Kinetic approach to relativistic heavy ion collisions

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- Introduction
- Relativistic Boltzmann equation (ReB).
- Two and three body collisions.
- EOS at zero baryon densities and finite T.
- Entropy production and its experimental determination.
- Conclusions and outlook

Kinetic approach

- JPCIAE model (old code)
- ReB model (new code)
- Preliminary results

both based on PYTHIA

D.M.Zhou, S.Terranova and A.B., nucl-th\0501083

☆ ReB model (new code)

⟹ based on mean free path idea

(1) Radial position of a nucleon in colliding nucleus A sampled in Woods-Saxon distribution.

(2) Solid angle of the nucleon sampled uniformly in 4π

(3) Beam momentum of each nucleon is given in *z* direction and zero initial momentum in x and y direction

(4) The origin of the time is set at the moment when the projectile and target nuclei touch

⟹

Using time evolution method

At each time step a two(three)-body collision takes place in this way:

1) For each particle i, find the closest particle j in phase space

2) A mean free path is defined as:

$$\lambda = \frac{1}{\bar{\sigma}\rho(1 + \rho\sigma^{3/2} + ..)\Pi_i(1 \pm f_i)}$$

$\bar{\sigma}$ ⟶ **Energy dependent cross section**

ρ ⟶ **density**

f_i ⟹ occupation function (+ bosons,-Fermions)

A. Bonasera, F. Gulminelli, J. Molitoris, Phys. Rep., 243(1&2), (1994) 1-124

3) A collision probability is:

$$\Pi_{i,j} = \frac{\Delta t}{\Delta t_{coll}} = \frac{\Delta t \upsilon_{ij}}{\lambda} = \Delta t \upsilon_{ij} \overline{\sigma} \rho (1 + ..)$$

Δt ⟹ is the time step interval

υ_{ij} ⟹ the relative velocity of particle i and j

4) A random number χ, in the $(0,1)$ interval, is compared

with Π_{ij} ,if $x < \Pi_{ij}$ the collision can occur .

After each collision the particle list is updated

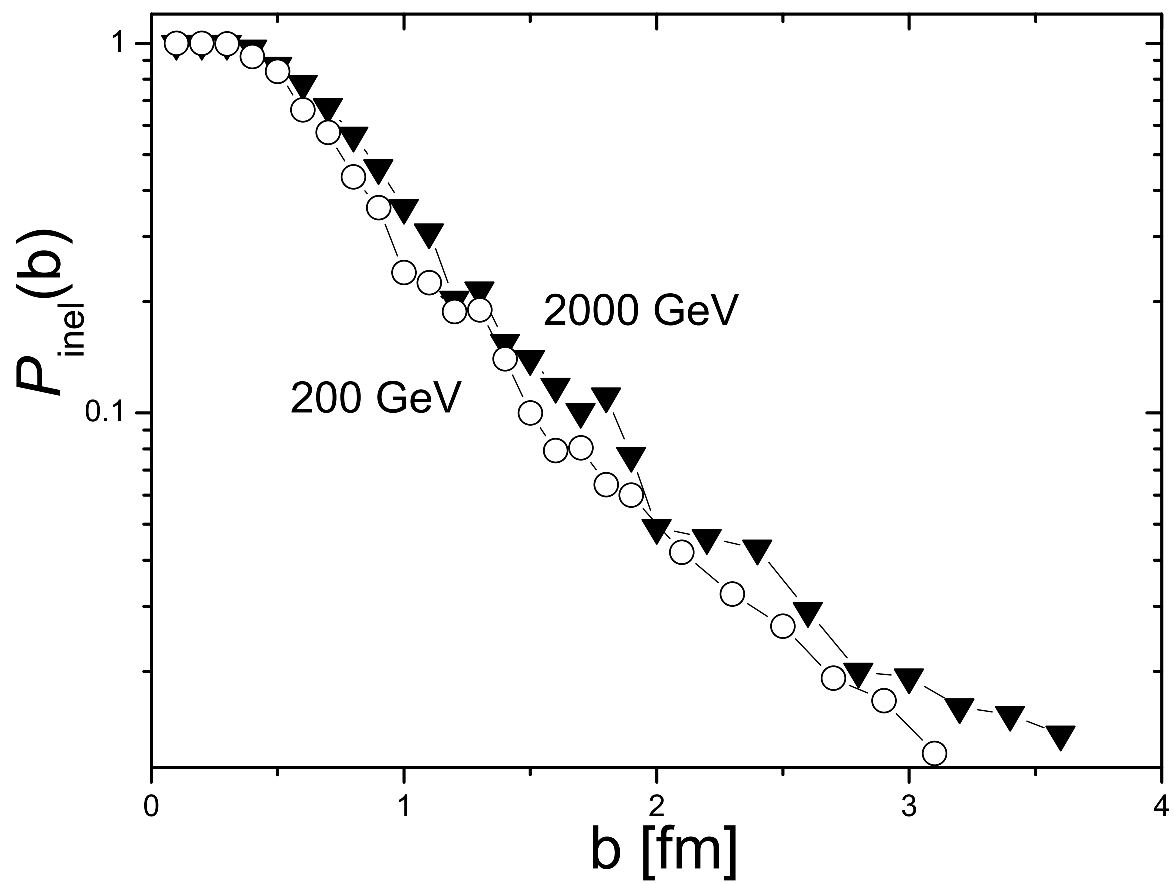
Both the old code and the new code:

For each collision pair:

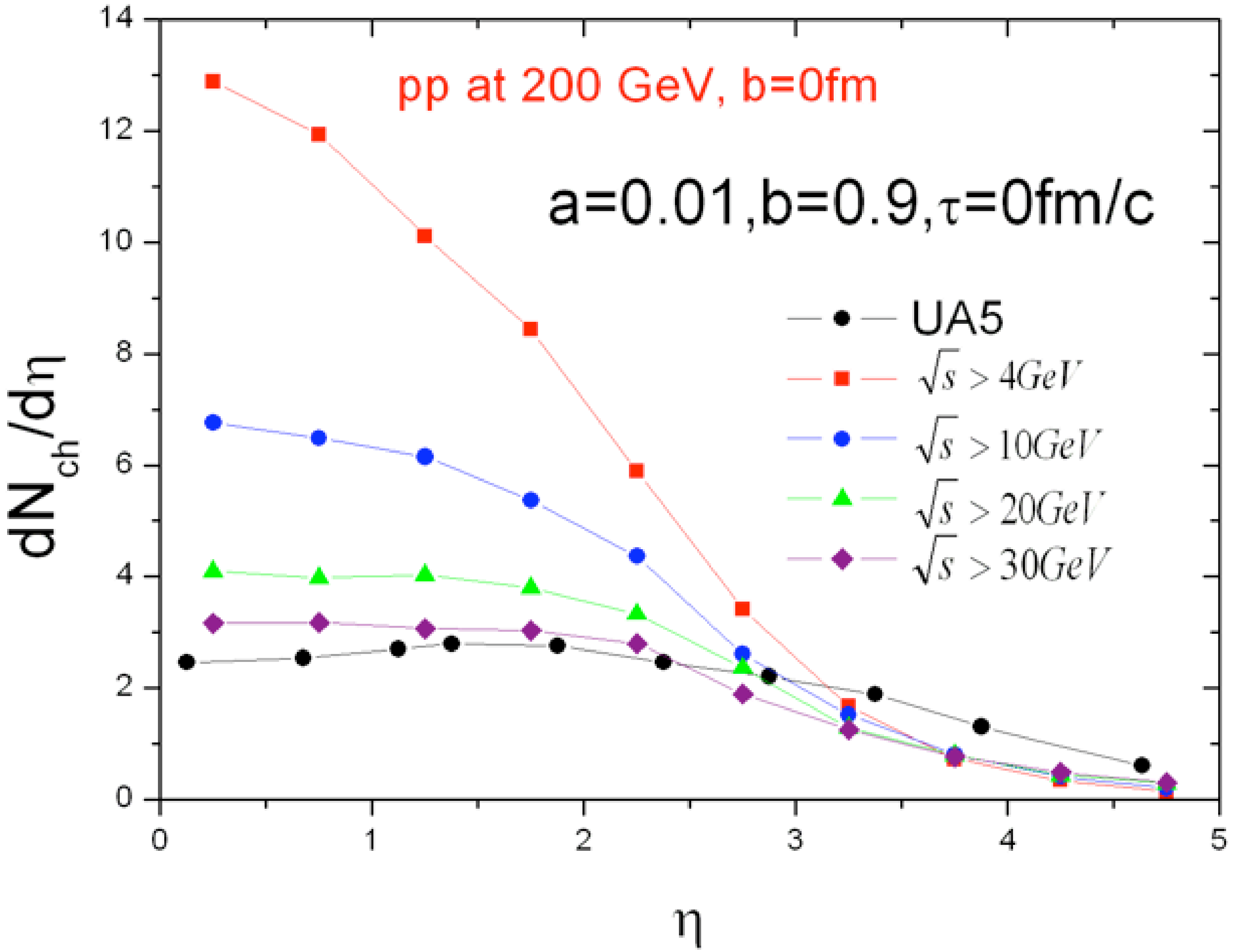
(a) If CMS energy > 4 GeV → strings are formed → PYTHIA is used to deal with particle production.

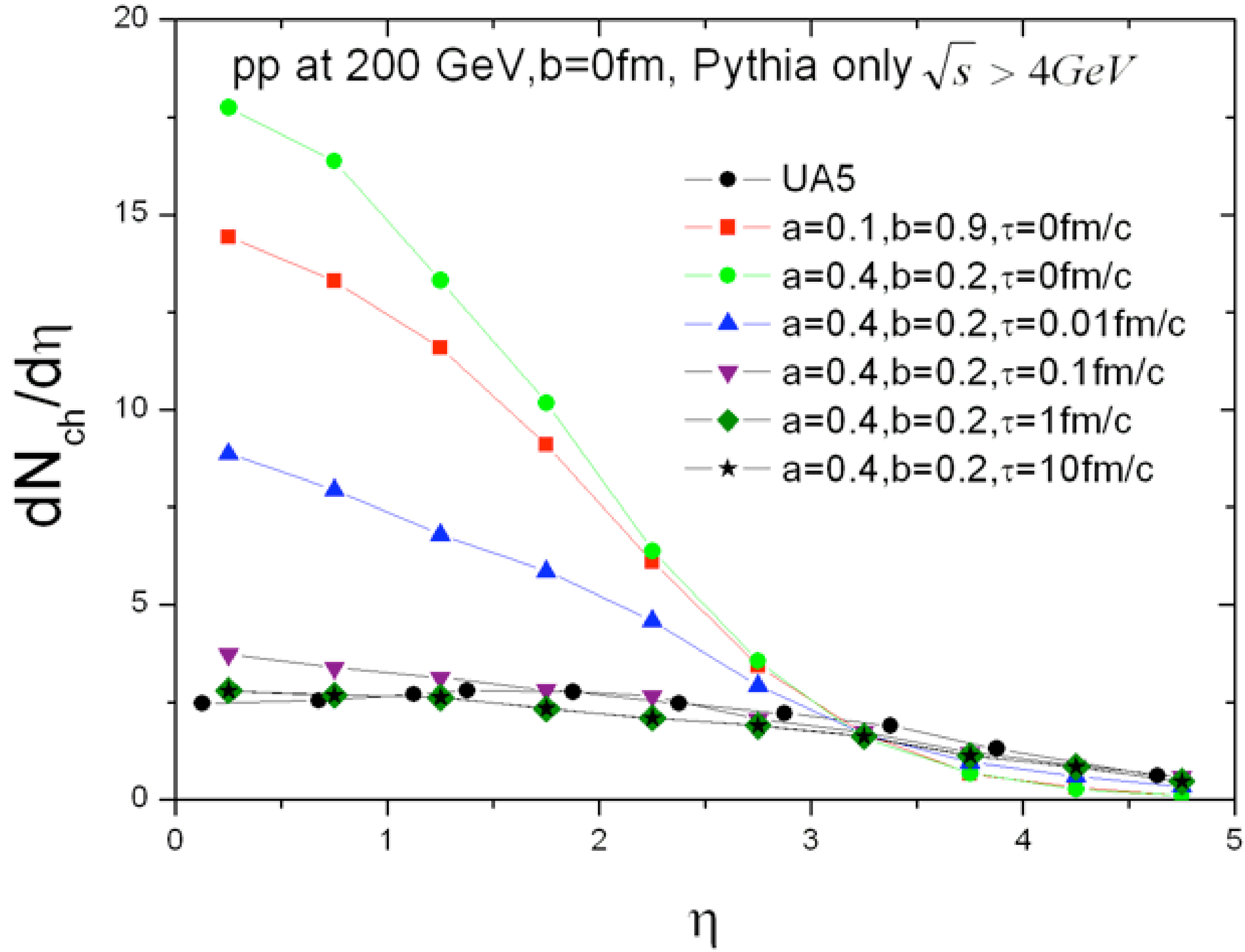
(b) Otherwise, the collision is treated as a two-body collision.

