

Scuola di Fisica Nucleare Teorica “Raimondo Anni”
Secondo Corso

Equazione di Stato ad Alta Densità' Barionica

- I. Dinamica della collisioni di ioni relativistici**
- II. Osservabili sensibili all'EOS.
Potenziale di Simmetria.**

Collisioni di Ioni Pesanti ad energie relativistiche: Test di Materia Nucleare ad Alta Densita' Barionica e di Isospin

*Collisioni "violente" di nuclei
ad energie "Intermedie"*



- varie regioni di densita'
- alti impulsi
- struttura covariante: "puri" effetti relativistici
- limiti della descrizione adronica

Campo Medio
Masse effettive
Sezioni d'urto nel mezzo

Self-energies →

Equazioni del Trasporto Relativistiche
Autoconsistenti

Osservabili sensibili all'Equazione di Stato

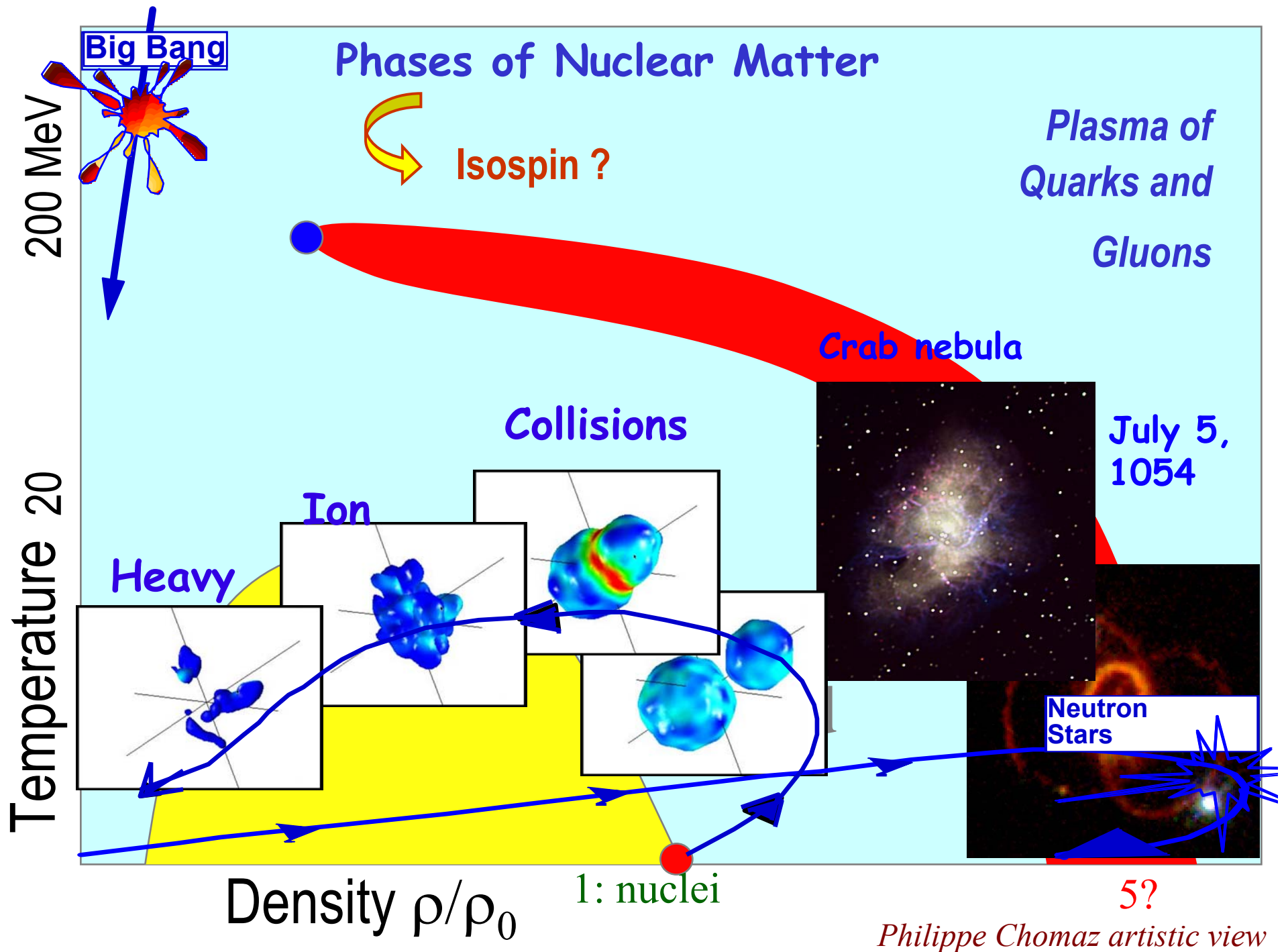
Produzione di Frammenti

Flussi Collettivi

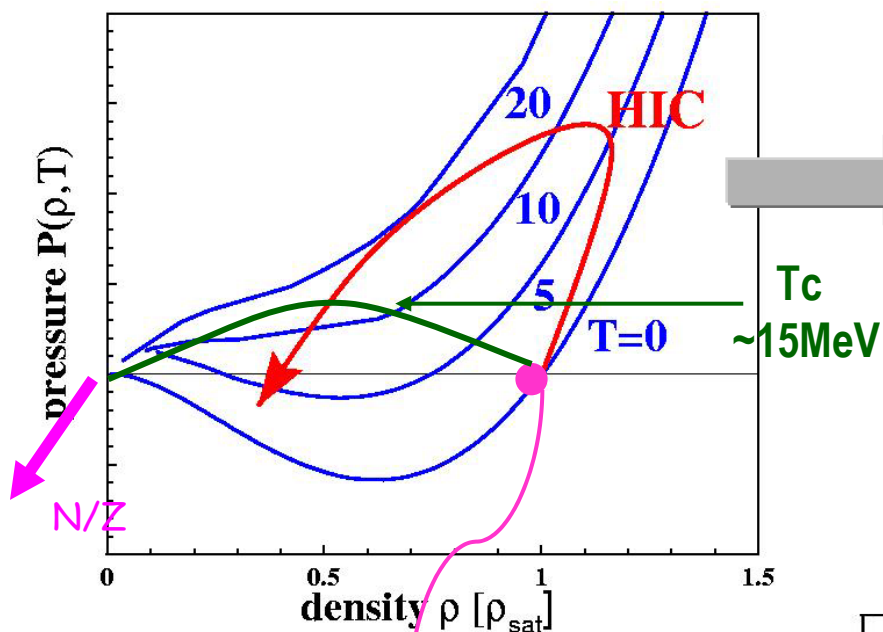
Produzione di Pioni/Kaoni

Trasparenza di Isospin

Segnali Precursori del Deconfinamento?



Terrestrial Labs.



Ground state
 $T, P=0 \text{ MeV}$
 $\rho_{\text{sat}} = 0.15 \text{ fm}^{-3}$
 $E/A = -16 \text{ MeV}$

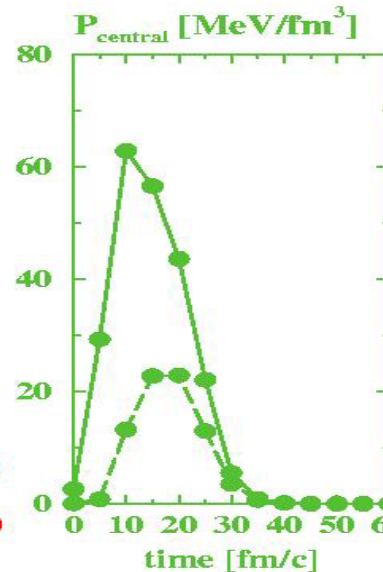
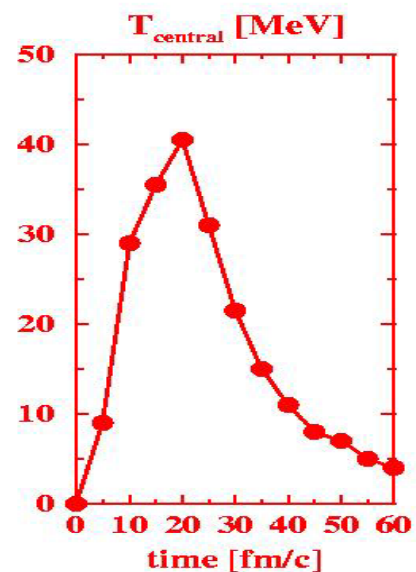
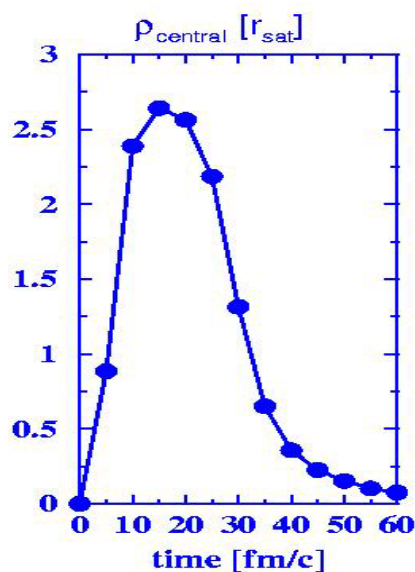
$$E_{\text{beam}} = 0.1-10 \text{ GeV/N}$$



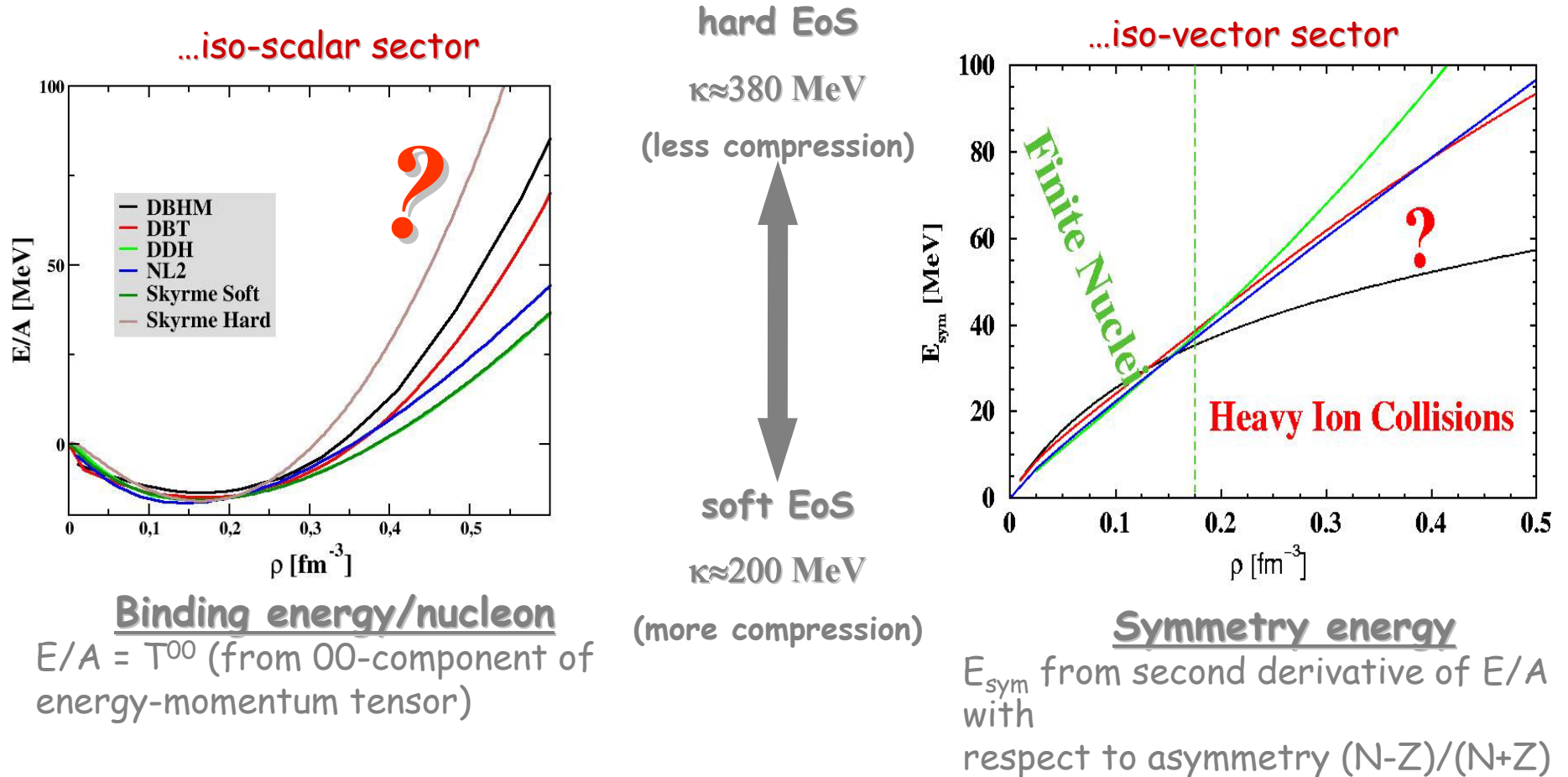
Explore matter under **extreme** Conditions

$$\rho = (2-3) \cdot \rho_{\text{sat}}, T \gg T_c \text{ and isospin}$$

Au ($E_{\text{beam}}=0.6 \text{ GeV/nucleon}$) + Au – central



The Nuclear EOS Uncertainties



**Nuclear matter at supra-normal densities not fixed
 (crucial differences between models)**

High density symmetry energy → neutron star: - structure (mass, hybrid)
 - cooling (proton fraction → direct URCA)

