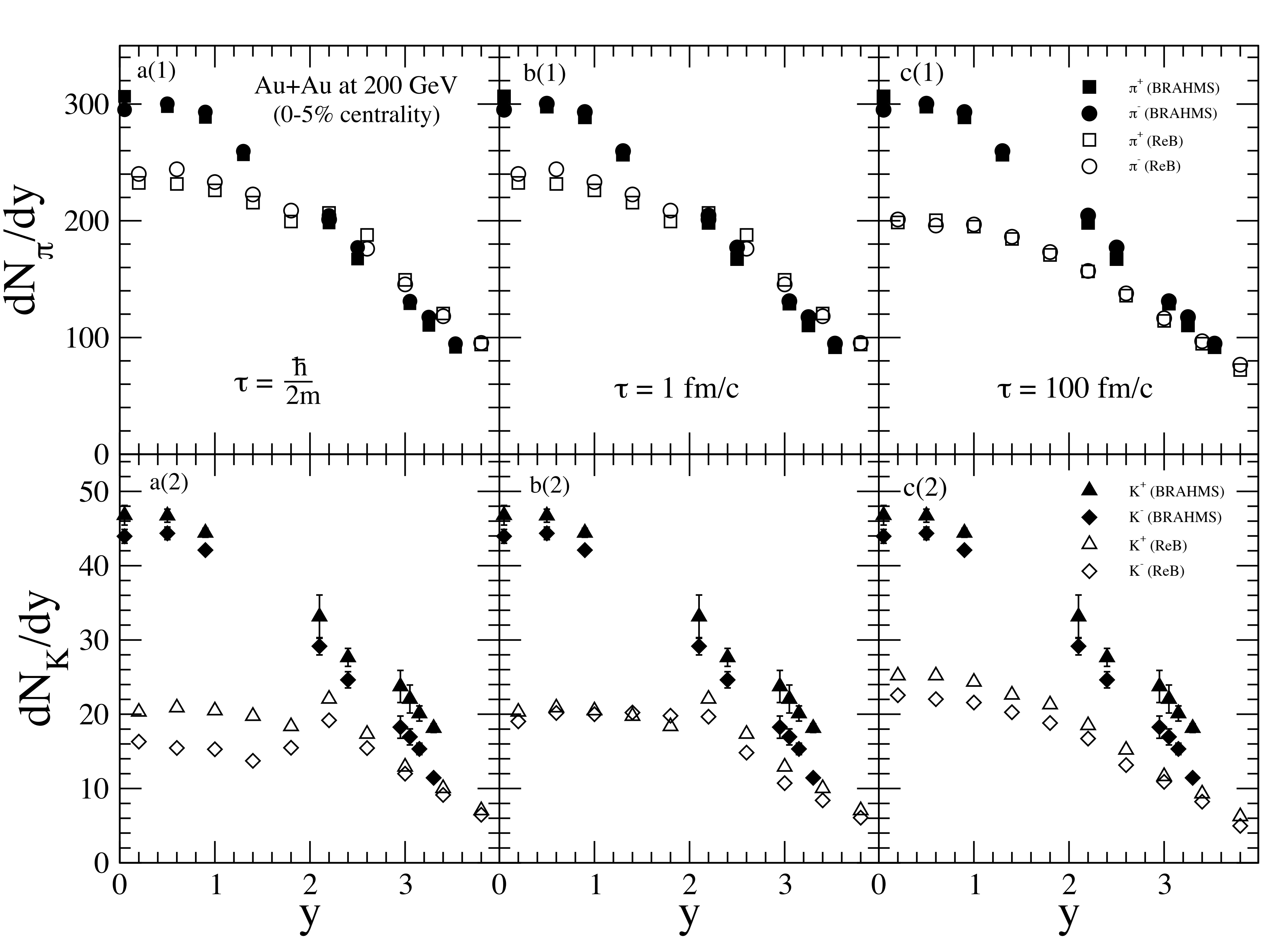
pp at 200 GeV
dN_ch/dη
η
UA5
ReB, $\tau = \frac{\hbar}{2m}$
Pythia (a=0.4,b=0.2)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5