The threshold , 4 GeV, is the minimum energy for which Pythia works. Implement cross sections for lower energies: very important for secondary processes producing low energy particles.

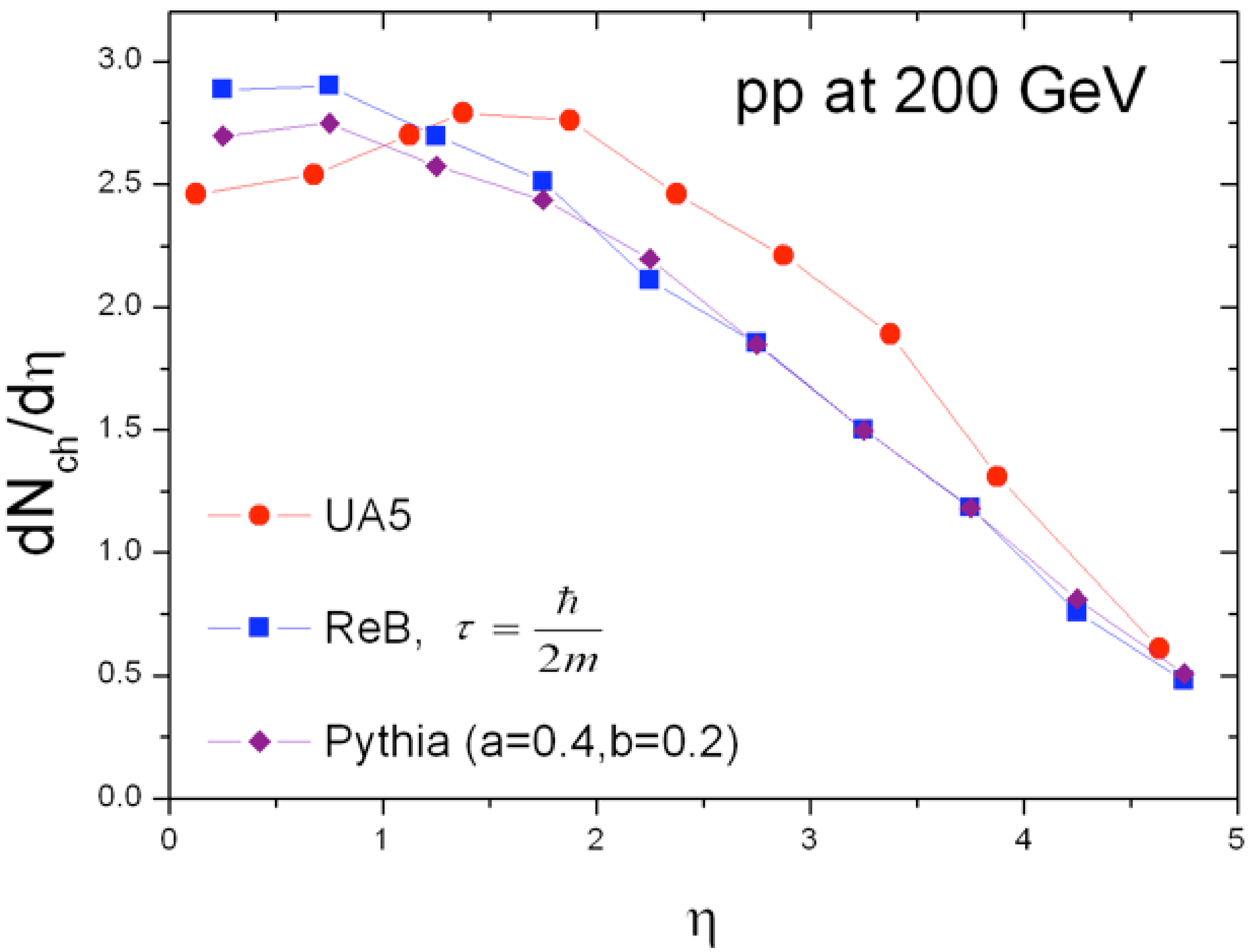


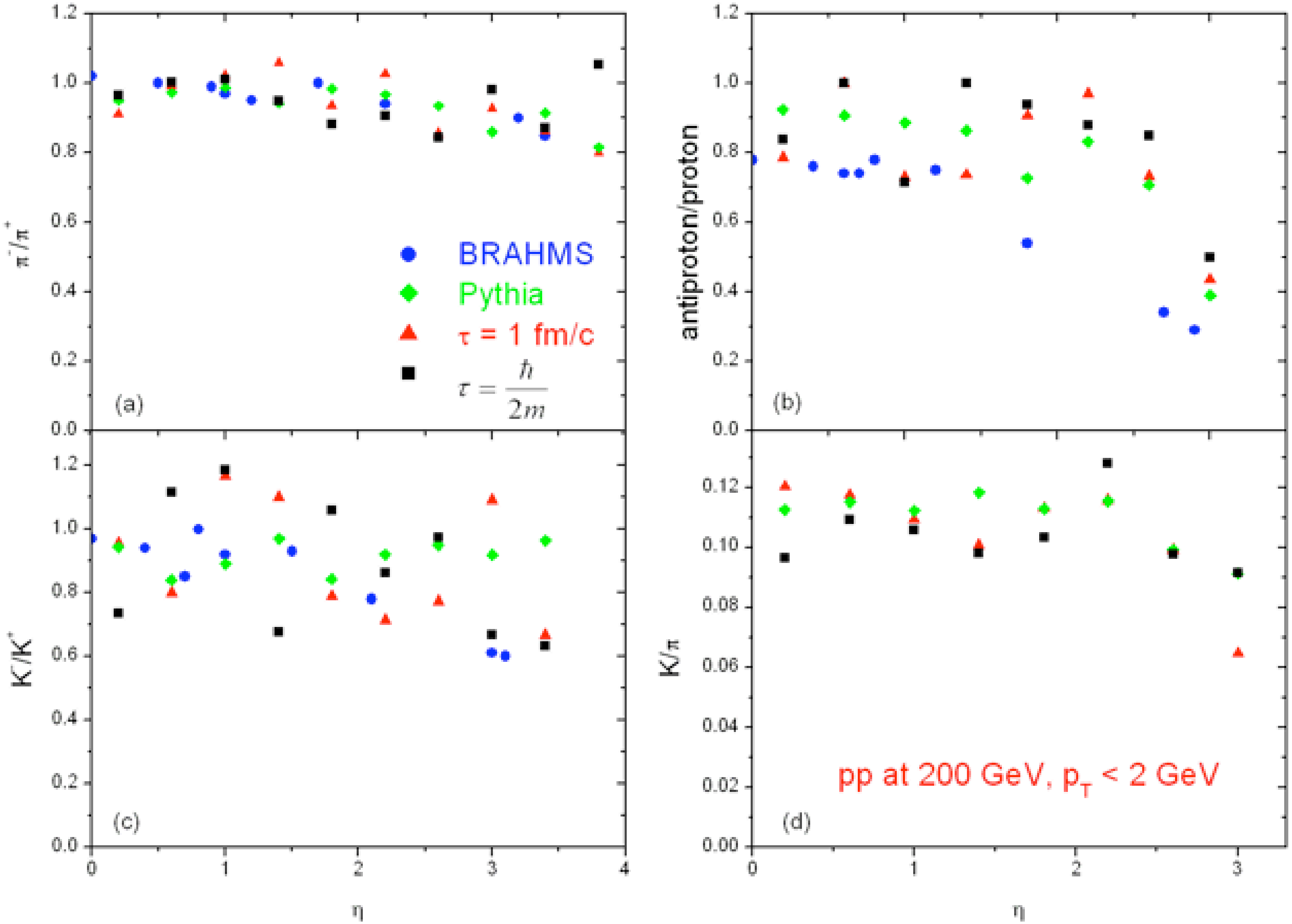
1
0.1
P_{inel}(b)
2000 GeV
200 GeV
0
1
2
3
4
b [fm]