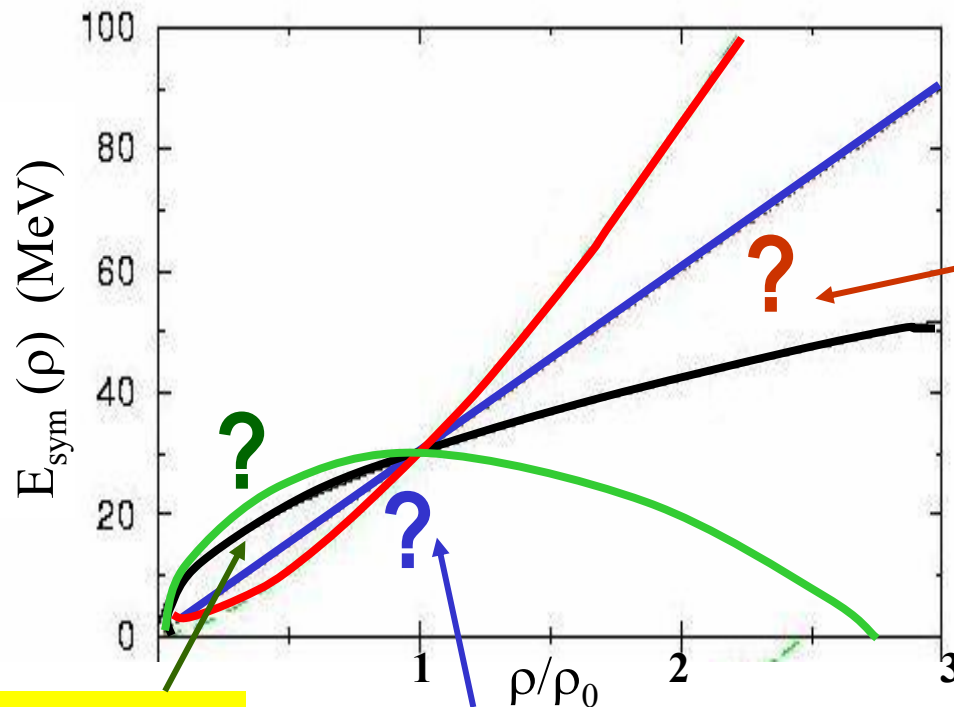
Symmetry Energy

$$\varepsilon(\rho, \rho_3) = \varepsilon(\rho) + \rho E_{\text{sym}}(\rho) \left(\frac{\rho_3}{\rho} \right)^2 + O(\rho_3^4) + \dots$$

$$\rho = \rho_p + \rho_n$$

$$\rho_3 = \rho_p - \rho_n$$

$$I \equiv \rho_3 / \rho$$

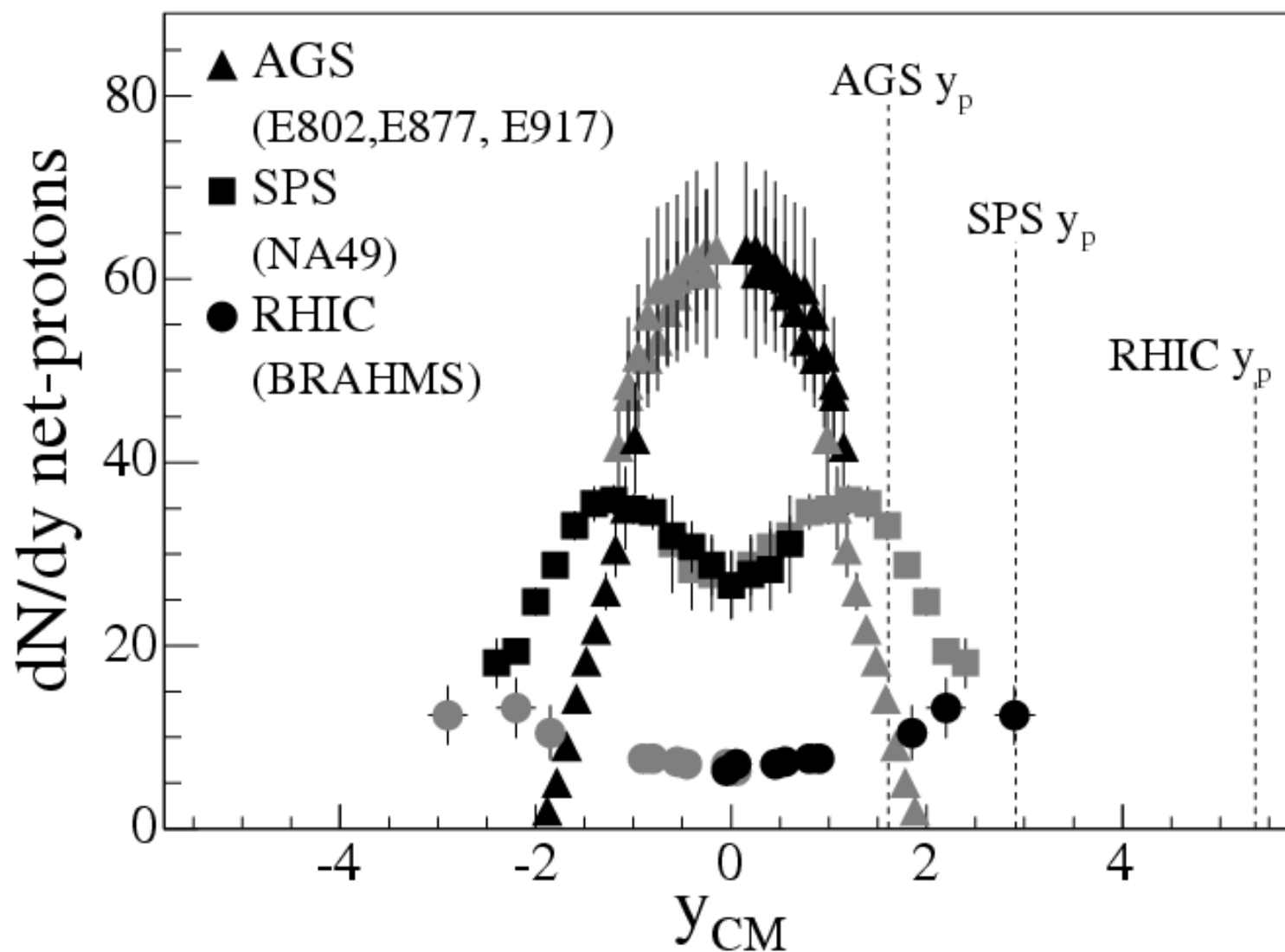


Neutron Stars
-formation
-mass/radius
-cooling
-"structure"