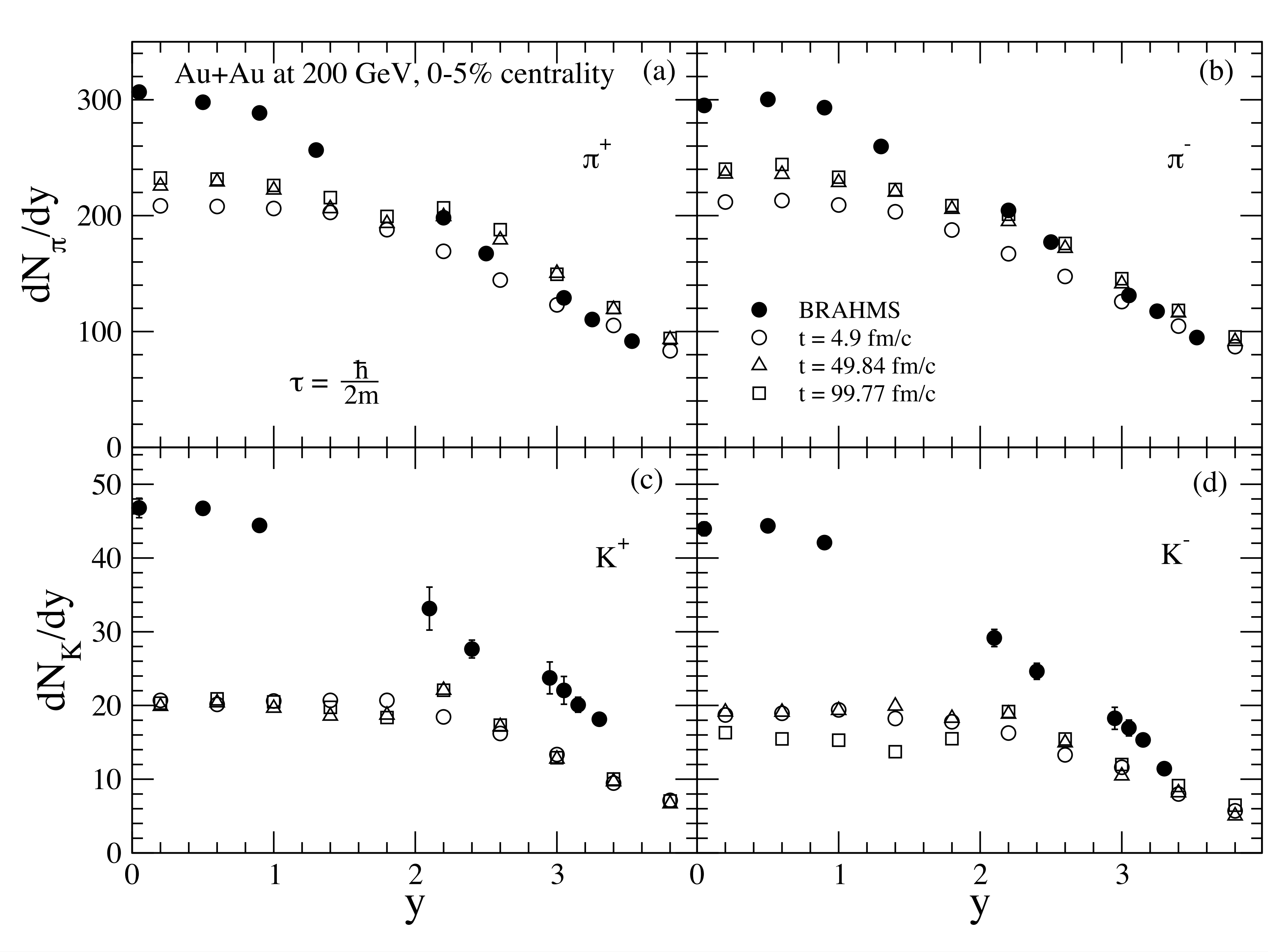
BRAHMS
Pythia
τ = 1 fm/c
$\tau = \frac{\hbar}{2m}$
(a)
(b)
(c)
(d)
π^-/π^+
antiproton/proton
K^-/K^+
K/π
η
pp at 200 GeV, p_T < 2 GeV