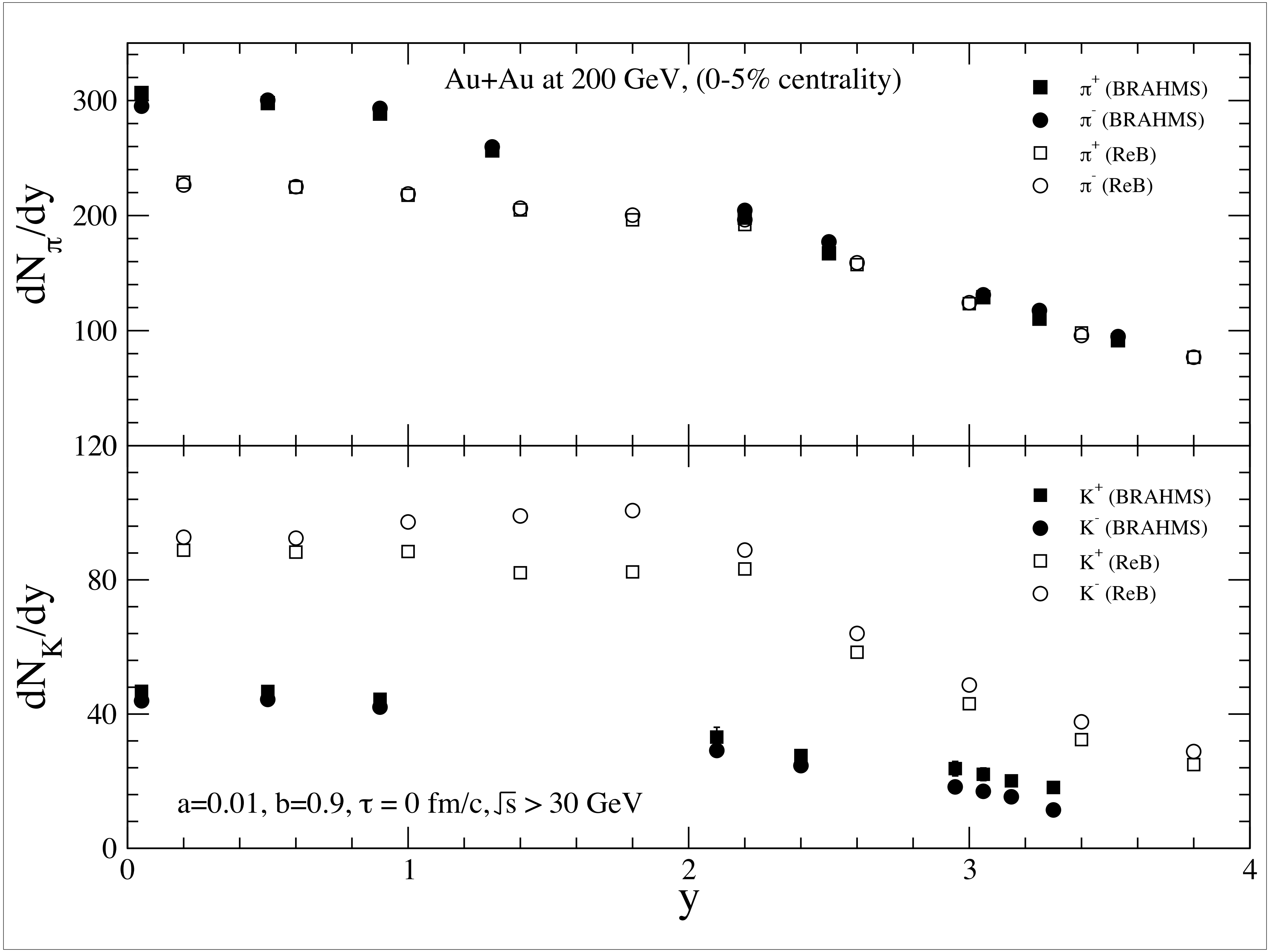

pp at 200 GeV, b=0fm
a=0.01,b=0.9,τ=0fm/c
UA5
$\sqrt{s} > 4GeV$
$\sqrt{s} > 10GeV$
$\sqrt{s} > 20GeV$
$\sqrt{s} > 30GeV$
$dN_{ch}/d\eta$
η
0
2
4
6
8
10
12
14
0
1
2
3
4
5



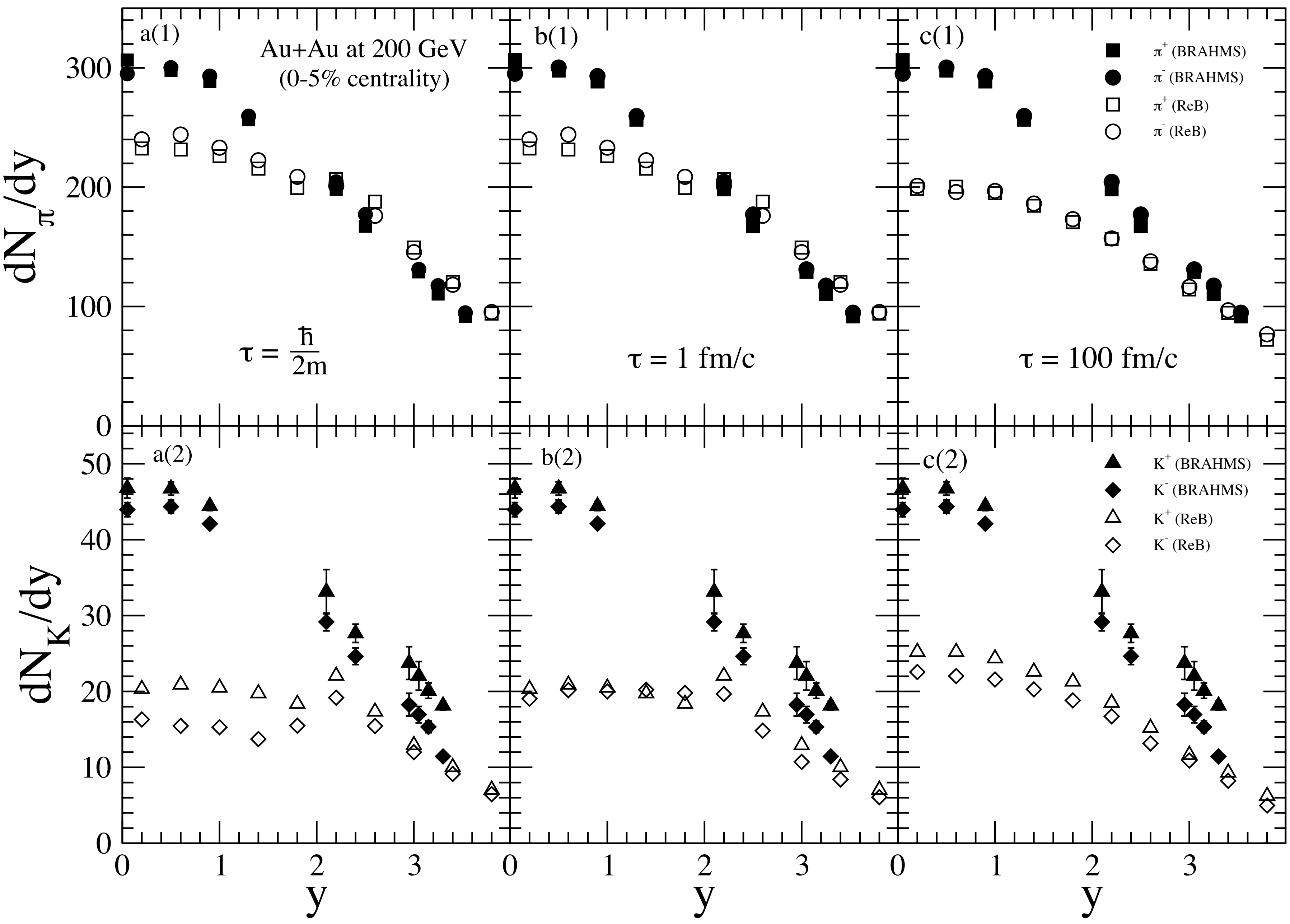
pp at 200 GeV,b=0fm, Pythia only $\sqrt{s} > 4GeV$
UA5
a=0.1,b=0.9,τ=0fm/c
a=0.4,b=0.2,τ=0fm/c
a=0.4,b=0.2,τ=0.01fm/c
a=0.4,b=0.2,τ=0.1fm/c
a=0.4,b=0.2,τ=1fm/c
a=0.4,b=0.2,τ=10fm/c
$dN_{ch}/d\eta$
η
0
5
10
15
20
0
1
2
3
4
5


pp at 200 GeV
dN_ch/dη
η
UA5
ReB, τ = ħ/2m
Pythia (a=0.4,b=0.2)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5

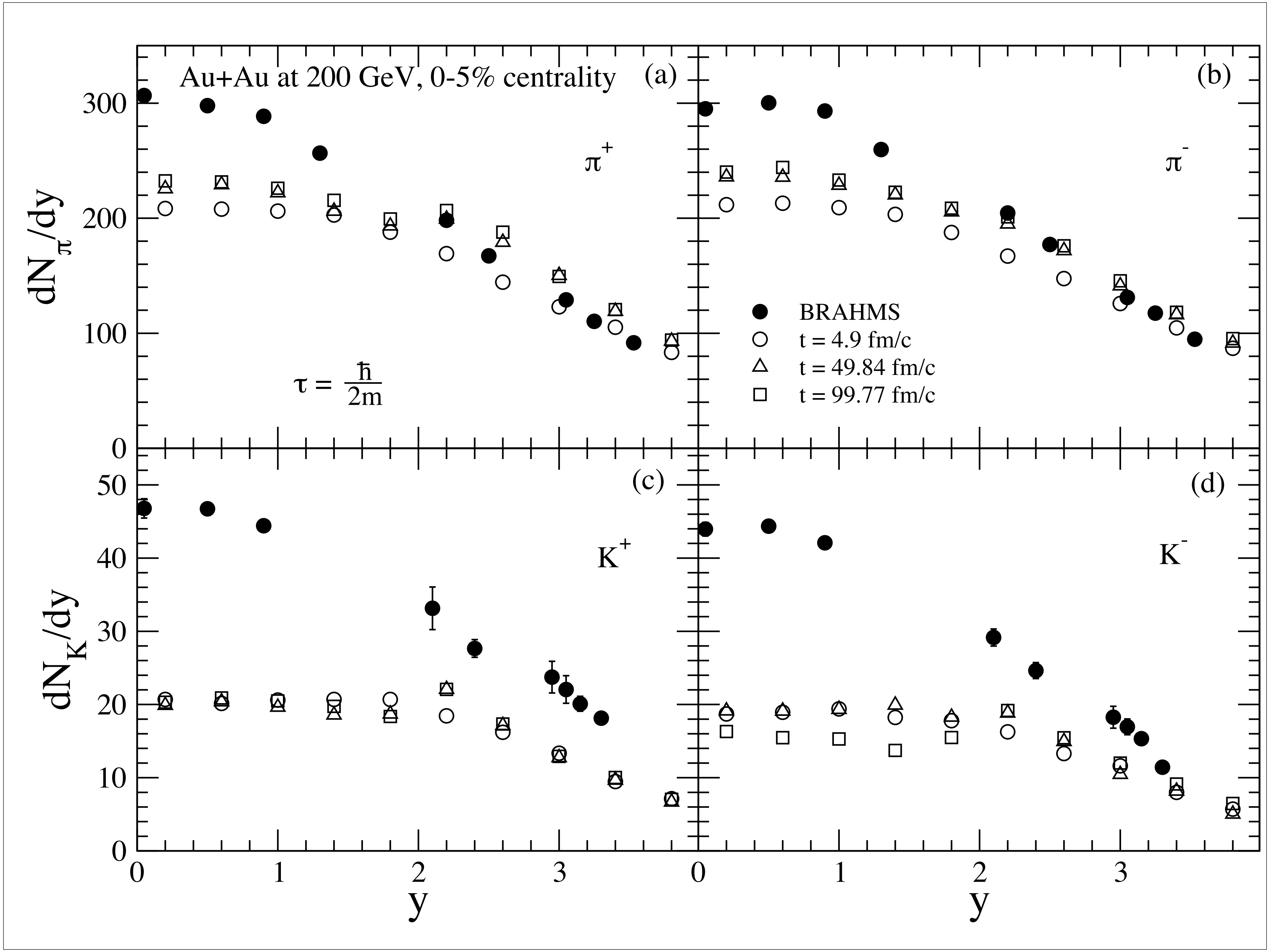

BRAHMS
Pythia
τ = 1 fm/c
$\tau = \frac{\hbar}{2m}$
(a)
(b)
(c)
(d)
π^-/π^+
antiproton/proton
K^-/K^+
K/π
η
η
pp at 200 GeV, p_T < 2 GeV