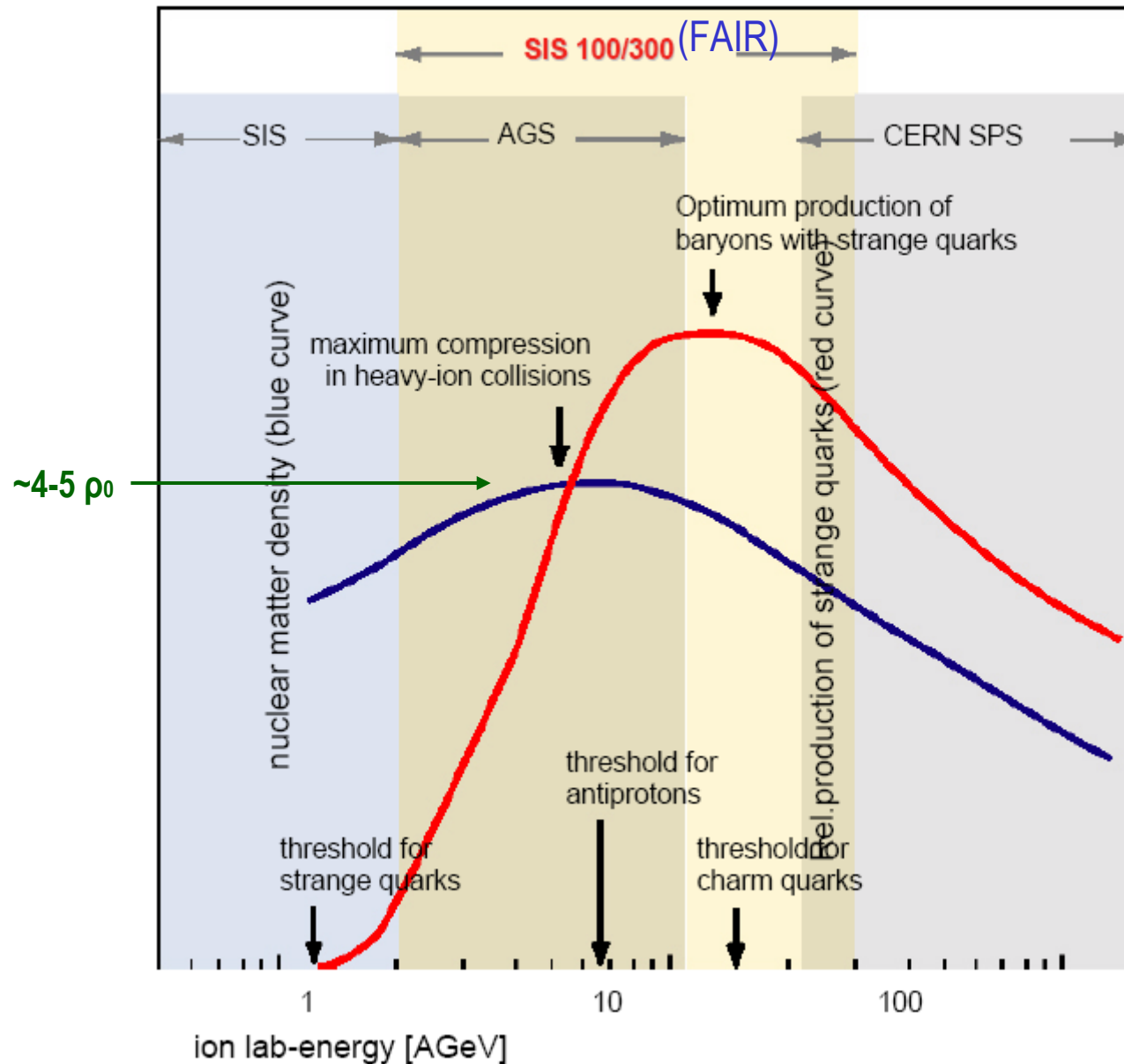
Neutron star
crust, merging

Nuclear structure
Giant Dipole
Neutron skin

Why Intermediate Energies?
Proton stopping at mid-rapidity: Au+Au central



Physics at High Baryon (and Isospin?) Density

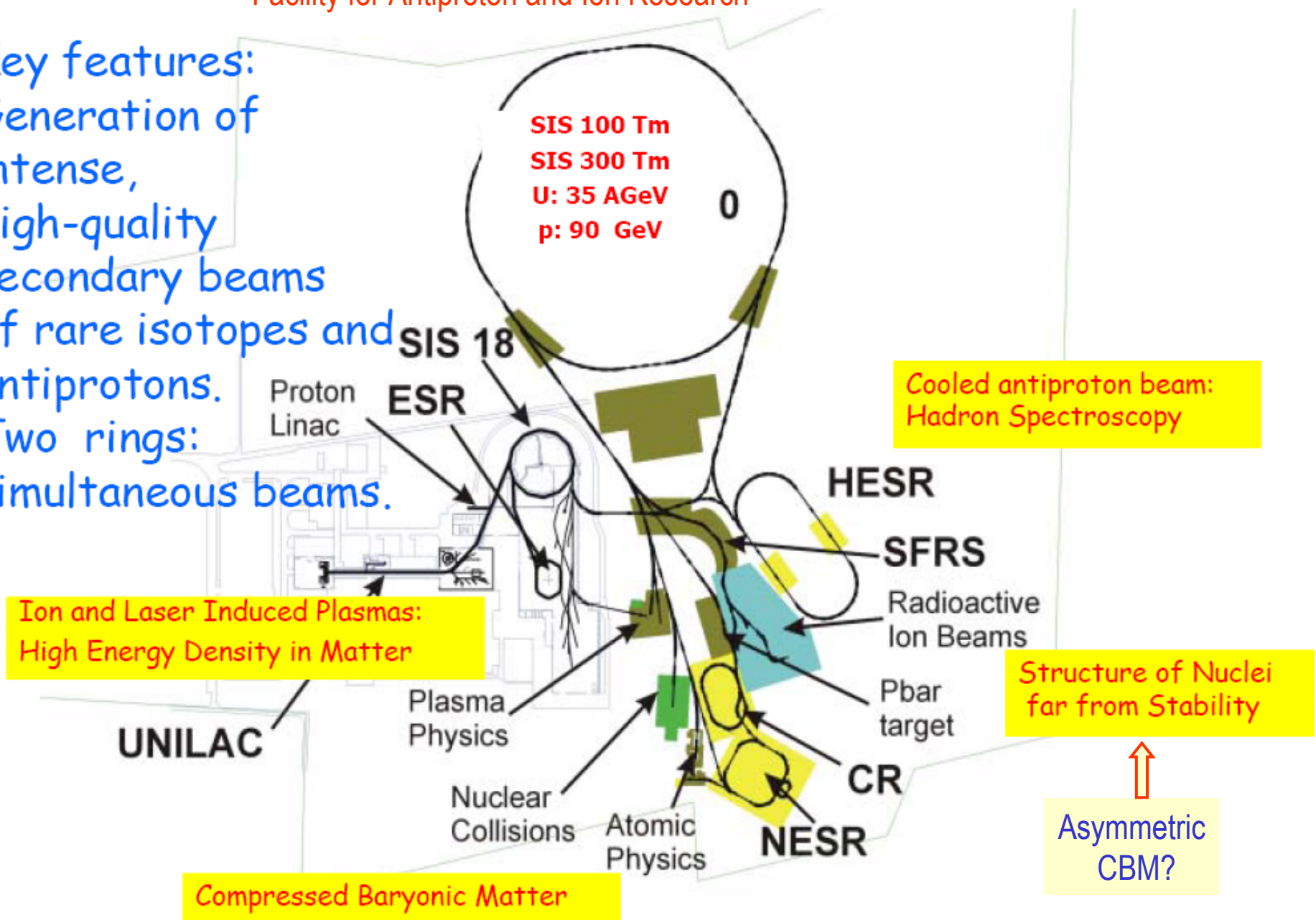


The future international accelerator facility FAIR

Facility for Antiproton and Ion Research

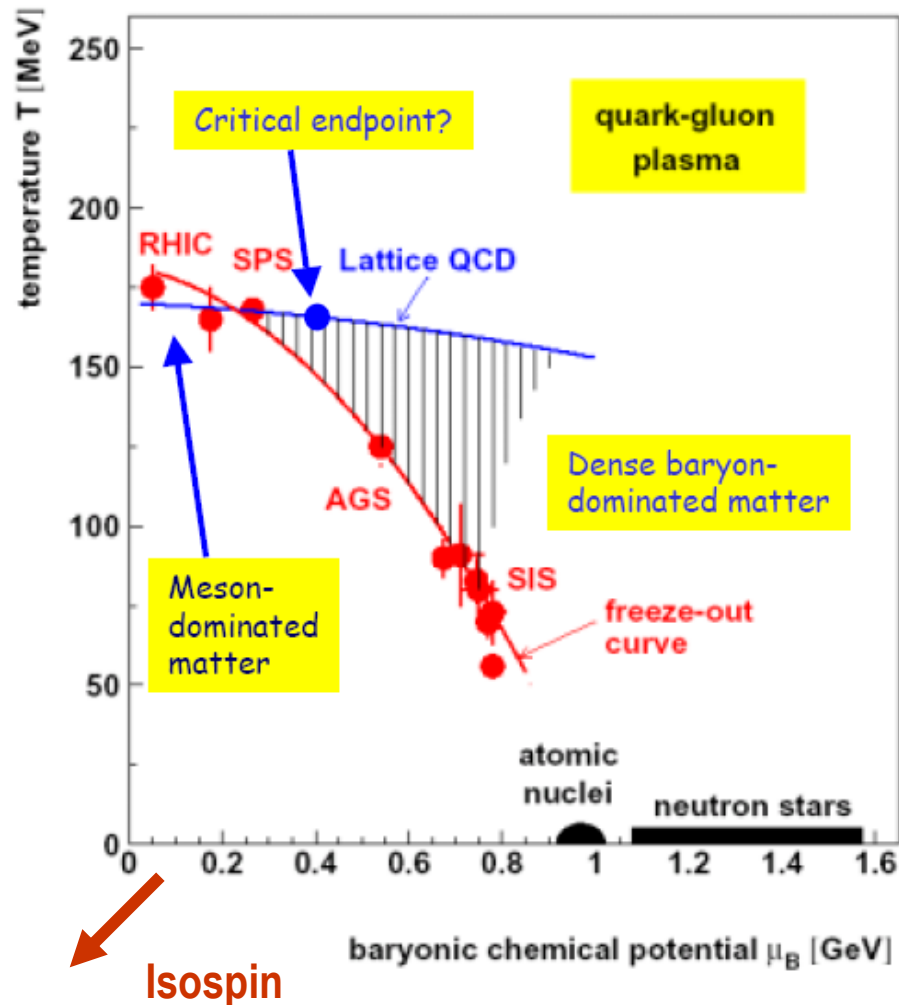


Key features:
Generation of
intense,
high-quality
secondary beams
of rare isotopes and
antiprotons.
Two rings:
simultaneous beams.



The QCD phase diagram

B.Friman,GSI



To map out the QCD phase diagram need:

High, intermediate and low energies

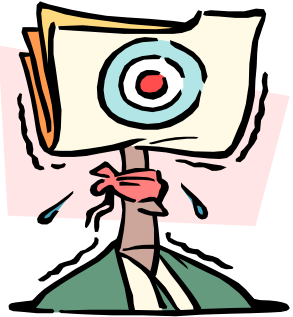
At $E = 10 - 45$ AGeV:

- Probe matter at high baryon densities
- Open charm near threshold
- Strangeness maximum
- Dileptons: ρ , ω , ϕ , J/Ψ in matter
- Collective flow
- Chiral restoration
- Deconfinement
- Critical end point?

Quantum Hadrodynamics (QHD)

NN scattering $\xrightarrow{\text{OBE}}$ nuclear interaction from meson exchange:
main channels (plus correlations)

$$\begin{array}{ccccc} \sigma(0^+,0) & \omega(1^-,0) & \delta(0^+,1) & \rho(1^-,1) \\ \underbrace{\text{Scalar}} & \underbrace{\text{Vector}} & \underbrace{\text{Scalar}} & \underbrace{\text{Vector}} \\ \text{Isoscalar} & & \text{Isovector} & \end{array}$$



Nuclear interaction by Effective Field Theory
as a covariant Density Functional Approach

⊕ Attraction & Repulsion



Saturation

$$L = \bar{\Psi} \left[\gamma_\mu (i\partial^\mu - g_\omega \hat{V}^\mu) - (M - g_\sigma \hat{\Phi}) \right] + \frac{1}{2} (\partial^\mu \hat{\Phi} \partial_\mu \hat{\Phi} - m_\sigma^2 \hat{\Phi}^2) - \frac{1}{4} \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{V}_\mu \hat{V}^\mu$$

$$\sigma: (\partial_\mu \partial^\mu + m_\sigma^2) \hat{\Phi} = g_\sigma \bar{\Psi} \Psi = g_\sigma \hat{\rho}_S$$

$$\omega: \partial_\mu \hat{W}^{\mu\nu} + m_\omega^2 \hat{V}^\nu = g_\omega \bar{\Psi} \gamma^\nu \Psi = g_\omega J^\nu$$

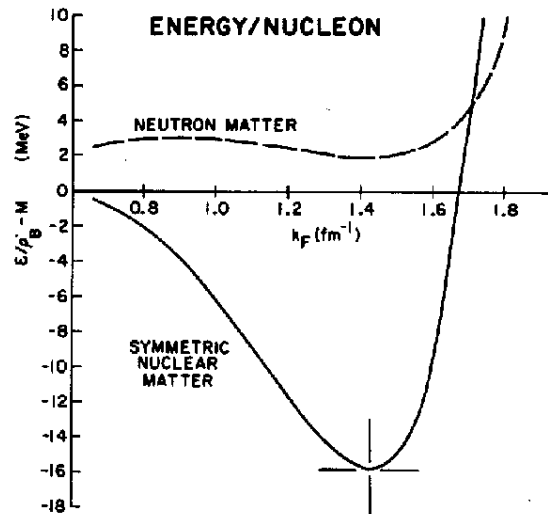
No linear

No perturbative

QHD Equation of State

$$\varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) n_i(k) + \frac{1}{2} \underbrace{\left(\frac{g_s^2}{m_\sigma^2} \right)}_{\mathbf{f}_\sigma} \rho_s^2 + \frac{1}{2} \underbrace{\left(\frac{g_\omega^2}{m_\omega^2} \right)}_{\mathbf{f}_\omega} \rho_B^2 \quad \text{QHD-I}$$

$$\varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) n_i(k) + \underbrace{\frac{1}{2} m_\sigma^2 \Phi^2 + U(\Phi)}_{\mathbf{f}_{\sigma\dots}} + \frac{1}{2} \underbrace{\left(\frac{g_\omega^2}{m_\omega^2} \right)}_{\mathbf{f}_\omega} \rho_B^2 + \frac{1}{2} \underbrace{\left(\frac{g_\rho^2}{m_\rho^2} \right)}_{\mathbf{f}_\rho} \rho_{B3}^2 + \frac{1}{2} \underbrace{\left(\frac{g_\delta^2}{m_\delta^2} \right)}_{\mathbf{f}_\delta} \rho_{S3}^2$$



QHD-I

- Self-interacting terms (NL models)
- Charged effective mesons (QHD-II)

- ✓ soft EoS ($K=220$ MeV)
- ✓ $E_{\text{bind}} = -16$ MeV
- ✓ $\rho_0 (\text{fm}^{-3}) \approx 0.15$
- ✓ $m^*/M = 0.75$
- ✓ $a_4 = 30.7$ MeV

Isospin degrees of freedom in QHD

➤ $\sigma - \omega$ model $\longrightarrow$ Only kinetic contribution to E_{sym}

➤ Charged mesons :

$\vec{\delta}[a_0(980)]$ (scalar isovector)

$\vec{\rho}(770)$ (vector isovector)

$$N: [\gamma_\mu i \partial^\mu - g_V \gamma_0 V^0 - g_\rho \gamma_0 \tau_3 b^0 - (M - g_s \Phi - g_\delta \tau_3 \delta_3)] \Psi = 0$$

$\longrightarrow$ Splitting n & p M^*

$$\vec{\rho}: b_0 = \frac{g_\rho}{m_\rho^2} (\rho_p - \rho_n)$$

$$\vec{\delta}: \delta_0 = \frac{g_\delta}{m_\delta^2} (\rho_{sp} - \rho_{sn})$$

Relativistic structure also
in isospin space !

$$E_{\text{sym}} = \text{cin.} + (\rho\text{-vector}) - (\delta\text{-scalar})$$

Nuclear Matter

$$f_{\rho,\delta} \equiv \left(\frac{g_{\rho,\delta}}{m_{\rho,\delta}} \right)^2$$

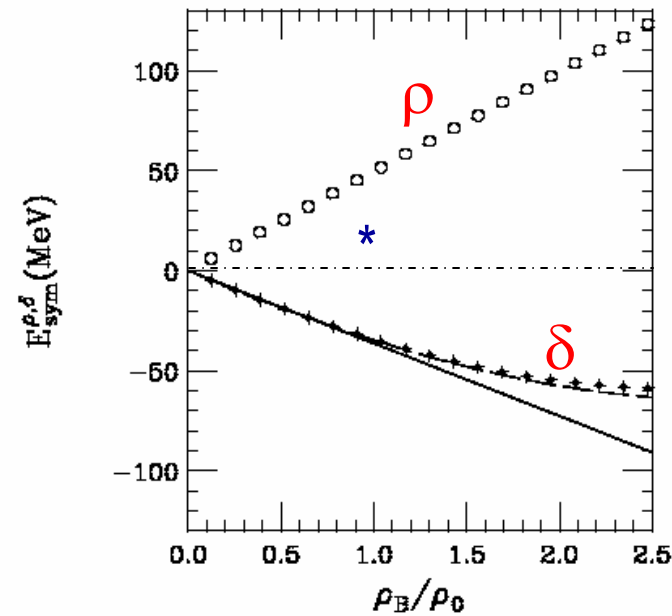
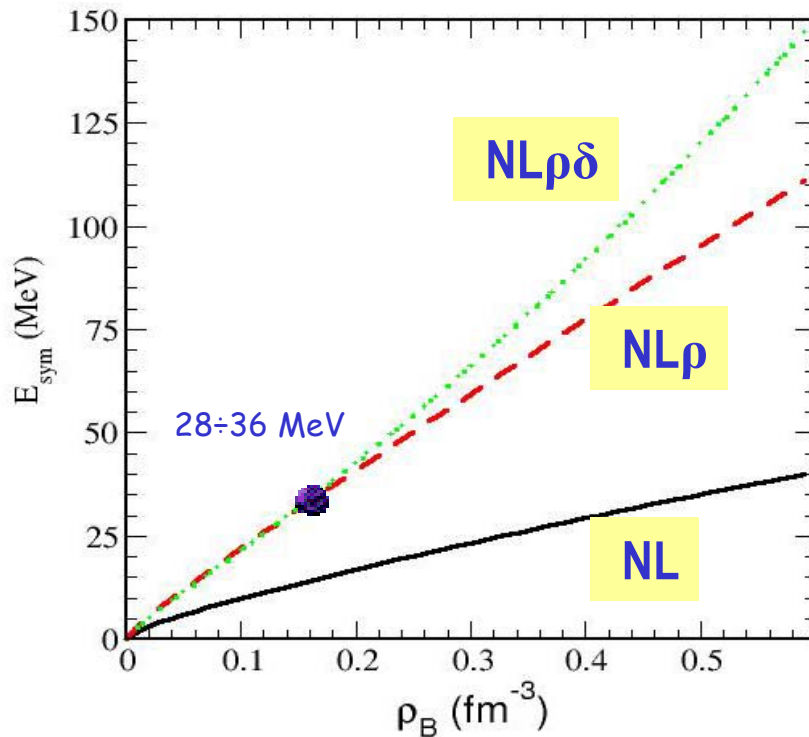
RMF Symmetry Energy: δ – contrib.