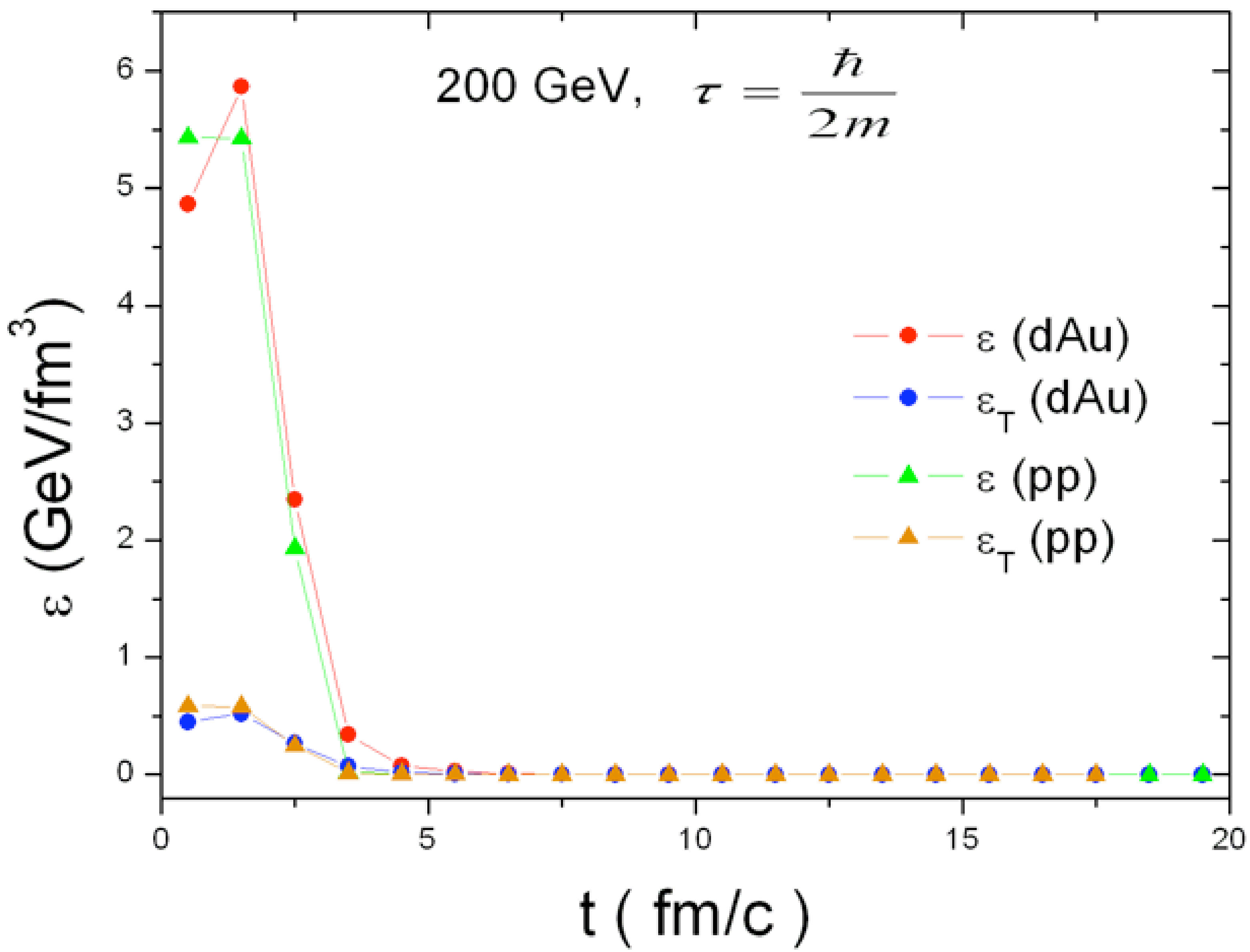
Au+Au at 200 GeV, (0-5% centrality)
π^+ (BRAHMS)
π^- (BRAHMS)
π^+ (ReB)
π^- (ReB)
dN_π/dy
300
200
100
120
80
40
0
K^+ (BRAHMS)
K^- (BRAHMS)
K^+ (ReB)
K^- (ReB)
dN_K/dy
a=0.01, b=0.9, τ = 0 fm/c, $\sqrt{s}$ > 30 GeV
0
1
2
3
4
y