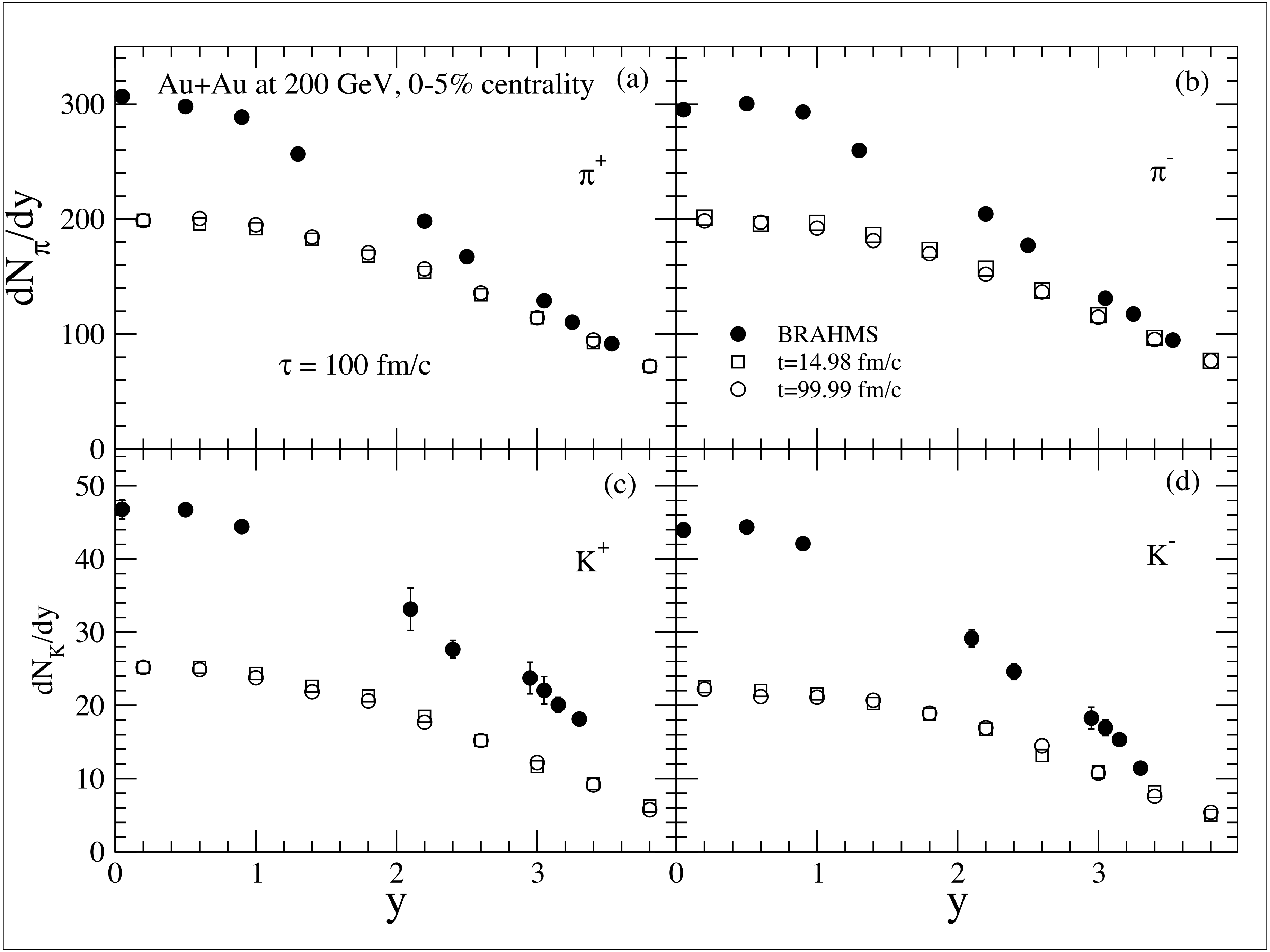


Au+Au at 200 GeV, (0-5% centrality)
π^+ (BRAHMS)
π^- (BRAHMS)
π^+ (ReB)
π^- (ReB)
dN_π/dy
300
200
100
120
80
40
0
K^+ (BRAHMS)
K^- (BRAHMS)
K^+ (ReB)
K^- (ReB)
dN_K/dy
a=0.01, b=0.9, τ = 0 fm/c, $\sqrt{s}$ > 30 GeV
0
1
2
3
4
y

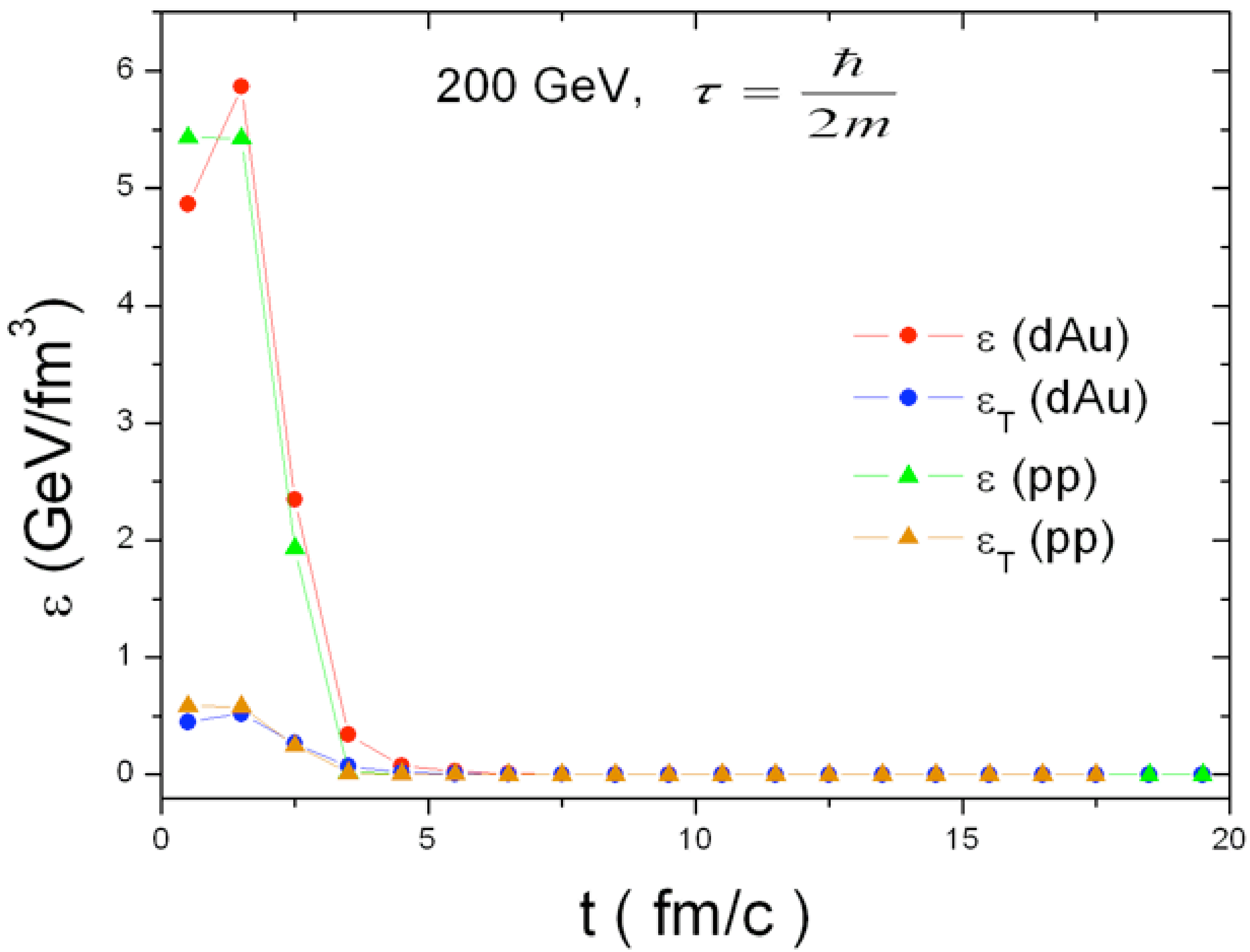


a(1)
Au+Au at 200 GeV
(0-5% centrality)
τ = ħ/2m
b(1)
τ = 1 fm/c
c(1)
τ = 100 fm/c
π+ (BRAHMS)
π- (BRAHMS)
π+ (ReB)
π- (ReB)
dNπ/dy
a(2)
b(2)
c(2)
K+ (BRAHMS)
K- (BRAHMS)
K+ (ReB)
K- (ReB)
dNK/dy
y

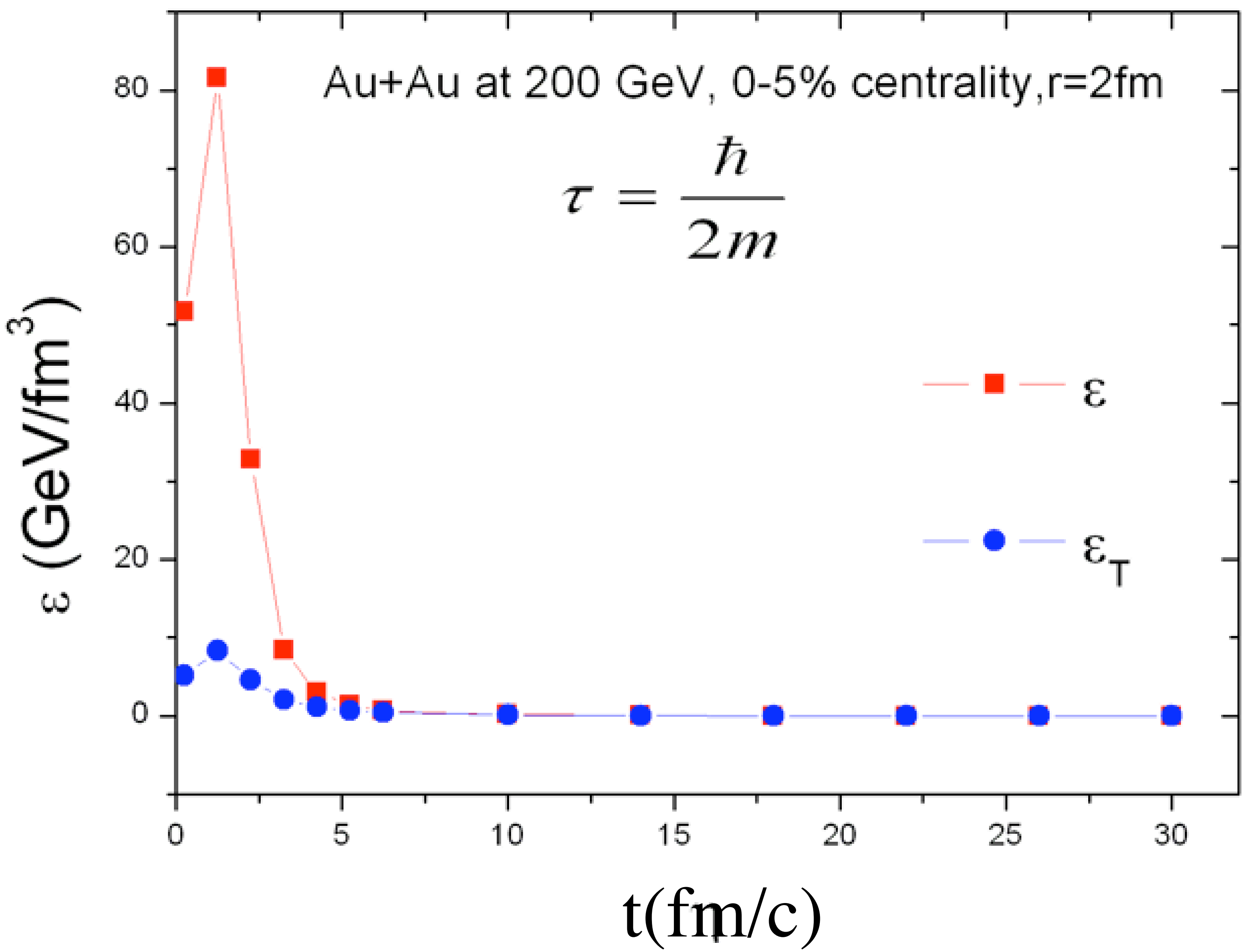

Au+Au at 200 GeV, 0-5% centrality
(a)
(b)
(c)
(d)
π^+
π^-
K^+
K^-
$\tau = \frac{\hbar}{2m}$
BRAHMS
t = 4.9 fm/c
t = 49.84 fm/c
t = 99.77 fm/c
dN_π/dy
dN_K/dy
y