$$E_{\text{sym}} = \frac{1}{6} \frac{k_F^2}{E_F^{*2}} + \frac{1}{2} \left[f_\rho - f_\delta \left(\frac{M^*}{E^*} \right)^2 \right] \rho_B$$

$\xrightarrow{\text{No } \delta} f_\rho \cong 1.5 f_\rho^{\text{FREE}}$
 $\xrightarrow{f_\delta = 2.5 \text{ fm}^2} f_\rho \cong 5 f_\rho^{\text{FREE}}$

$a_4 = E_{\text{sym}}(\rho_0)$ fixes (f_ρ, f_δ)

DBHF } $f_\delta \approx 2.0 \div 2.5 \text{ fm}^2$
 DHF }



Self-Energies: kinetic momenta and (Dirac) effective masses

$$k_i^{*\mu} \equiv k_i^\mu - \Sigma_i^\mu$$

$$m_i^* \equiv M - \Sigma_{s,i}$$

$$\Sigma_s(n, p) = f_\sigma \sigma(\rho_s) \mp f_\delta \rho_{s3}$$

$$\Sigma^\mu(n, p) = f_\omega j^\mu \mp f_\rho j_3^\mu$$

Upper sign: n

Dirac dispersion relation: single particle energies

$$(\rho, j)_3 \equiv (\rho, j)_p - (\rho, j)_n$$

$$\rho_{B3} \equiv \rho_{Bp} - \rho_{Bn} < 0, n\text{-rich}$$

$$\varepsilon_i + M = +\Sigma_i^0 + \sqrt{k^2 + m_i^{*2}}$$

Chemical Potentials (zero temp.)

$$\mu_i = \sqrt{k_F^2 + m_i^{*2}} + f_\omega \rho_B \mp f_\rho \rho_{B3}$$

Symmetry Energy

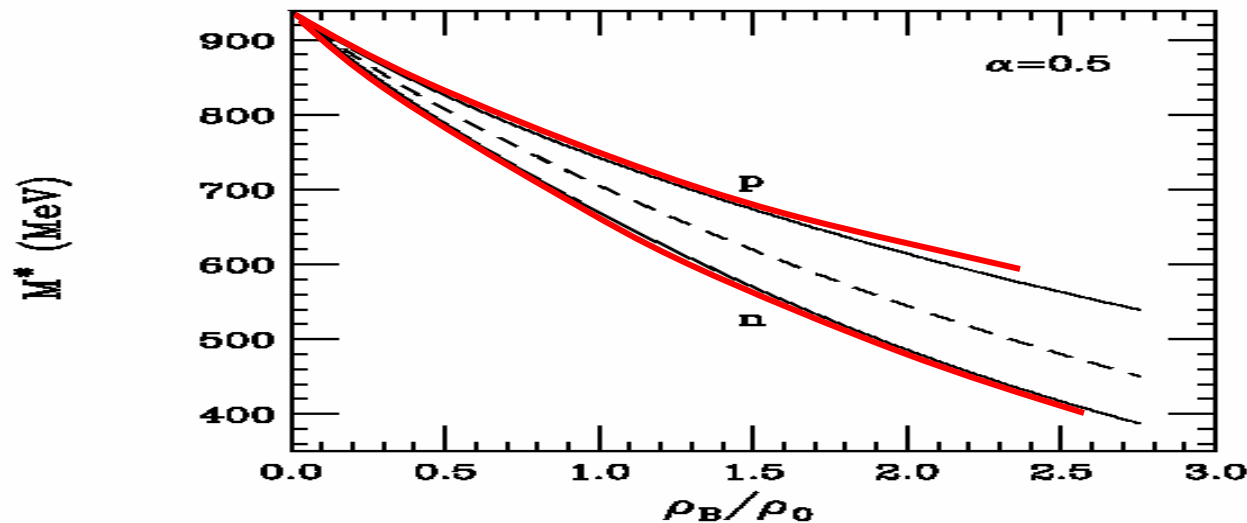
$$\mu_n - \mu_p = 4E_{\text{sym}} I$$

$$I \equiv \frac{\rho_n - \rho_p}{\rho}$$

Asymmetry parameter

Effective Mass Splitting: Dirac Masses

Minimal Effective Field Approach: $(\sigma, \omega, \delta, \rho)$



$$m_D^*(q) = m - \Sigma_s(\sigma) \pm f_\delta \rho_{S3}$$

$$\rightarrow +n, -p$$

$$\rho_3 \equiv \rho_p - \rho_n, < 0 \Rightarrow n\text{-rich}$$

— RMF-($\rho+\delta$)
 - - - RMF- ρ

Splitting sign $(m_D^*(n) - m_D^*(p))$

Agree { RMFT, DHF (V. Greco et al., PRC63, PRC64 (2001))
 DBHF (F. Hofmann et al., PRC64 (2001))
 SLy (E. Chabanat et al., NPA 627 (1997)) non rel.

Disagree { BHF (W. Zuo, PRC60 (1999) 24605)
 "Old" Skyrme non rel.

DIRAC OPTICAL POTENTIAL

Dispersion relation $(E - \Sigma_0)^2 = p^2 + (m - \Sigma_s)^2$

$$\downarrow$$

$$\varepsilon + m \Rightarrow \sqrt{k_\infty^2 + m^2}$$

$$\longrightarrow V_{opt} = \Sigma_0 - \Sigma_s + \frac{1}{2m} (\Sigma_s^2 - \Sigma_0^2) + \frac{\Sigma_0}{m} \varepsilon$$

$\longrightarrow$ RMF

Schroedinger mass

$$\frac{m_q^*}{m} = \left[1 + \frac{m}{\hbar^2 k} \frac{\partial U_q}{\partial k} \right]^{-1}$$

$$m_S^* = \frac{m}{1 + \frac{\Sigma_0}{m}} \approx m - \Sigma_0 = m_D^* + (\Sigma_s - \Sigma_0)$$

$\nearrow$
~50 MeV

Dirac mass

$$m_D^* = m - \Sigma_s$$

Asymmetric Matter

$(\sigma, \omega, \rho, \delta)$

$$\Sigma_0 \Rightarrow \Sigma_0 \mp f_\rho \rho_{B3}$$

$$\Sigma_s \Rightarrow \Sigma_s \mp f_\delta \rho_{S3}$$

upper signs: neutron

$$\rho_{B3} \equiv \rho_{Bp} - \rho_{Bn} < 0, n\text{-rich}$$

$$m_S^*(n, p) = m_{Dsym}^* + (\Sigma_s - \Sigma_0)_{sym} \pm \rho_{B3} \left(f_\rho - \frac{m_D^*}{E_F^*} f_\delta \right) \rightarrow \begin{matrix} m_D^*(n) < m_D^*(p) \\ m_S^*(n) < m_S^*(p) \end{matrix}$$

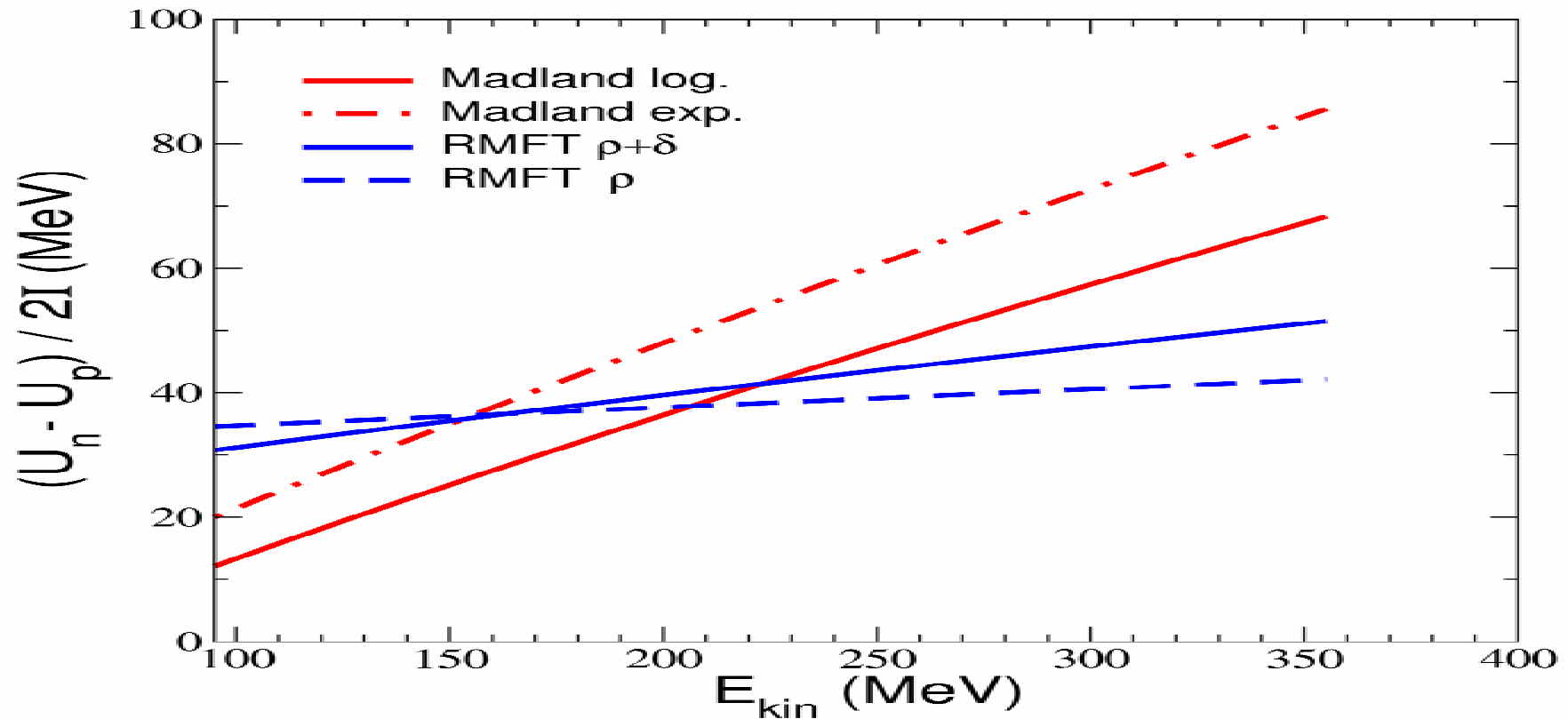
DIRAC LANE POTENTIAL

$$\frac{U_n - U_p}{2\beta} = E_{sym,pot}(\rho_0) + f_\rho \frac{\rho_0}{m} \varepsilon$$

ρ
 $\rho + \delta$

$\nearrow$
 $\searrow$

$+ 0.04\varepsilon$
 $+ 0.12\varepsilon$



BEYOND RMF: k-dependence of the Self-Energies

$$f(\sigma, \omega, \rho, \delta) \equiv f_i(\rho_B, k) \quad \Leftrightarrow \quad \text{DBHF}$$

Schroedinger mass

$$m_S^* = m_D^* + (\Sigma_s - \Sigma_0) + (m - \Sigma_s)\Sigma_s' - (m + \varepsilon - \Sigma_0)\Sigma_0'$$

$$\Sigma_0' \equiv \frac{d\Sigma_0}{d\varepsilon} < 0 \quad \text{High momentum saturation of the optical potential}$$

$$\Sigma_s' \equiv \frac{d\Sigma_s}{d\varepsilon} < 0 \quad \text{High momentum increase of the Dirac Mass}$$

Asymmetric Matter

$$m_D^*(n) < m_D^*(p)$$

but

$$m_S^*(n) >, < m_S^*(p)$$

Analysis of DB self energies

Decomposition of DB self energy

$$\Sigma(p) = \Sigma^s(p) - \gamma^0 \Sigma^0(p) + \vec{\gamma} \cdot \vec{p} \Sigma^v.$$