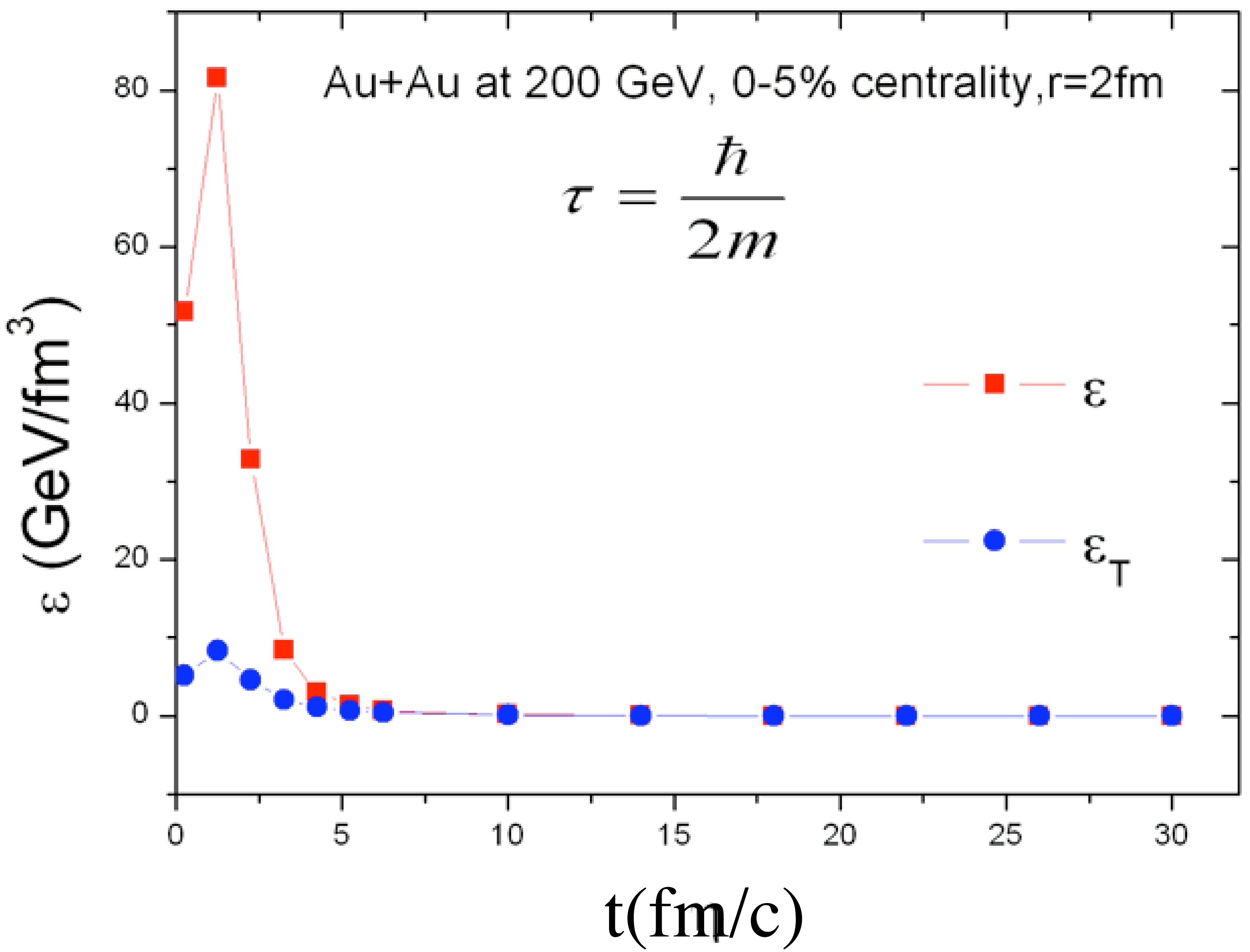
a(1)
Au+Au at 200 GeV
(0-5% centrality)
b(1)
c(1)
π+ (BRAHMS)
π- (BRAHMS)
π+ (ReB)
π- (ReB)
τ = ħ/2m
τ = 1 fm/c
τ = 100 fm/c
dNπ/dy
a(2)
b(2)
c(2)
K+ (BRAHMS)
K- (BRAHMS)
K+ (ReB)
K- (ReB)
dNK/dy
y