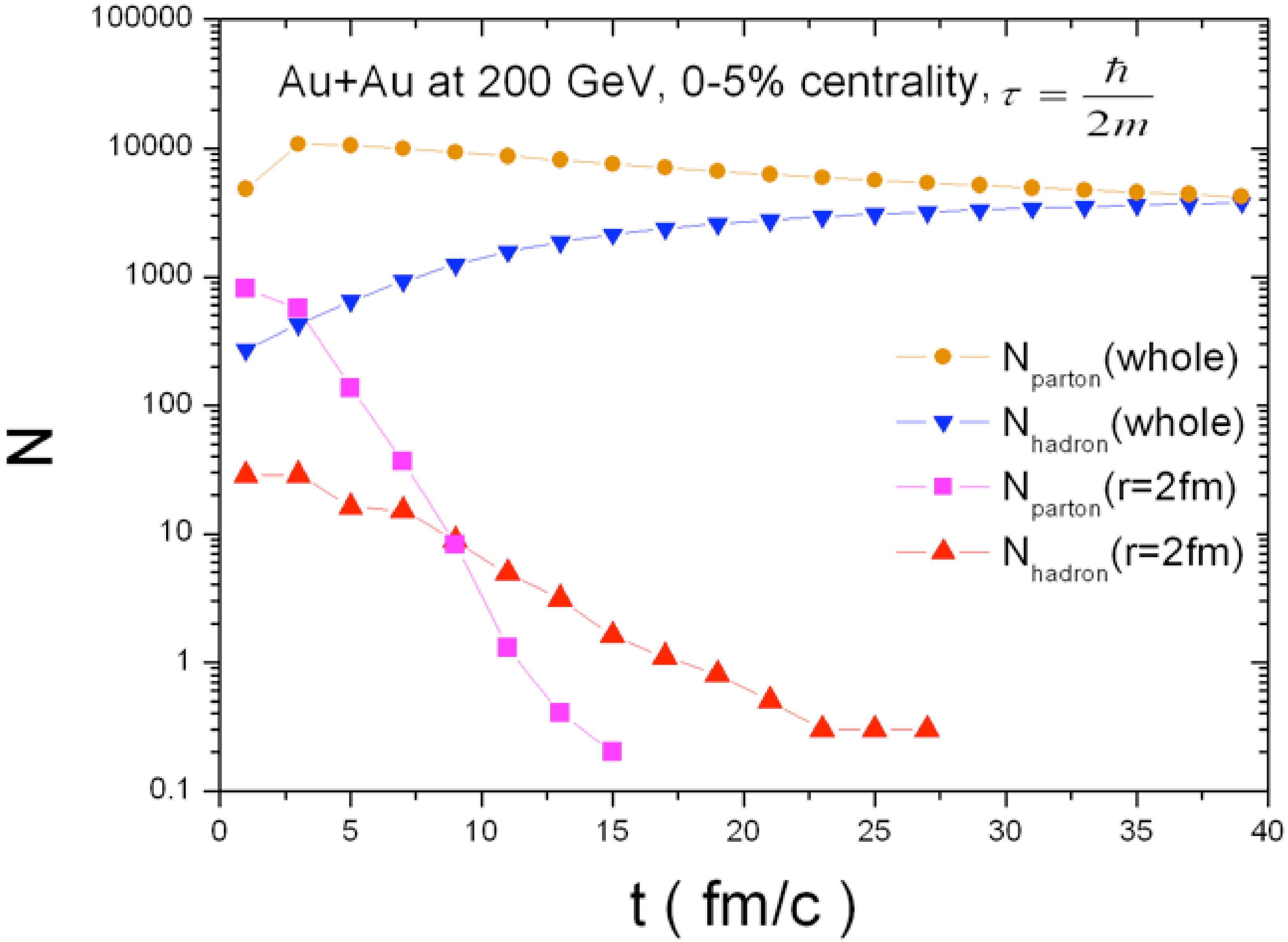
Au+Au at 200 GeV, 0-5% centrality
(a)
(b)
(c)
(d)
π+
π-
K+
K-
τ = 100 fm/c
BRAHMS
t=14.98 fm/c
t=99.99 fm/c
dNπ/dy
dNK/dy
y



200 GeV, $\tau = \frac{\hbar}{2m}$
ε (dAu)
ε_T (dAu)
ε (pp)
ε_T (pp)
ε (GeV/fm^3)
t (fm/c)
0
1
2
3
4
5
6
0
5
10
15
20

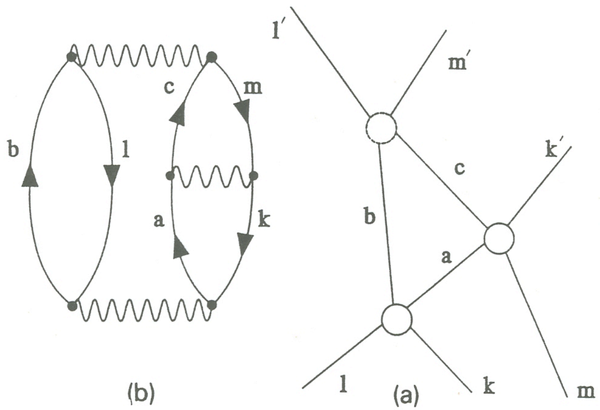


Au+Au at 200 GeV, 0-5% centrality,r=2fm
$\tau = \frac{\hbar}{2m}$
ε
ε_T
ε (GeV/fm^3)
t(fm/c)
0
20
40
60
80
0
5
10
15
20
25
30

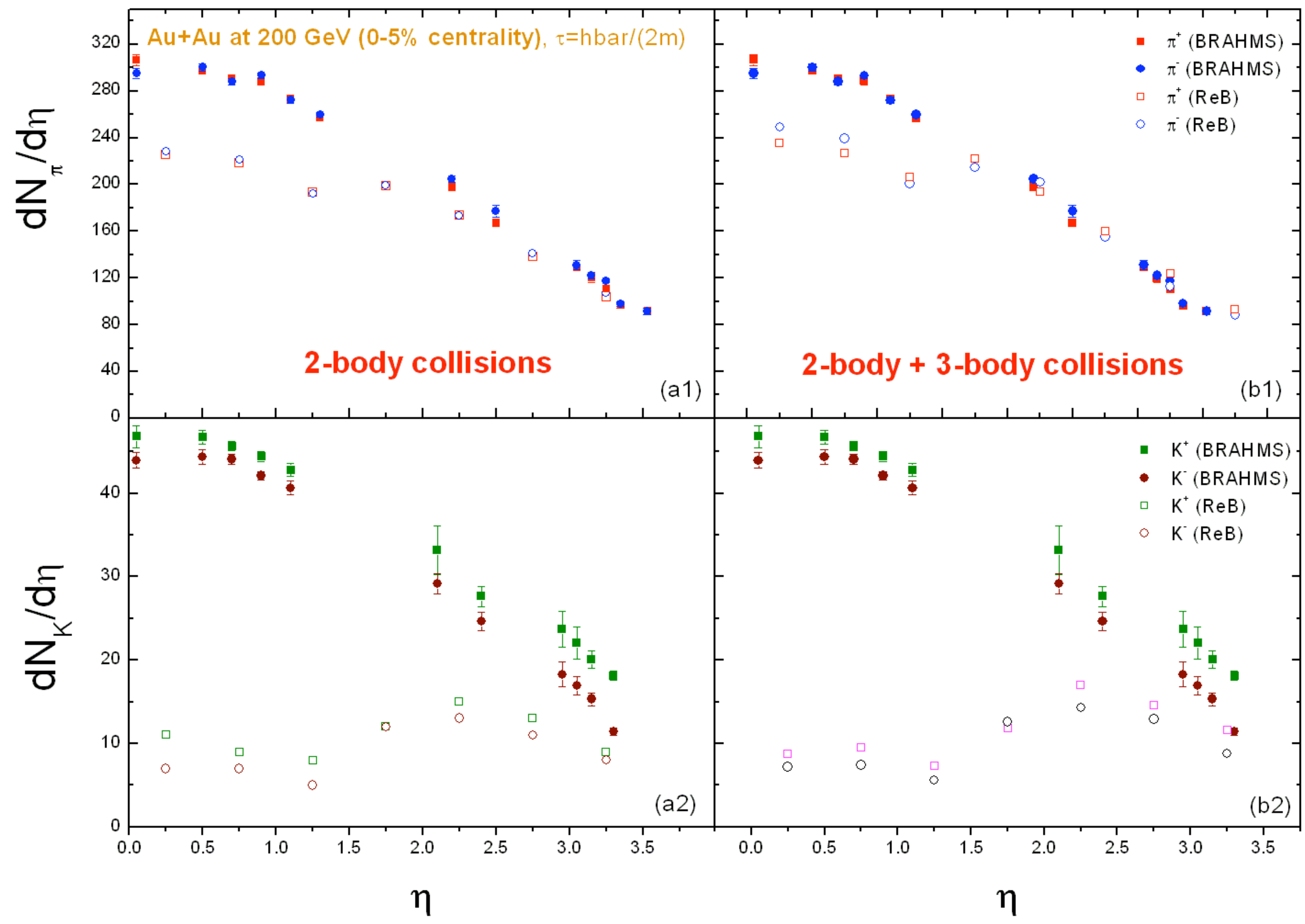


Au+Au at 200 GeV, 0-5% centrality, $\tau = \frac{\hbar}{2m}$
100000
10000
1000
100
10
1
0.1
N
0
5
10
15
20
25
30
35
40
t (fm/c)
N_{parton}(whole)
N_{hadron}(whole)
N_{parton}(r=2fm)
N_{hadron}(r=2fm)