Density (and momentum) dependent coupling coeff.
(at k_F)

$$\begin{aligned} \left(\frac{g_\sigma^*}{m_\sigma}\right)^2 &= -\frac{1}{2} \frac{\Sigma_n^s(\bar{p}_f) + \Sigma_p^s(\bar{p}_f)}{\rho_n^s + \rho_p^s} \\ \left(\frac{g_\omega^*}{m_\omega}\right)^2 &= -\frac{1}{2} \frac{\Sigma_n^0(\bar{p}_f) + \Sigma_p^0(\bar{p}_f)}{\rho_n^v + \rho_p^v} \\ \left(\frac{g_\delta^*}{m_\delta}\right)^2 &= -\frac{1}{2} \frac{\Sigma_n^s(\bar{p}_f) - \Sigma_p^s(\bar{p}_f)}{\rho_n^s - \rho_p^s} \\ \left(\frac{g_\rho^*}{m_\rho}\right)^2 &= -\frac{1}{2} \frac{\Sigma_n^0(\bar{p}_f) - \Sigma_p^0(\bar{p}_f)}{\rho_n^v - \rho_p^v}, \end{aligned}$$

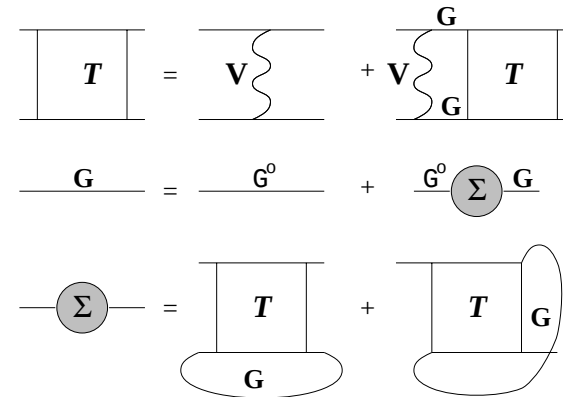
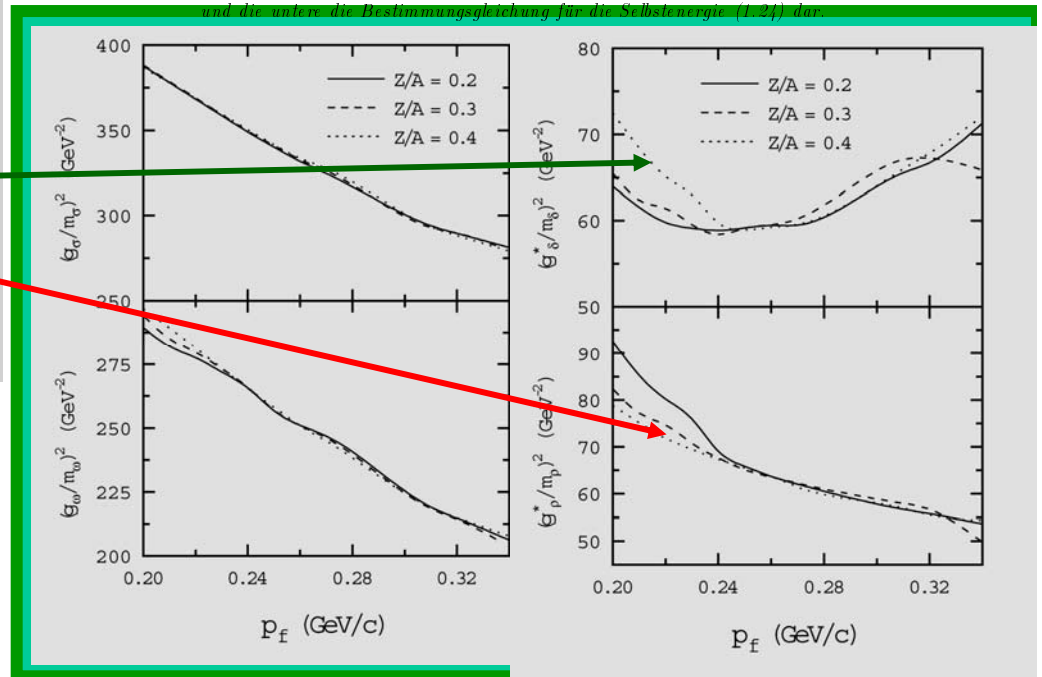


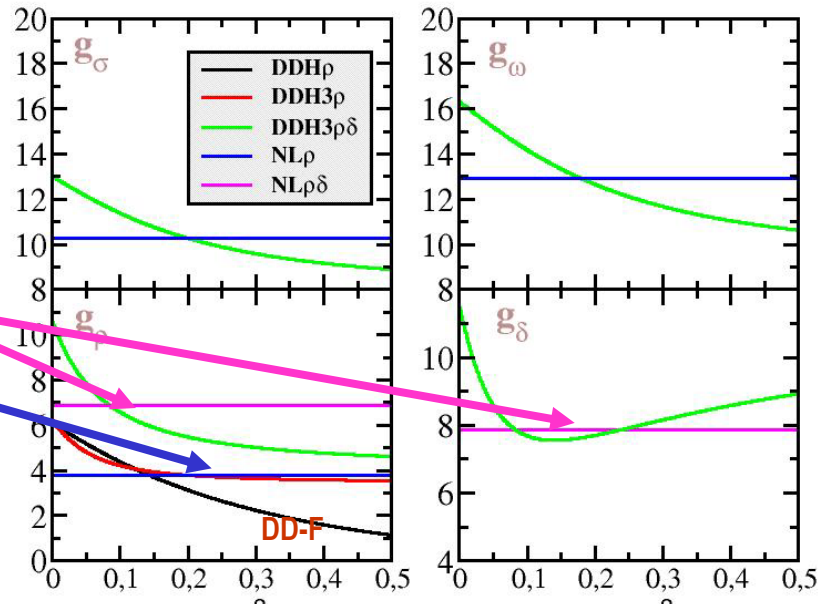
Abbildung 1.2: Diagrammatische Darstellung der DB-Methode. Die oberste Reihe stellt die Bethe-Salpeter-Gleichung (1.23), die mittlere die Dyson-Gleichung (1.25) und die untere die Bestimmungsgleichung für die Selbstenergie (1.24) dar.



A $\rho\delta$ parametrization of the isovector dependence

NL $\rho\delta$

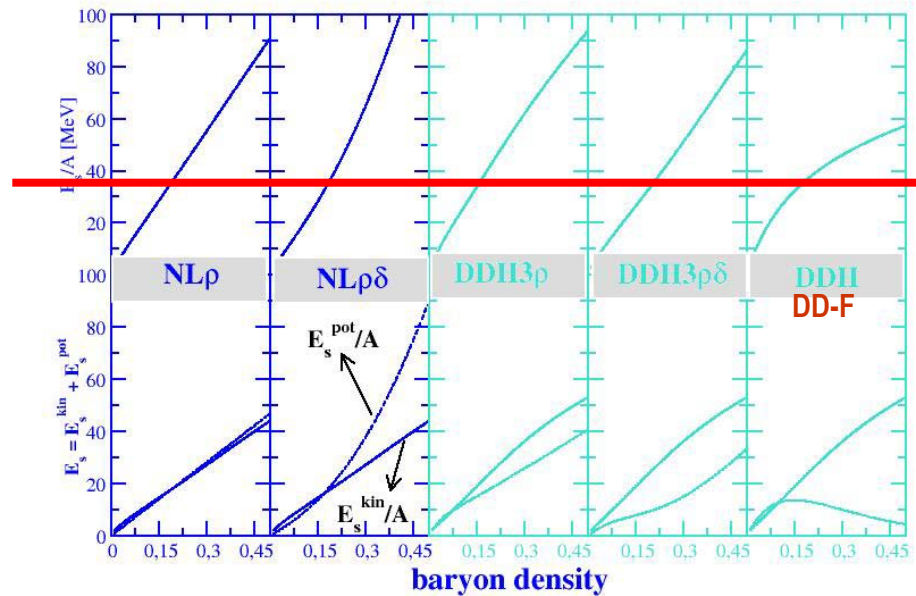
NL ρ



Walecka-Type (NL)

DDH Models

$$a_4 = E_{\text{sym}}(\rho_0) \approx 33 \text{ MeV}$$



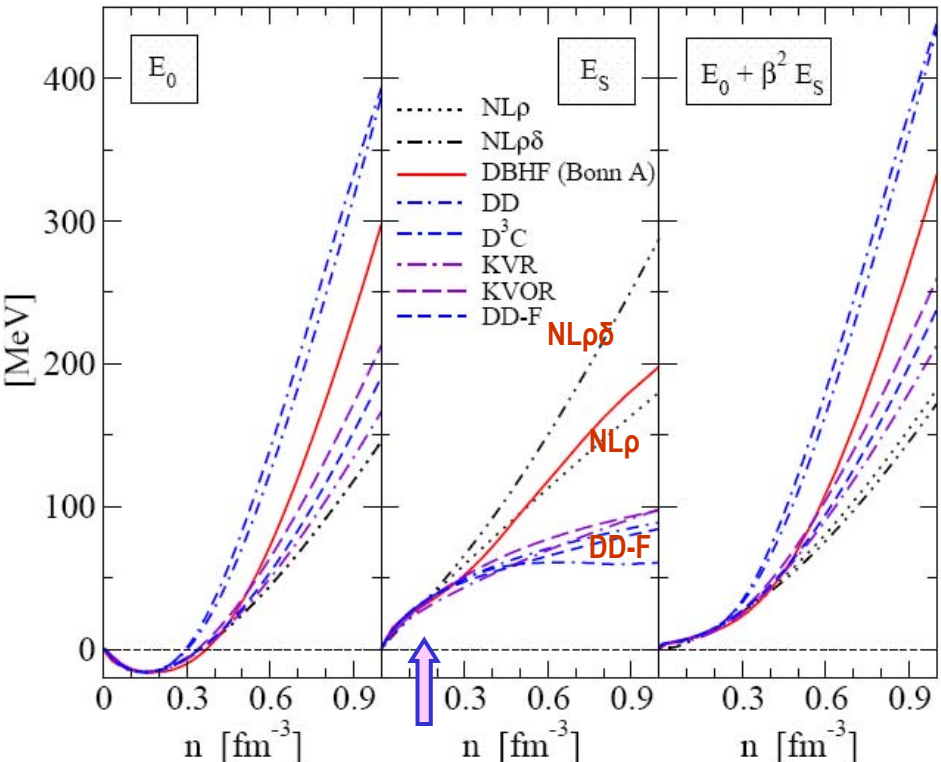
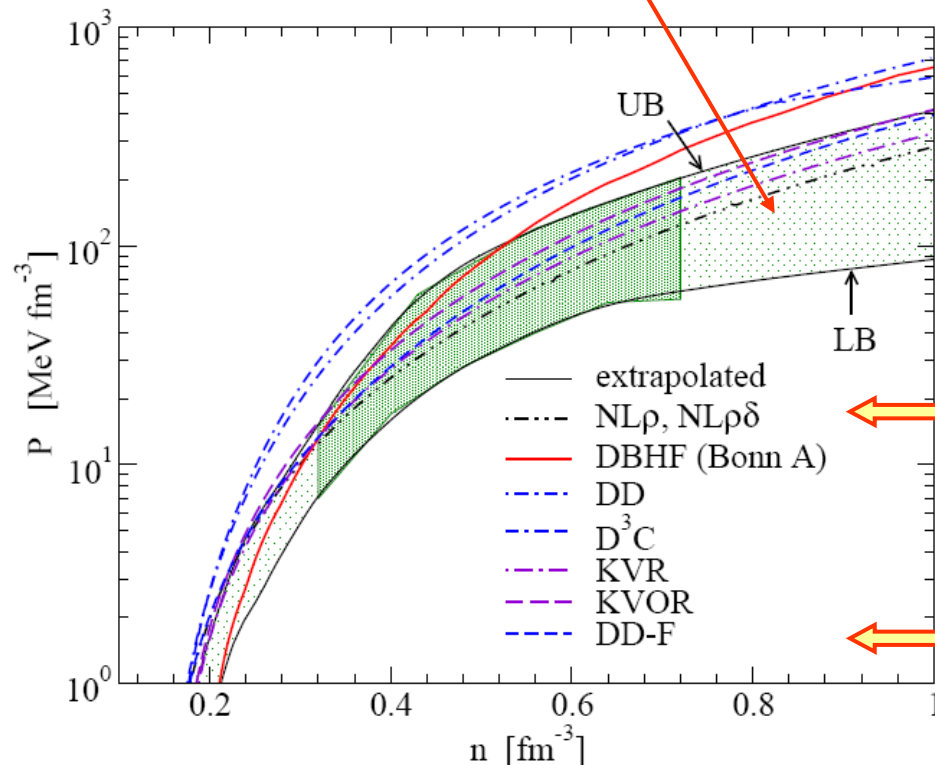
Collection of EOS "Realistic" Covariant Models