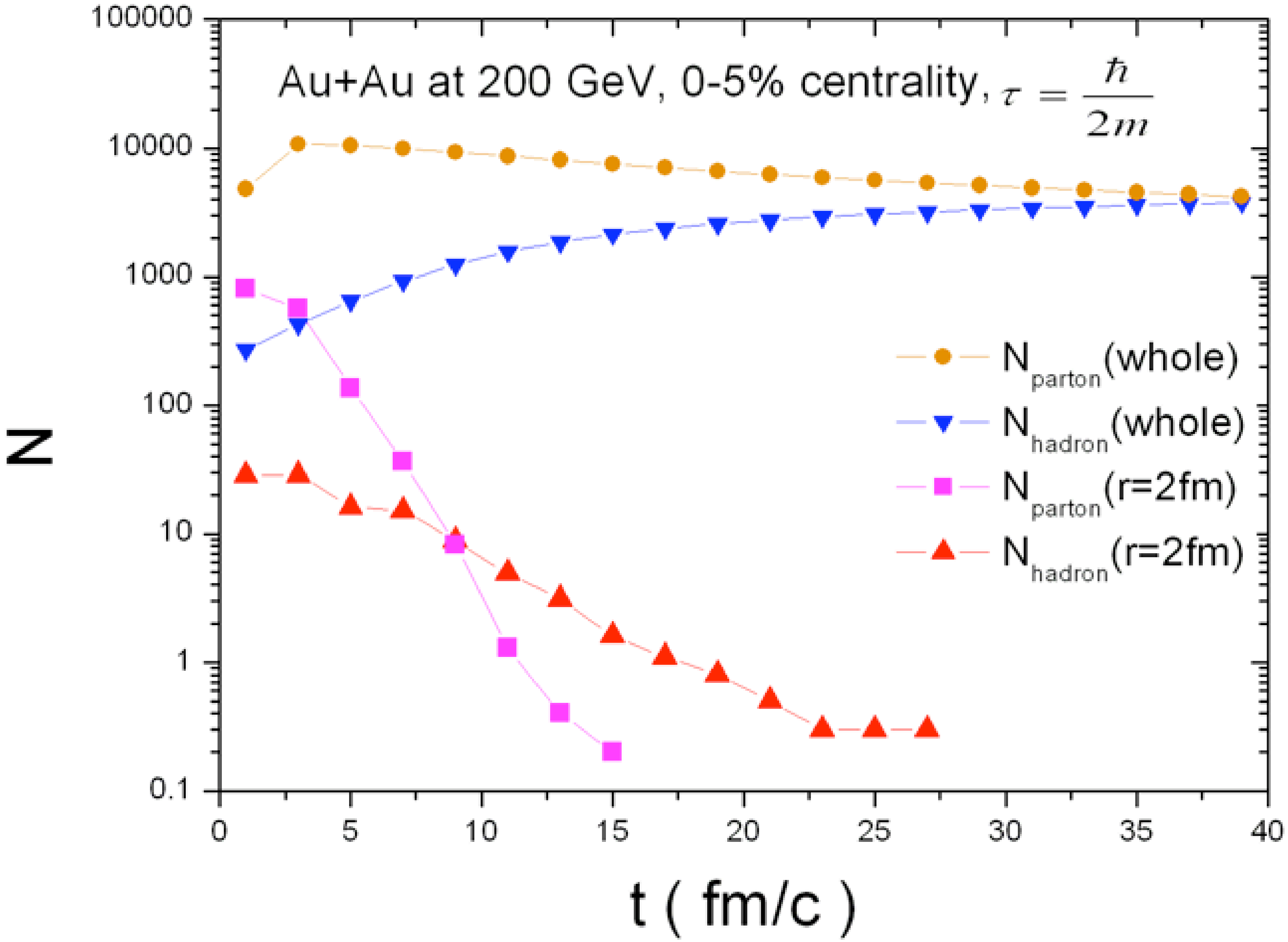


Au+Au at 200 GeV, 0-5% centrality
(a)
(b)
(c)
(d)
π^+
π^-
K^+
K^-
$\tau = \frac{\hbar}{2m}$
BRAHMS
t = 4.9 fm/c
t = 49.84 fm/c
t = 99.77 fm/c
dN_π/dy
dN_K/dy
y



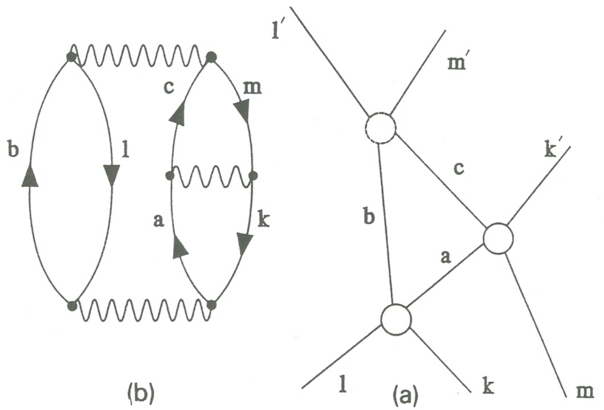
200 GeV, $\tau = \frac{\hbar}{2m}$
ε (dAu)
ε_T (dAu)
ε (pp)
ε_T (pp)
ε (GeV/fm^3)
t (fm/c)
0
1
2
3
4
5
6
0
5
10
15
20


Au+Au at 200 GeV, 0-5% centrality,r=2fm
$\tau = \frac{\hbar}{2m}$
ε
ε_T
ε (GeV/fm^3)
t(fm/c)
0
20
40
60
80
0
5
10
15
20
25
30