2+3 body collisions



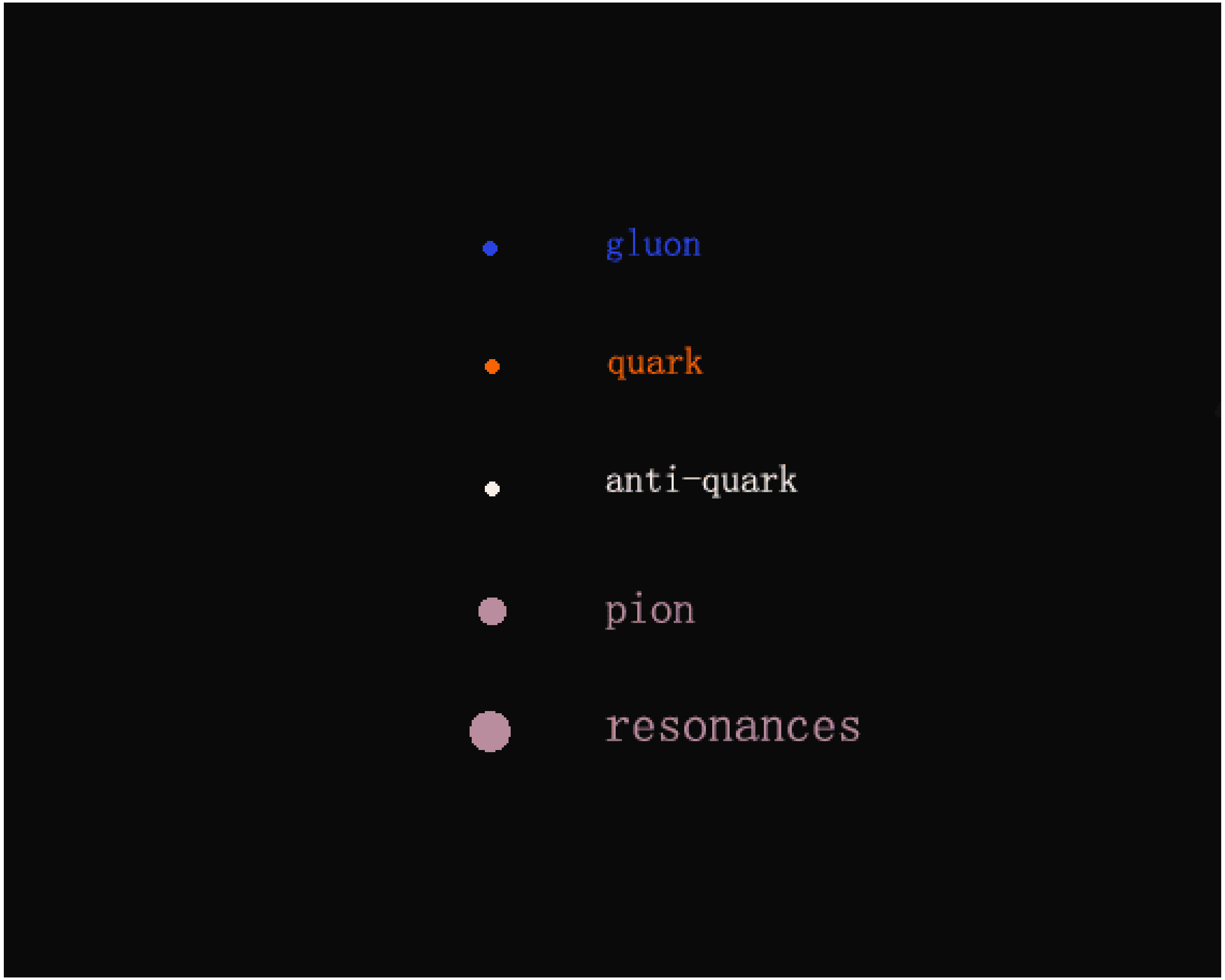
In the last collision only particle production is possible



Au+Au at 200 GeV (0-5% centrality), τ=hbar/(2m)
2-body collisions
2-body + 3-body collisions
(a1)
(b1)
(a2)
(b2)
π+ (BRAHMS)
π- (BRAHMS)
π+ (ReB)
π- (ReB)
K+ (BRAHMS)
K- (BRAHMS)
K+ (ReB)
K- (ReB)
dNπ/dη
dNK/dη
η

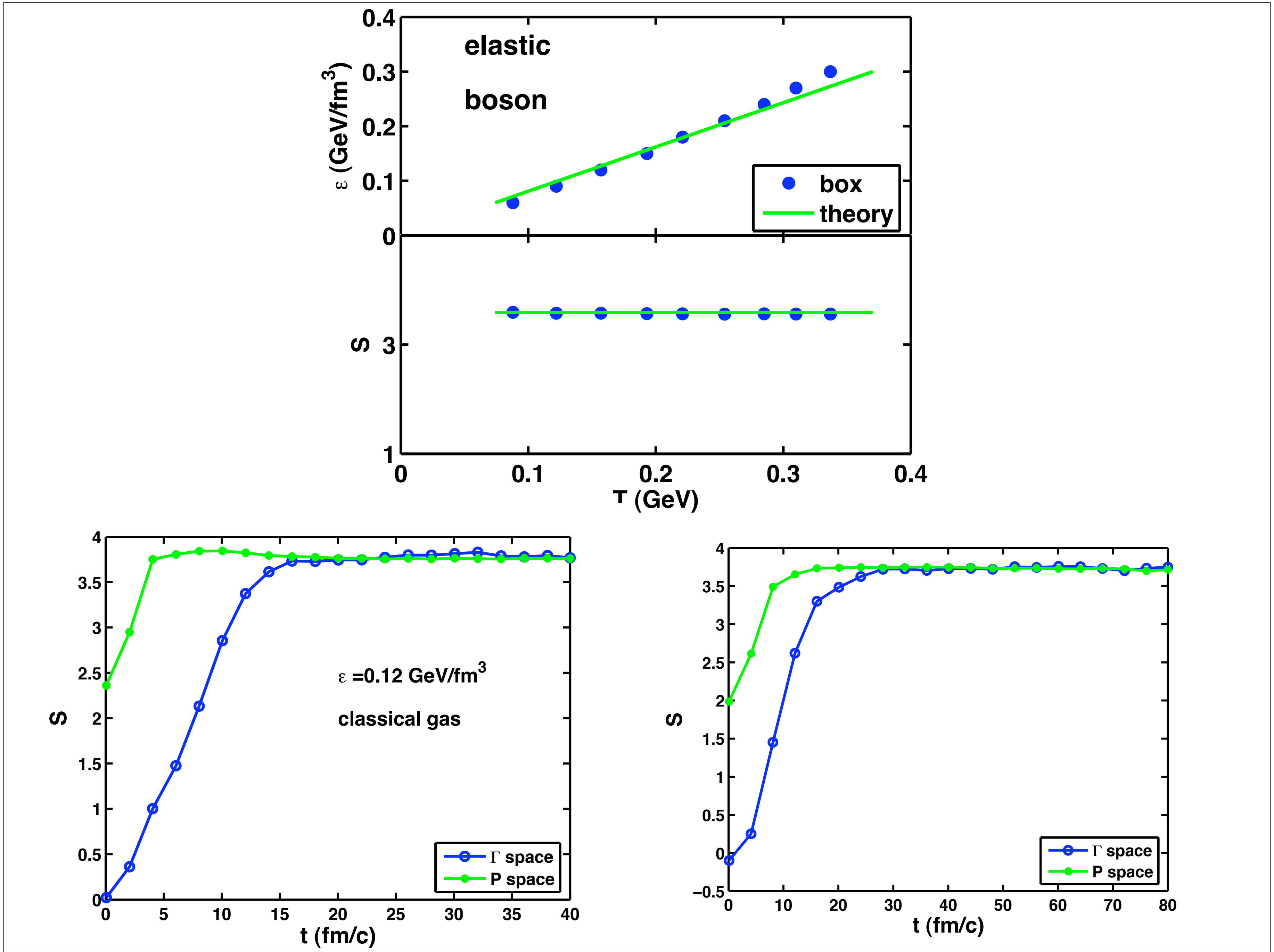
EOS at zero Baryon densities

- Initially prepare two colliding pion systems in a box with periodic boundary conditions.
- Include all possible elastic and inelastic channels from data (if available) or theory.
- Calculate temperature and entropy after equilibrium has been reached.
- Define entropy in p-space only (which could be measured) and compare.
- Include possibility for a QGP based on the Bag model (SU2).
- Compare to LQCD results

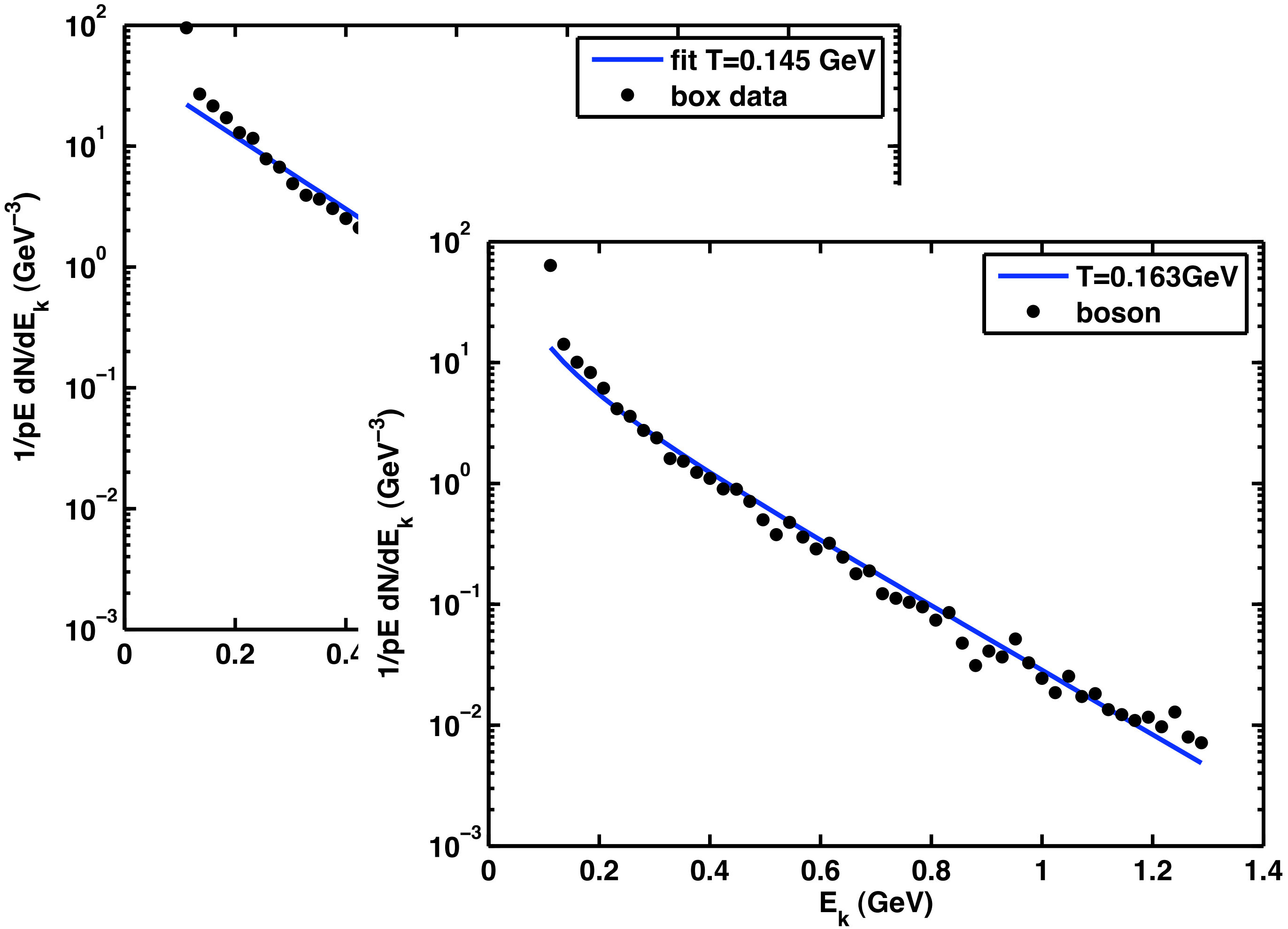

gluon
quark
anti-quark
pion
resonances

Entropy

- Define entropy: $S=-1/N\sum(f \ln f \pm (1\pm f)\ln(1\pm f))$; N=number of particles ×number of events (time). Normalize $\sum f(i)=1$
- $f(i)=cnst/d^3r_{ij}d^3p_{ij}$;where j is the closest particle in phase space to i
- Reduced entropy $Sp=-1/N\sum(g \ln g \pm (1\pm g)\ln(1\pm g))$; Normalize $\sum g(i)=1$
- $g(i)=cnst\varrho/d^3p_{ij}$;where j is the closest particle to i
- S proportional to S_p if the system reaches equilibrium at a freeze-out density.


elastic
boson
ε (GeV/fm³)
box
theory
S
T (GeV)
ε =0.12 GeV/fm³
classical gas
Γ space
P space
t (fm/c)
S
Γ space
P space
t (fm/c)

How did we determine the Temperature?