Symmetric Matter | Symmetry Energy | Neutron Matter

compact stars & heavy ion data
T.Klaen et al. Nucl-th/0602038

Pressure

Exp boundaries (flow, kaon multiplicities)



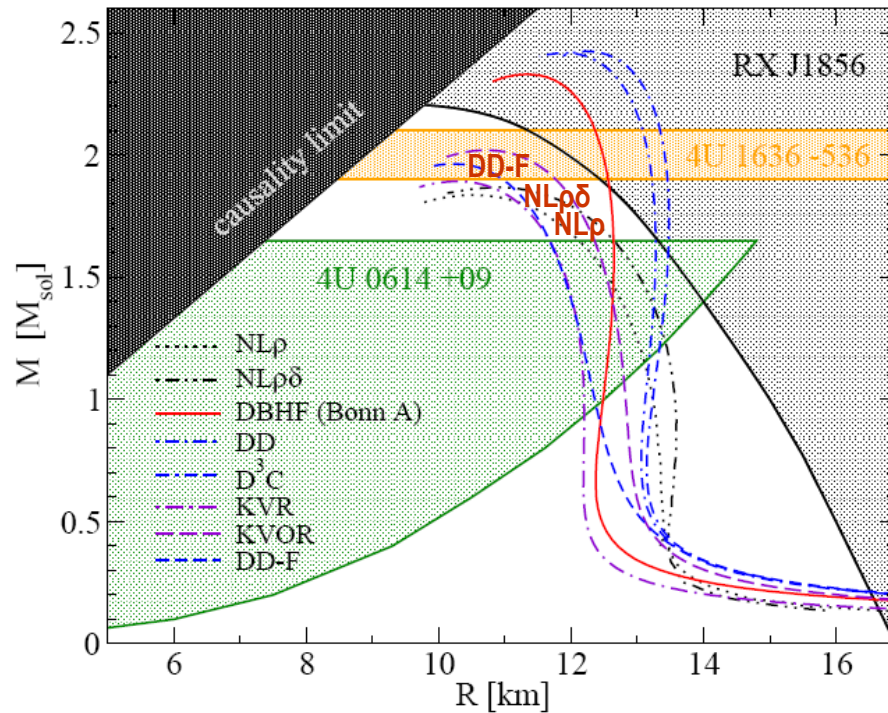
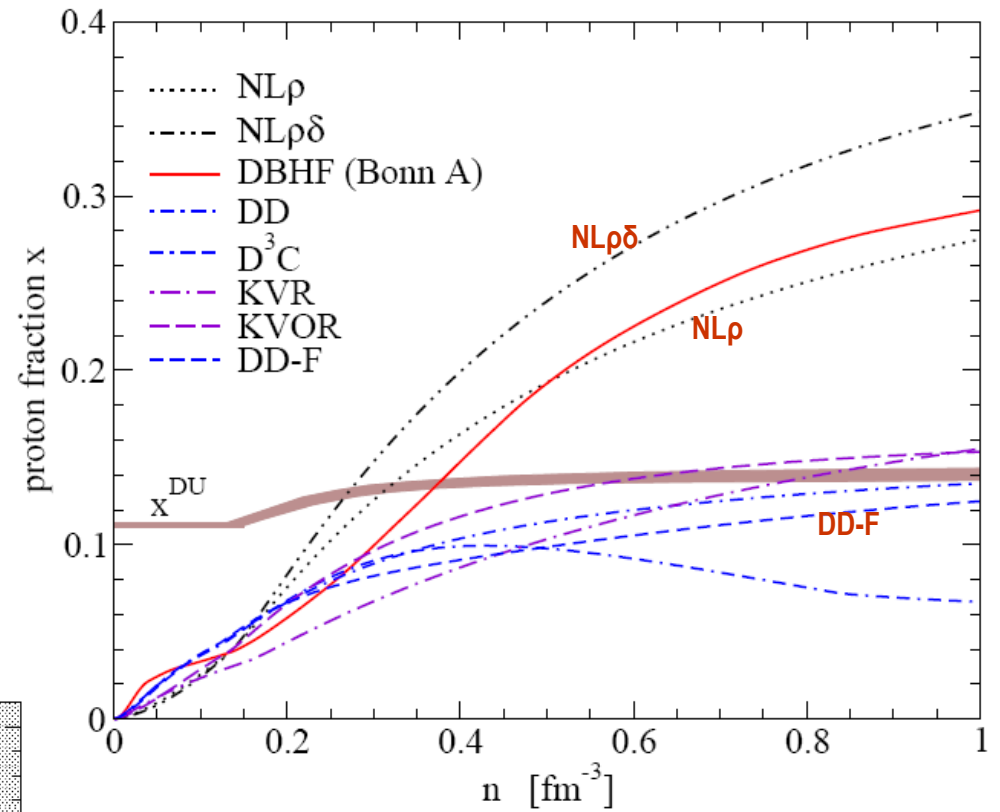
Slope at normal density:
Isospin transport at Fermi energies

Soft Symm Matter, Stiff (super-stiff) Symmetry Energy

Soft Symm Matter, Soft Symmetry Energy

Neutron Star ($npe\mu$) properties

Direct URCA threshold



Mass/Radius relation

RBUU transport equation

Wigner transform $\cap$ Dirac + Fields Equation $\Rightarrow$ Relativistic Vlasov Equation + Collision Term...

$$\left[\frac{p_i^{*\mu}}{M_i^*} \partial_\mu + \left(\frac{p_{\nu i}^*}{M_i^*} \mathcal{F}_i^{\mu\nu} + \partial^\mu M_i^* \right) \partial_\mu^{(p^*)} \right] f_i(x, p^*) = \mathcal{I}_c$$

drift

mean field

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_r f + \vec{\nabla}_r U \cdot \vec{\nabla}_p f = I_{coll}$$

Non-relativistic Boltzmann-Nordheim-Vlasov

**“Lorentz Force” $\rightarrow$ Vector Fields
pure relativistic term**

$$k_i^{*\mu} \equiv k_i^\mu - \Sigma_i^\mu$$

$$m_i^* \equiv M - \Sigma_{s,i}$$

$$F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu$$

Collision term:

$$\mathcal{I}_c = \frac{g}{(2\pi)^3} \int \frac{dp_2^*}{p_2^{*0}} \frac{dp_3^*}{p_3^{*0}} \frac{dp_4^*}{p_4^{*0}} \int d\Omega (p^* + p_2^*)^2 \frac{d\sigma}{d\Omega} \delta^4(p^* + p_2^* - p_3^* - p_4^*) \\ \times \{ f_3 f_4 [1 - f][1 - f_2] - f f_2 [1 - f_3][1 - f_4] \}$$

Relativistic Landau Vlasov Propagation

Discretization of $f(x, p^*) \rightarrow$ Test particles represented by covariant Gaussians in xp -space

$$f(x, p^*) = \sum_{i=1}^{AN_{test}} \int_{-\infty}^{+\infty} d\tau \, g(x - x_i(\tau)) g(p^* - p_i^*(\tau))$$

$\rightarrow$ **Relativistic** Equations of motion for x^μ and $p^{*\mu}$ for centroids of Gaussians

$$\frac{d}{d\tau} x_i^\mu = \frac{p_i^{*}(\tau)}{M_i^{*}(x_i)},$$

$$\frac{d}{d\tau} p_i^{*\mu} = \frac{p_{i\nu}^{*}(\tau)}{M_i^{*}(x_i)} \mathcal{F}_i^{\mu\nu}(x_i(\tau)) + \partial^\mu M_i^{*}(x_i)$$

u_ν Test-particle 4-velocity $\rightarrow$ Relativity: - momentum dependence always included due to the Lorentz term $(u_\nu F^{\mu\nu})$
 - E^*/M^* boosting of the vector contributions

Collision Term: local Montecarlo Algorithm imposing an average Mean Free Path plus Pauli Blocking
 $\rightarrow$ **in medium reduced Cross Sections**

Rel. Transport 1

1 ONE-BODY DENSITY MATRIX ($4 \times 4 + \text{ISOSPIN}$)

$$\rho_{\alpha\beta}(x_1, x_2) \equiv \langle \bar{\Psi}_\alpha(x_1) \Psi_\beta(x_2) \rangle$$

2 WIGNER TRANSFORM $x = \frac{1}{2}(x_1 + x_2)$

$$R = (x_1 - x_2)$$

$$F_{\alpha\beta}(x, p) \equiv \frac{1}{(2\pi)^4} \int d^4 R e^{-ipR} \rho_{\alpha\beta}(x + \frac{R}{2}, x - \frac{R}{2})$$

$$\Rightarrow \text{scalar density } \rho_s(x) \equiv \langle \bar{\Psi} \Psi \rangle = \int d^4 p \text{Tr} [F(x, p)]$$

$$\text{vector current } j_\mu(x) \equiv \langle \bar{\Psi} \gamma_\mu \Psi \rangle = \int d^4 p \text{Tr} [\gamma_\mu F(x, p)]$$

3 EQ.-OF-MOTION FOR $F(x, p) : (\sigma, \omega)$

$$[\gamma_\mu (\partial^\mu - 2ip^\mu) + 2im] F(x, p) = 2ie^{-\frac{i}{2}\Delta} (\not{\Sigma}_3 - \gamma_\mu \not{\Sigma}^\mu) F$$

$$\Delta \equiv \partial_\mu^P \partial_x^\mu$$

$\Downarrow$ on $F(x, p)$ $\Downarrow$ on fields

$\Rightarrow$ FIRST ORDER
EXPANSION

$$\Rightarrow \left\{ \frac{i}{2} [\gamma_\mu \partial^\mu - \Delta (\not{\Sigma}_3 - \gamma_\mu \not{\Sigma}^\mu)] + \gamma_\mu (p^\mu - \Sigma^\mu) - (m - \Sigma_3) \right\} F(x, p) = 0$$

$\downarrow$ $\downarrow$
 $|p^{*\mu}|$ $|m^*|$

Rel. Transport 2

4 REAL PART

$$(\gamma_\mu p^{*\mu} - m^*) F(x, p) = 0$$

$$\Rightarrow \text{MASS SHELL CONSTRAINT} \quad [(\gamma_\mu p^{*\mu} + m^*)] \times \text{left}$$

$$\boxed{(p^{*2} - m^{*2}) F = 0} \quad 8 \rightarrow 7 \text{ variables}$$

$$\Rightarrow \text{LORENTZ COMPONENTS OF } F(x, p)$$

$$F(x, p) = F_S(x, p) + \gamma_\mu F^\mu(x, p) + \frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu}(x, p) +$$

→ tensor ~ 0

$$+ \gamma^5 A(x, p) + \gamma^5 \gamma_\mu A^\mu(x, p)$$

pseudoscalar = 0 axial vector = 0 locally spin-saturated

$$\Rightarrow p^{*\mu} F_S = m^* F^\mu$$

$$\boxed{F^\mu(x, p) \equiv p^{*\mu} f(x, p)} \quad f(x, p) \equiv \frac{F_S}{m^*} \Rightarrow \rho_s(x)$$

SCALAR ⇒ $j_\mu(x)$

5 IMAGINARY PART

$$\partial_\mu F^\mu + \Delta(\Sigma_\mu F^\mu) - \Delta(\Sigma_S F_S) = 0$$

$$(x_\mu, p_\mu) \Rightarrow (x_\mu, p_\mu^*)$$

$$\Rightarrow p_\mu^* \partial_\mu f(x, p^*) + \left[p_\nu^* F^{\mu\nu} + m^* \partial^\mu m^* \right] \partial_\mu^{(p^*)} f(x, p^*) = 0$$

$$\boxed{F^{\mu\nu} \equiv \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu} \quad \begin{array}{l} \downarrow \\ \text{Lorentz force} \\ \text{FROM VECTOR FIELD} \end{array} \quad \begin{array}{l} \downarrow \\ \text{force FROM} \\ \text{SCALAR FIELD} \end{array}$$

Rel. Transport 3

6

RELATIVISTIC VLASOV EQUATION

$$\frac{P_\mu^*}{m^*} \partial_{(x)}^\mu f(x, p^*) + \left[\frac{P_\nu^*}{m^*} F^{\mu\nu} + \partial_{(x)}^\mu m^* \right] \partial_\mu^{(p^*)} f(x, p^*) = 0$$

① NON RELATIVISTIC

$$\partial_t f + \frac{\vec{p}}{m} \cdot \vec{\nabla} f$$

pure relativistic term

$$-\vec{\nabla} u \cdot \vec{\nabla}^{(p)} f$$

② LIOUVILLE THEOREM

$$\Rightarrow \left[\frac{\partial x^\mu}{\partial z} \frac{\partial}{\partial x^\mu} + \frac{\partial p^{*\mu}}{\partial z} \frac{\partial}{\partial p^{*\mu}} \right] f(x, p^*) = 0$$

$$\frac{d}{dz} f[x(z), p^*(z)] = 0 \quad \text{CONSTANT ALONG } x(z), p^*(z) \text{ P.S.}$$