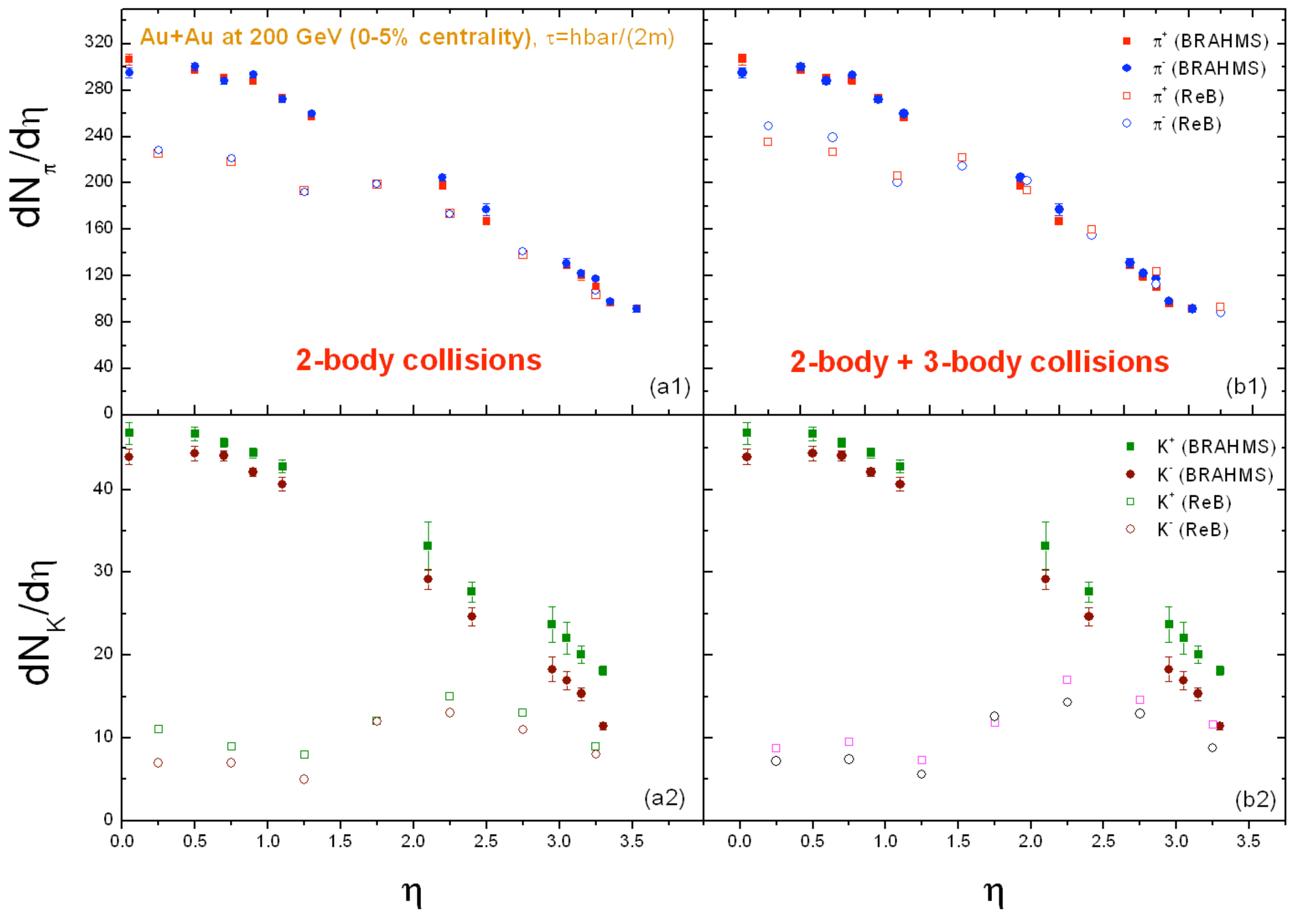


Au+Au at 200 GeV, 0-5% centrality, $\tau = \frac{\hbar}{2m}$
100000
10000
1000
100
10
1
0.1
N
0
5
10
15
20
25
30
35
40
t (fm/c)
N_{parton}(whole)
N_{hadron}(whole)
N_{parton}(r=2fm)
N_{hadron}(r=2fm)

2+3 body collisions



In the last collision only particle production is possible



Au+Au at 200 GeV (0-5% centrality), τ=hbar/(2m)
2-body collisions
2-body + 3-body collisions
(a1)
(b1)
(a2)
(b2)
dNπ/dη
dNK/dη
η
π+ (BRAHMS)
π- (BRAHMS)
π+ (ReB)
π- (ReB)
K+ (BRAHMS)
K- (BRAHMS)
K+ (ReB)
K- (ReB)

Summary and outlook

- CoMD results confirm and suggest new observables for the phase transition in particular the variances in p-space of D-mesons.

- Proposed a new method to solve the relativistic kinetic equation with 2 and 3 body collisions:

  Results critically dependent on the hadrons formation time

  Need to include collisions at the parton level

  Bose-Einsten and Fermi statistics are still lacking

  Include a phase transition in the model in a (possibly) realistic way