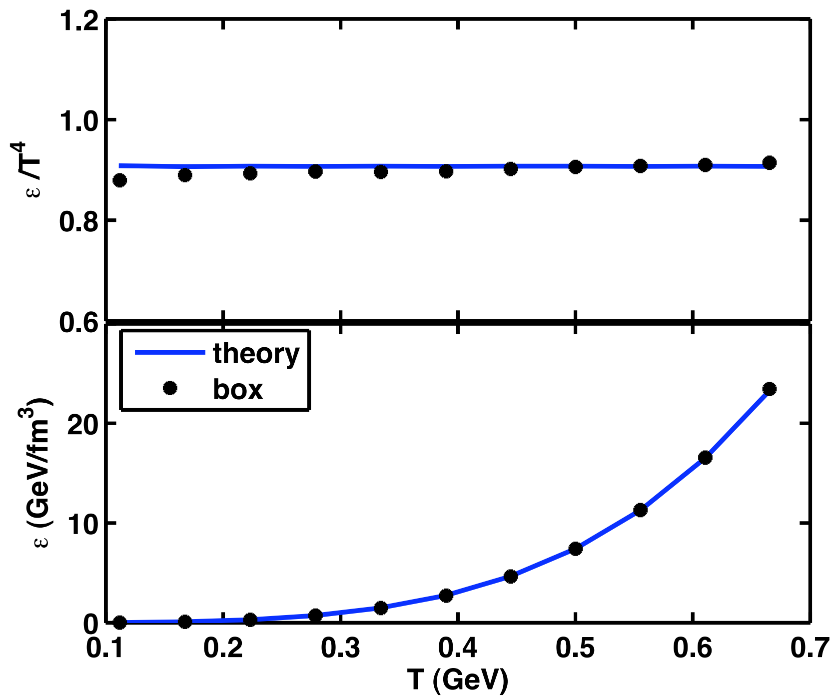
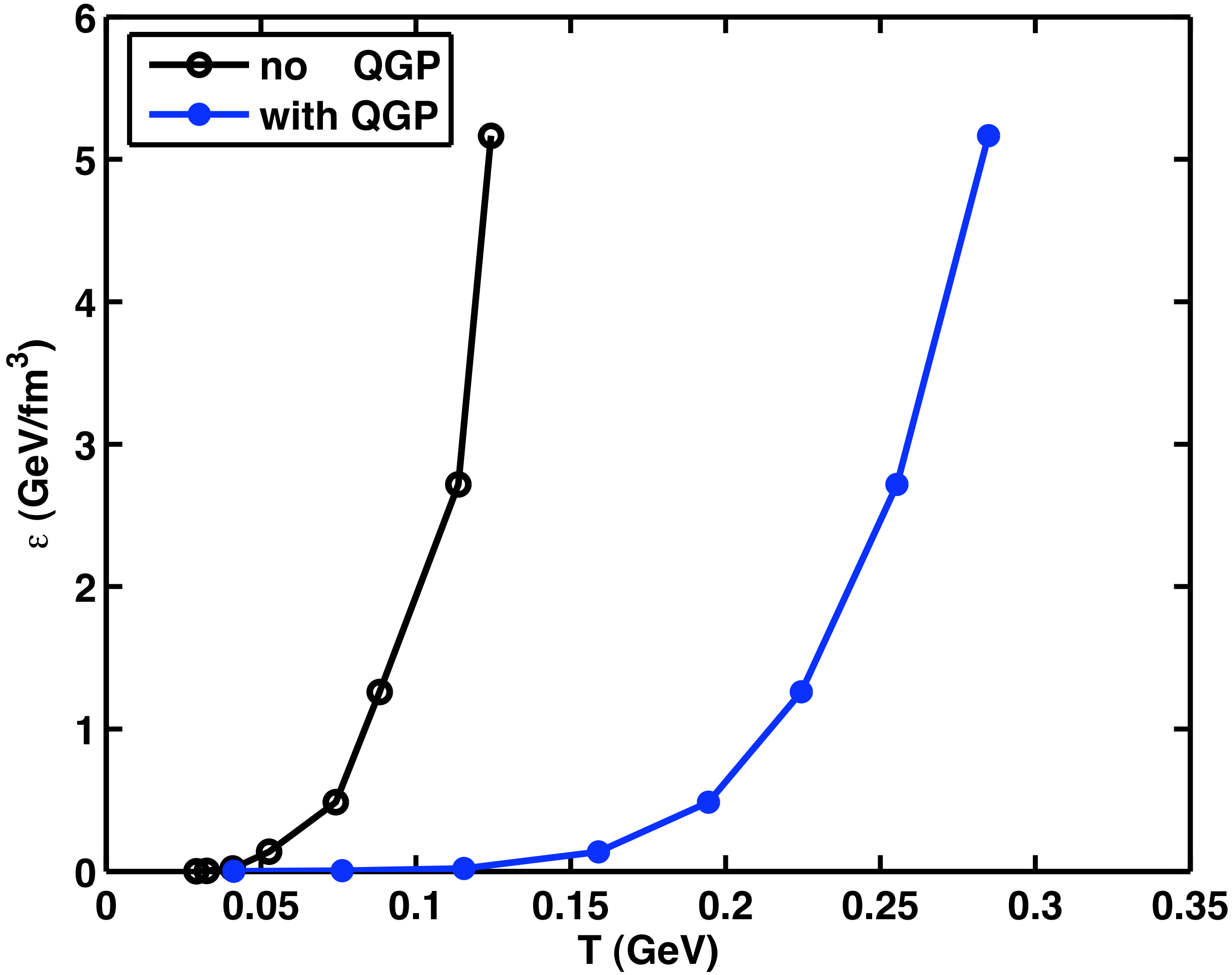


FIG. 2: Energy density divided by T^4 (top) and energy density (bottom) versus temperature for a classical ideal gas of finite mass. The full lines represents the analytical, while the dots are our numerical result.

QGP

- Massless quark and gluon gas:
  $P=g_{tot}(\pi^2/90)T^4$; $\varepsilon=3P$; $g_{tot}=37$ (2 flavors)
- Define the critical pressure and energy density in the Bag model: $\varepsilon_c=3B=0.71$ GeV/fm^3 ($B^{1/4}=206$ MeV)
- If in a h-h collision $\varepsilon \geq \varepsilon_c$, quarks and gluons are liberated: $n_q=n_{qbar}=f(\varepsilon)$; $n_g=g(\varepsilon)$
- quarks and gluons can collide elastically (to reach equilibrium), and also decay (g) or combine to form new hadrons (q-qbar).



no QGP
with QGP
ε (GeV/fm3)
T (GeV)
0
1
2
3
4
5
6
0.05
0.1
0.15
0.2
0.25
0.3
0.35

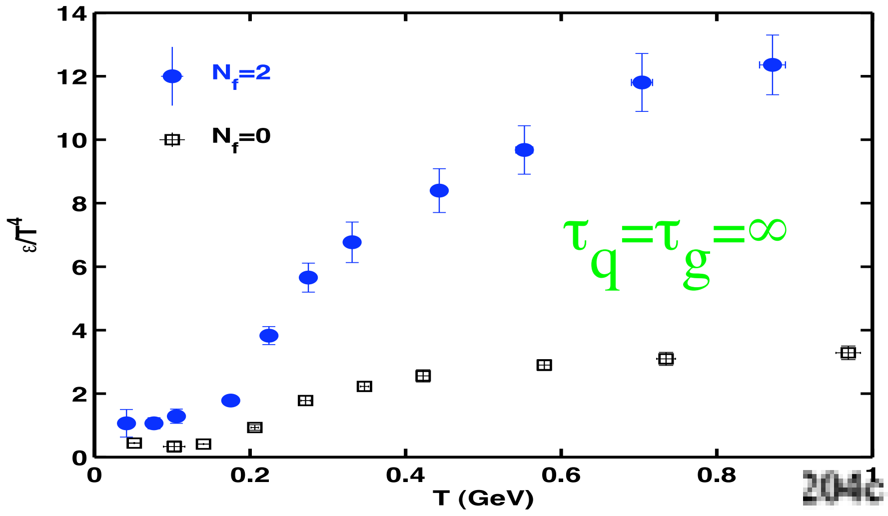


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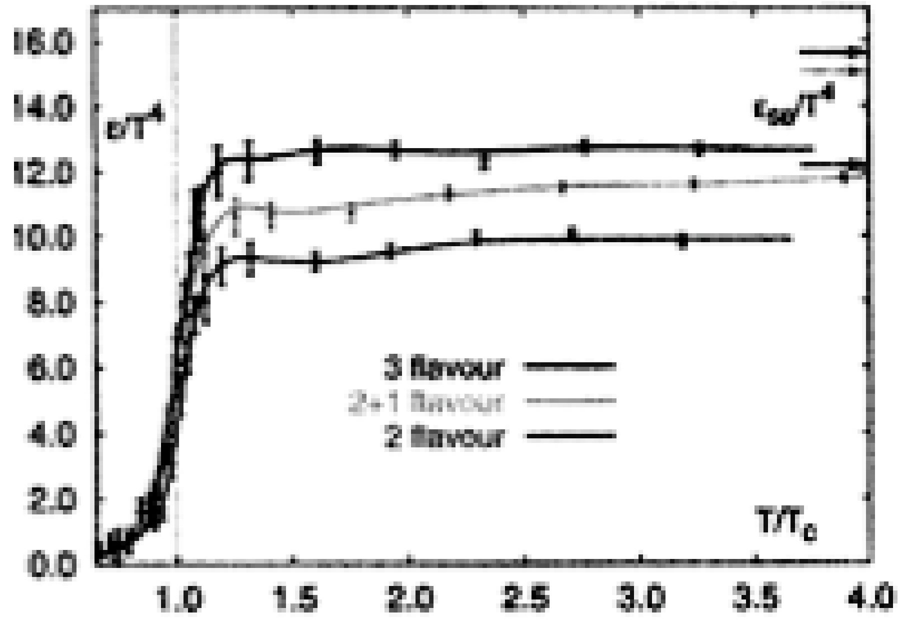
FIG. 6: Energy density divided T^4 versus temperature for $N_f = 2$ (full circles). The LQCD results are given by the squares.