TRAJECTORIES

WITH

$$\begin{cases} \frac{\partial x^\mu}{\partial z} = \frac{p^{*\mu}}{m^*} \\ \frac{\partial p^{*\mu}}{\partial z} = \frac{P_\nu^*}{m^*} F^{\mu\nu} + \partial_{(x)}^\mu m^* \end{cases} \quad \begin{array}{l} \text{MASS} \\ \text{SHELL} \end{array} \quad dx_\mu^2 = dz^2$$

EIGENTIME OF THE "PARTICLES"

$\Rightarrow$ TEST PARTICLE APPROACH

⊕ PROPAGATION BY A SYSTEM TIME

Rel. Transport 4

MEAN FIELD PROPAGATION :

"GENUINE" RELATIVISTIC EFFECTS FROM THE
LORENTZ STRUCTURE OF THE FIELDS

1 SCALAR AND BARYON DENSITIES (σ, ω)

$$\Sigma_S = \int_\sigma \rho_S(x) = \int_\sigma \int d^4p^* u^*(x) f(x, p^*)$$

$$\Sigma_\mu = \int_\omega J_\mu(x) = \int_\omega \int d^4p^* p_\mu^* f(x, p^*)$$

$$\Rightarrow \Sigma_0 = \int_\omega \rho_B(x) \quad ; \quad \rho_B(x) = \int d^4p^* p_0^* f(x, p^*)$$

$$\rho_S(x) \approx \frac{u^*}{p_0^*} \rho_B = \frac{\rho_B}{\gamma} < \rho_B \quad \text{AT HIGH DENSITIES (MOMENTA)}$$

$\rightarrow E^* = \sqrt{p^{*2} + u^{*2}}$

2 LORENTZ BOOSTING OF THE VECTOR FIELD CONTRIBUTION TO THE "FORCE" ACTING ON THE PARTICLE

$$\frac{d\vec{p}^*}{dz} = \int_\omega \frac{p_\nu^*}{u^*} (\vec{\nabla}_\nu J_\nu - J_\nu \vec{\nabla}_\nu) - \int_\sigma \vec{\nabla} \rho_S$$

$$\approx \int_\omega \frac{E^*}{u^*} \vec{\nabla} \rho_B - \int_\sigma \vec{\nabla} \rho_S \approx \left(\int_\omega \frac{E^*}{u^*} - \frac{u^*}{E^*} \int_\sigma \right) \vec{\nabla} \rho_B$$

(neglecting $\vec{p}_\nu J_\mu$)

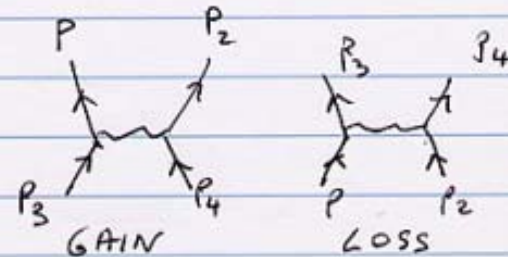
γ $\downarrow$
 $\frac{1}{\gamma}$

$\Rightarrow$ ENHANCED EFFECT OF ρ -MESON FIELD (ISOVECTOR-VECTOR)
ON NEUTRON VS PROTON DYNAMICS AT HIGH MOMENTA

Rel. Transport 5

COLLISION TERM

$$\frac{d}{dz} f(x, p^*) = I_{\text{coll}}[f]$$



⇒ STOSSZAHLANSATZ ⊕ LOCAL

$$N_{\text{coll}}(1,2) \sim f_1(x, p_1^*) f_2(x, p_2^*)$$

$$I_{\text{coll}}[f] = \int \frac{d^3 p_1^*}{E_1^*} \frac{d^3 p_3}{E_3^*} \frac{d^3 p_4}{E_4^*} W(p_1^* p_2^* | p_3^* p_4^*)$$

$$\times [f_3 f_4 (1-f_1)(1-f_2) - f_1 f_2 (1-f_3)(1-f_4)]$$

GAIN LOSS PAULI BLOCKING

$$W(p_1^* p_2^* | p_3^* p_4^*) \approx \frac{d\sigma}{d\Omega}(s, t) \delta^4(p_1 + p_2 - p_3 - p_4)$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

SIMULATION : MONTECARLO ALGORITHM

PROBABILITIES GIVEN BY IN-MEDIUM
REDUCED CROSS SECTIONS

Rel. Transport 6

INELASTIC COLLISIONS : ISOSPIN EFFECTS

$$P_1 + P_2 = P_3 + P_4$$

$$\Rightarrow P_1^* + \Sigma_1^* + P_2^* + \Sigma_2^* = P_3^* + \Sigma_3^* + P_4^* + \Sigma_4^*$$

$$\text{REFERENCE FRAME } \vec{P}_1^* + \vec{P}_2^* \approx \vec{P}_3^* + \vec{P}_4^* = 0 \quad (\Delta \vec{\Sigma} \approx 0)$$

$$\Rightarrow \boxed{E_1^* + \Sigma_1^0 + E_2^* + \Sigma_2^0 = E_3^* + \Sigma_3^0 + E_4^* + \Sigma_4^0}$$

$$\boxed{> m_3^* + \Sigma_3^0 + m_4^* + \Sigma_4^0}$$

THRESHOLD

$$\text{NEUTRON RICH SYSTEM : } \left[\begin{array}{c} \Sigma_u^0 \uparrow \\ \Sigma_p^0 \downarrow \end{array} \right] \begin{array}{l} (\text{line } \Sigma_{\Delta^-}^0) \\ (\text{line } \Sigma_{\Delta^{++}}^0) \end{array}$$

$$\Rightarrow nn \rightarrow p \Delta^- \quad \text{vs} \quad pp \rightarrow n \Delta^{++}$$

$$\sim 2(E_n^* + \Sigma_u^0) > m_p^* + m_{\Delta^-}^* \quad 2(E_p^* + \Sigma_p^0) > m_n^* + m_{\Delta^{++}}^*$$

$\sim 2m^*$ (symmetric matter) $\sim 2m^*$ (symmetric matter)

INCREASING

$$nL \rightarrow nLp \rightarrow nLp\delta$$

DECREASING

$$nL \rightarrow nLp \rightarrow nLp\delta$$

$\Rightarrow$ LARGER π^-/π^+ AND ISOVECTOR CHANNELS DEPENDENCE

Fragment Formation in Central Collisions at Relativistic Energies

Au+Au, Zr+Zr, Ni+Ni at 400 AMeV → Central

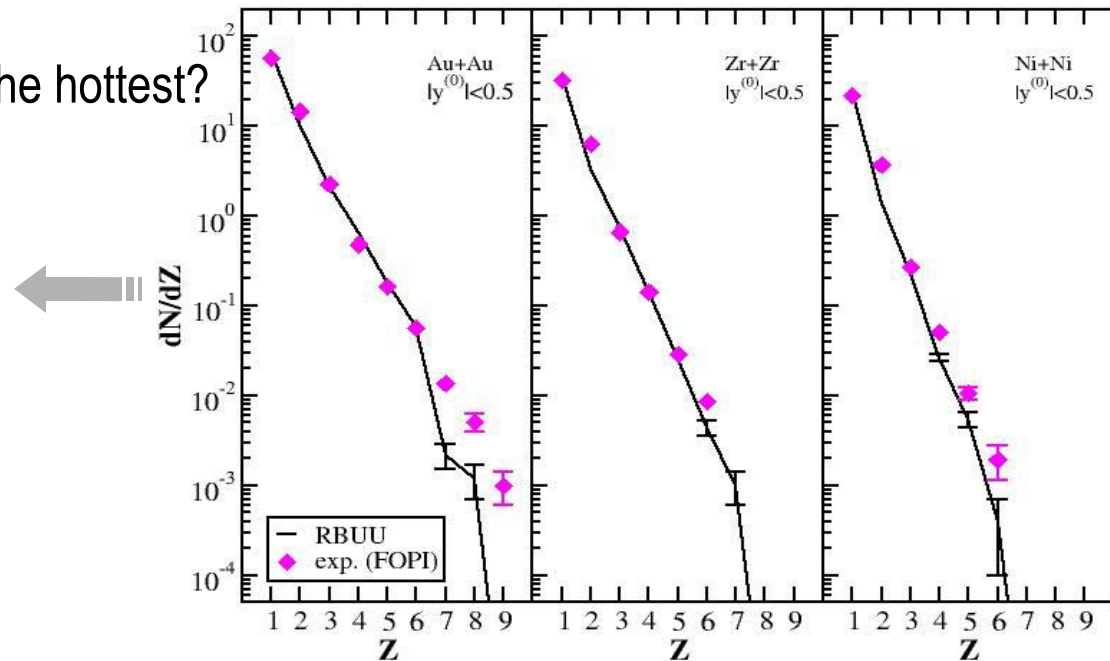
Stochastic RBUU + Phase Space Coalescence

➤ Global fit to experimental charge distributions

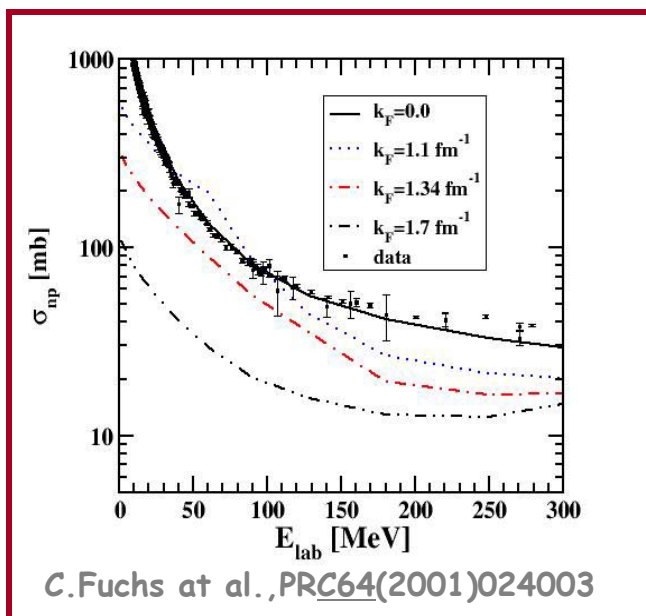
Size dependence: the lightest is the hottest?

No but

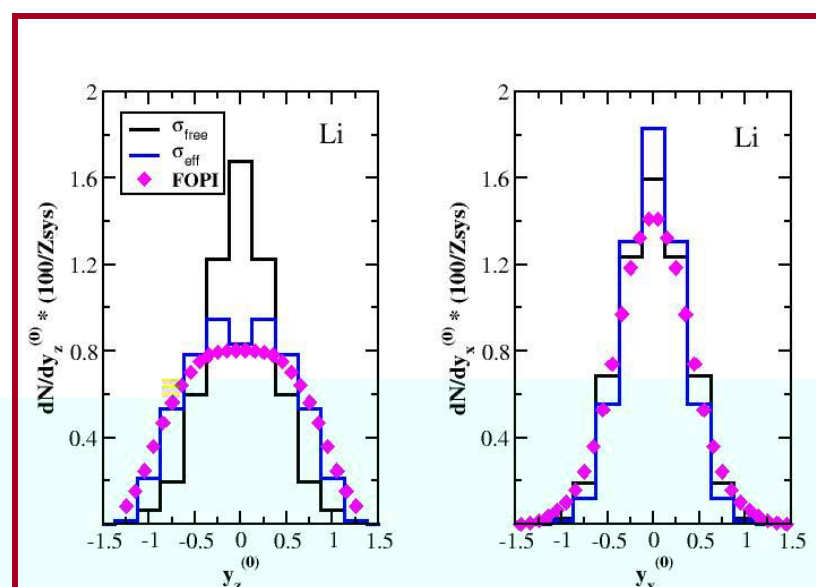
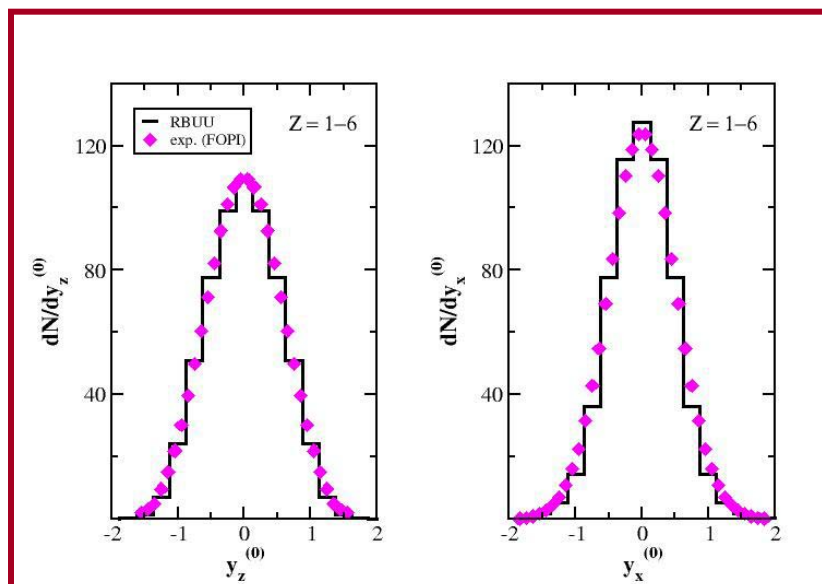
Fast clusterization in the high density phase