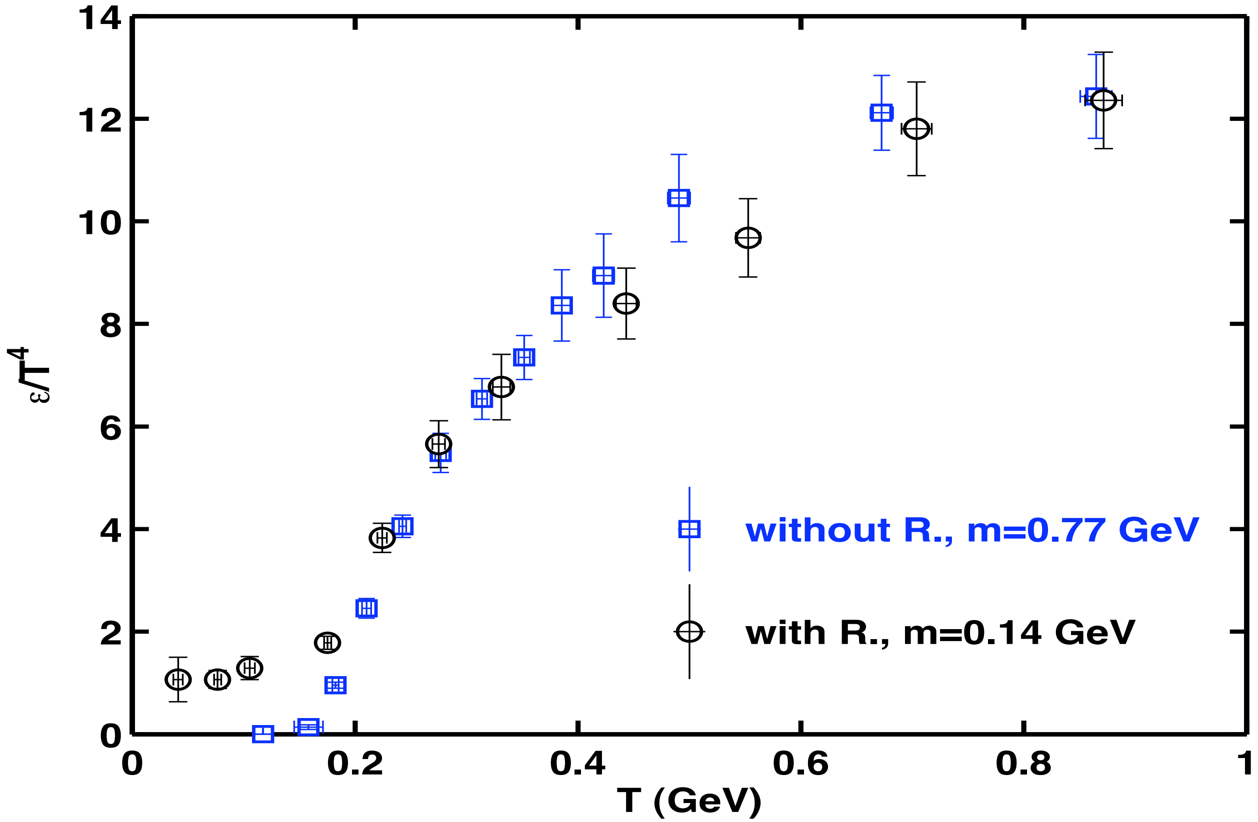
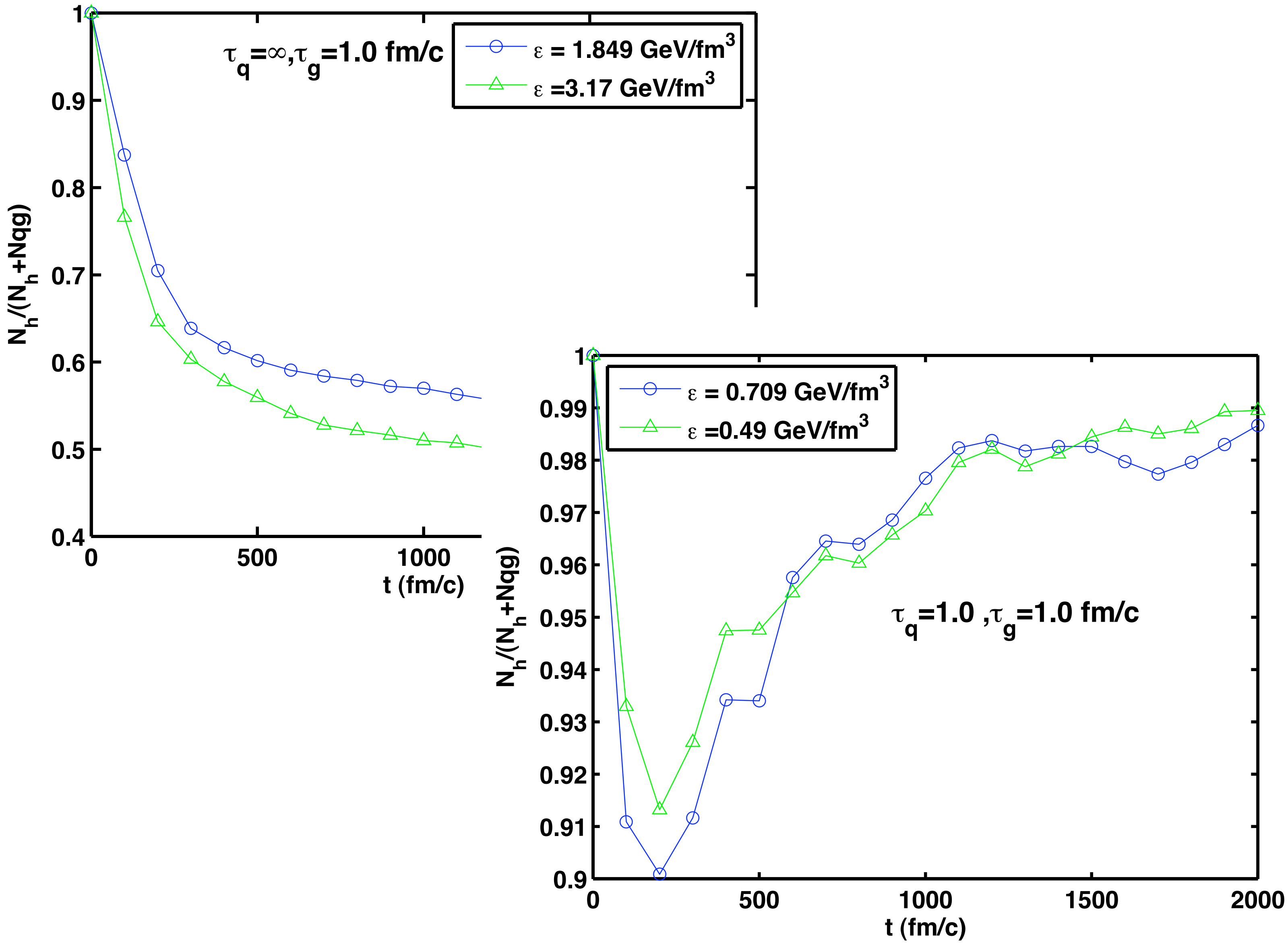
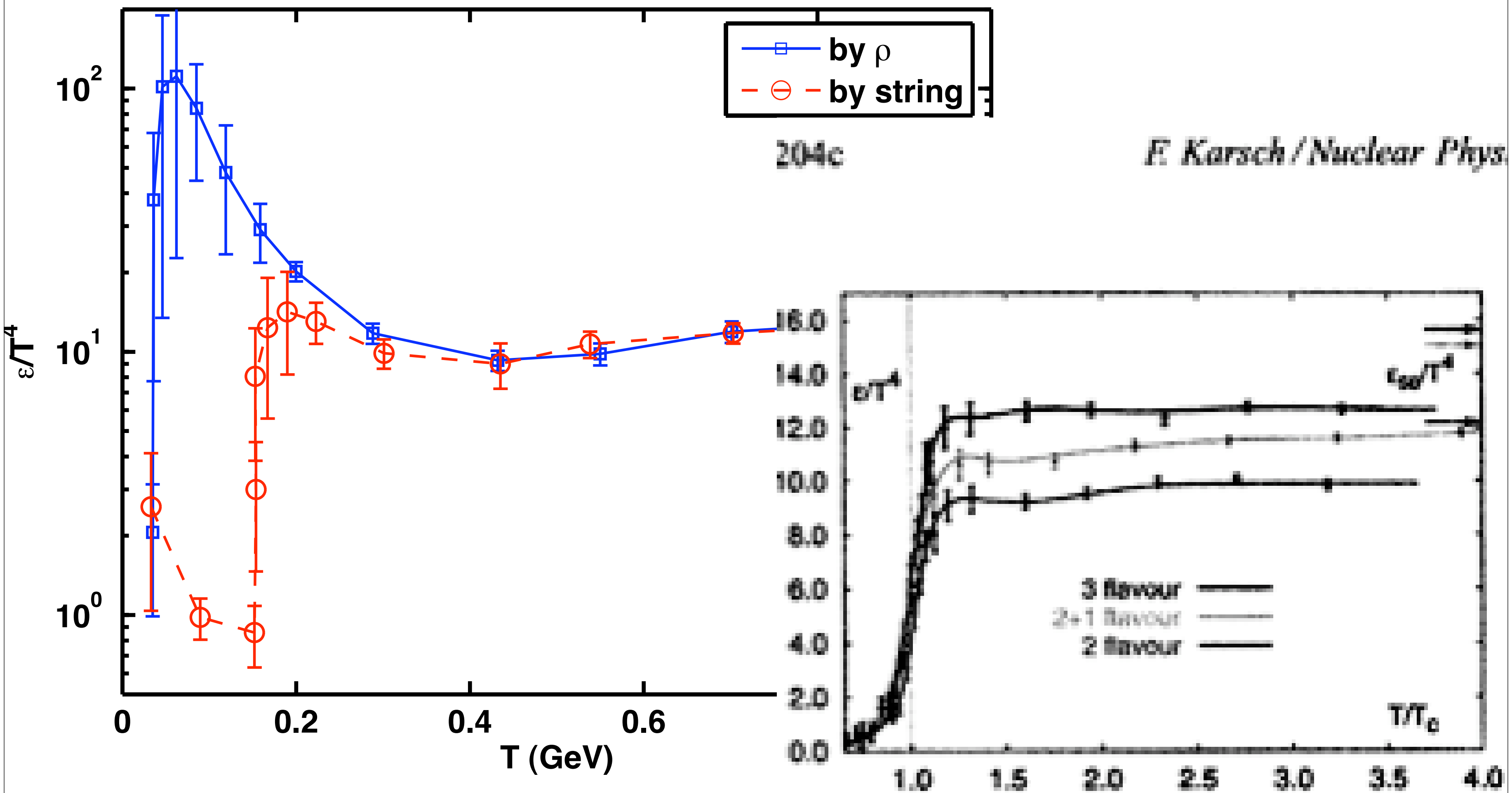


FIG. 7: Equation of state including QGP formation and for pion masses equal to 0.14 GeV (open circles) and 0.77GeV (open squares).



τq=∞,τg=1.0 fm/c
ε = 1.849 GeV/fm3
ε =3.17 GeV/fm3
Nh/(Nh+Nqg)
t (fm/c)
ε = 0.709 GeV/fm3
ε =0.49 GeV/fm3
τq=1.0 ,τg=1.0 fm/c
Nh/(Nh+Nqg)
t (fm/c)

τ_q=1.0fm/c, τ_g=0.5fm/c, $\varrho<\varrho_c$(B)



Summary and outlook

- Proposed a new method to solve the relativistic kinetic equation with 2 and 3 body collisions:

Results critically dependent on the hadrons formation time

Need to include collisions at the parton level☑

Bose-Einsten and Fermi statistics are still lacking☑

Include a phase transition in the model in a (possibly) realistic way☑