In-medium effects of σ_{NN} on stopping



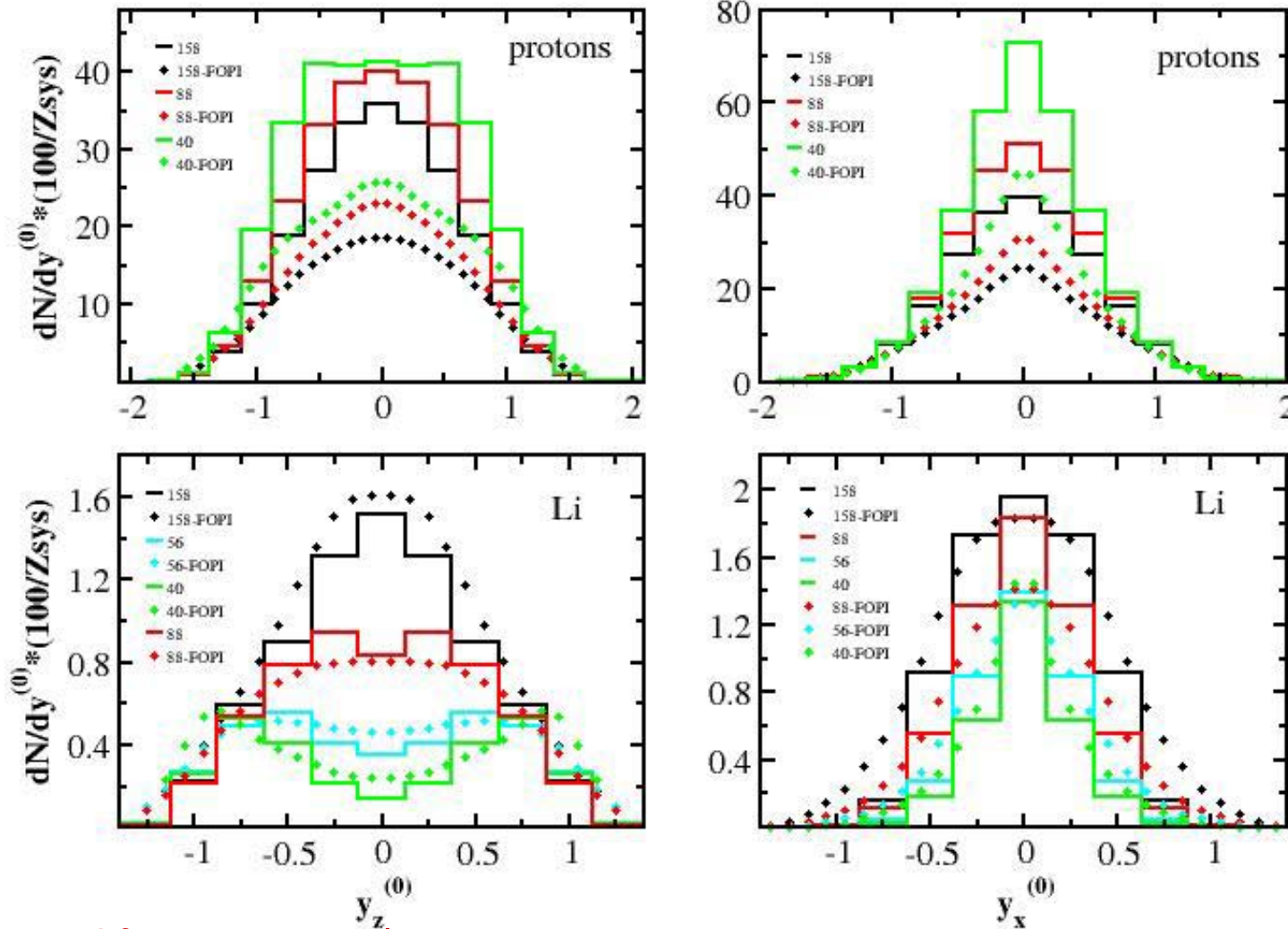
Fragment
z- and x- rapidity distributions



(E. Santini et al., NPA756(2005)468)

Transparency properties of protons & clusters

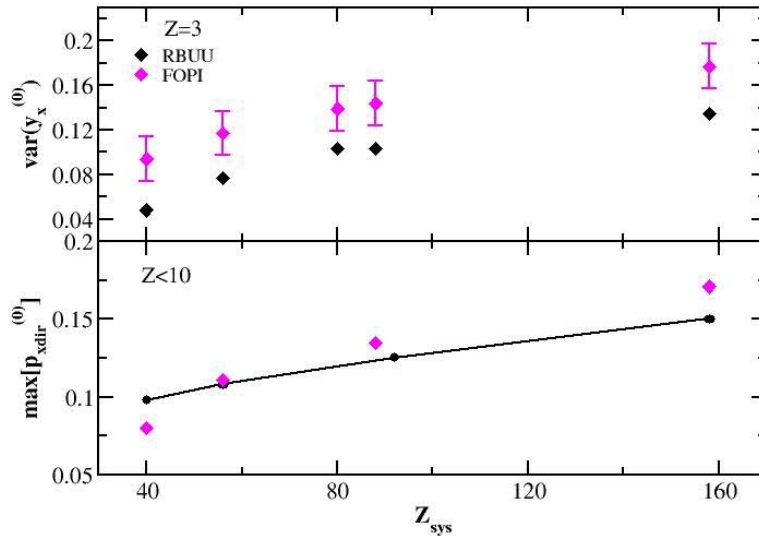
(E. Santini et al., NPA756(2005)468)



Opposite trend for protons vs clusters:
correlation between fragment multiplicity and stopping

Deuteron coalescence problem: protons overestimated (but not $Z=1-6$)

System size behaviour of clusters



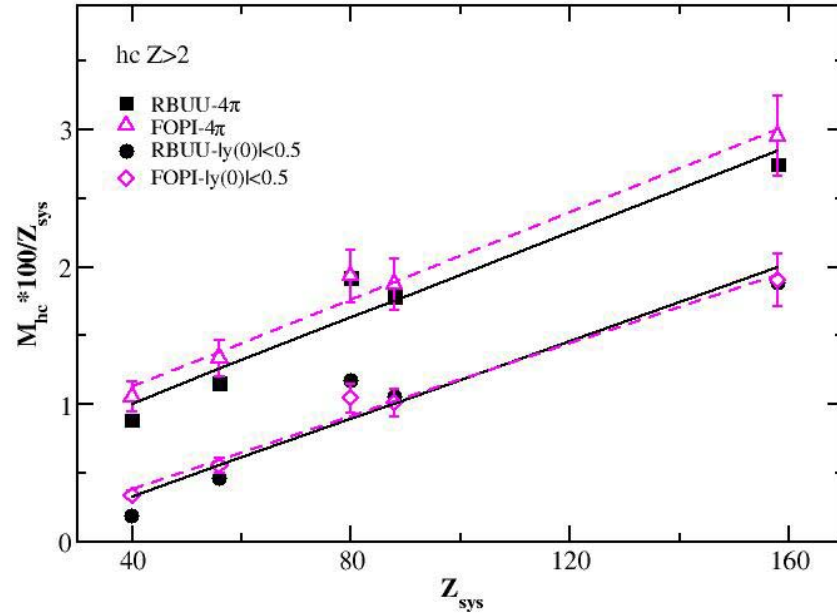
Li-stopping correlation

$Z=1-10$ cluster directed flow

Size (Z_{sys}^2)-dependence of heavy cluster multiplicity



Seeds: two-body correlations

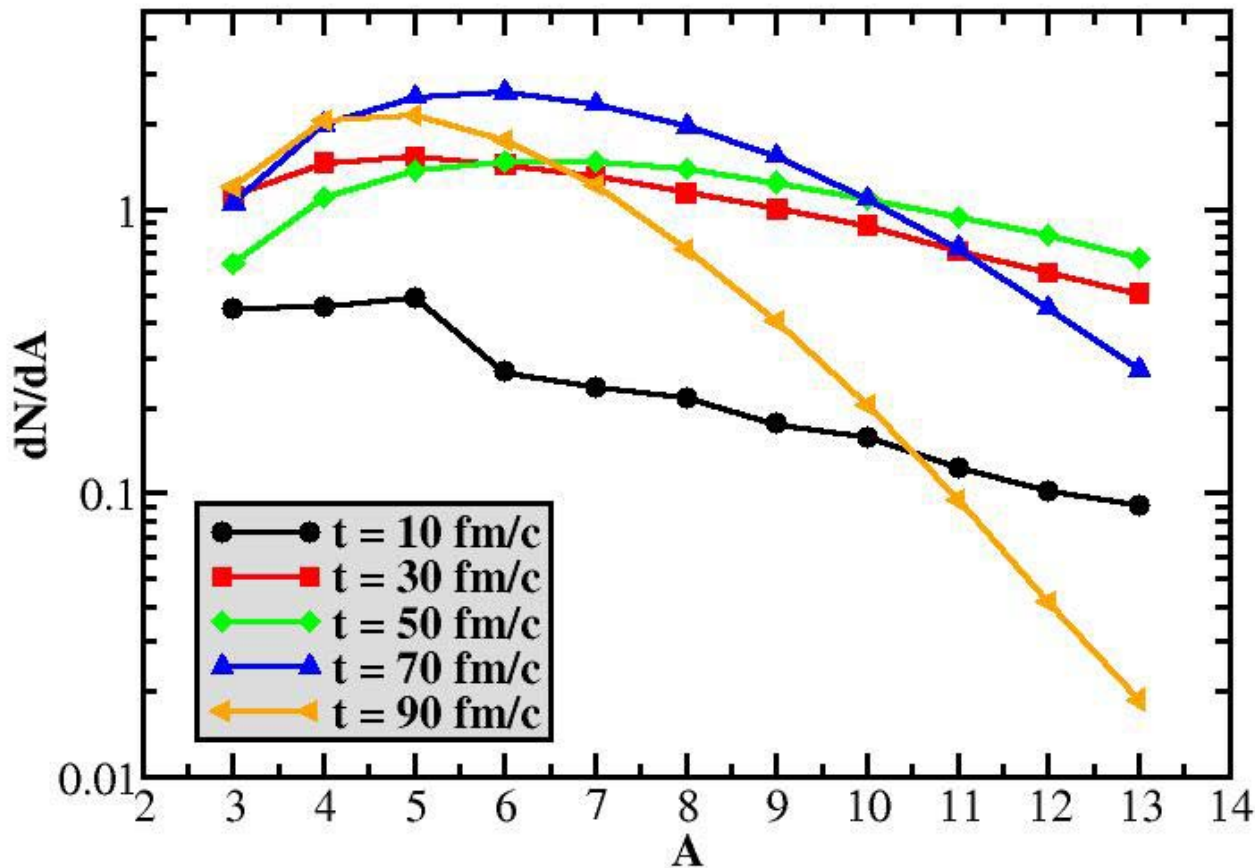


Time-evolution of fragment formation

(E. Santini et al., NPA756(2005)468)

Au+Au 0.4 AGeV Central

Z=3,4



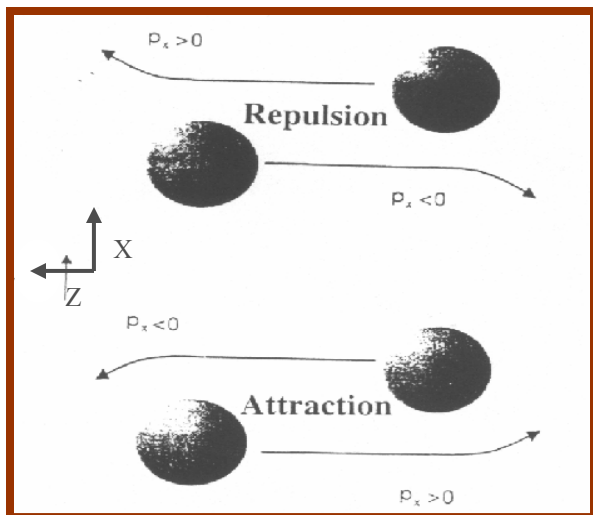
Heavier fragments: “relics” of the high density phase



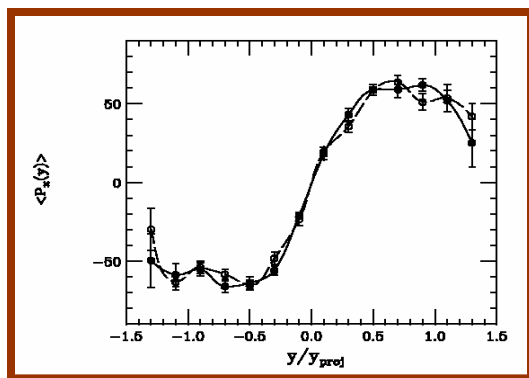
Isospin Content vs. Symmetry Term ?

Collective flows

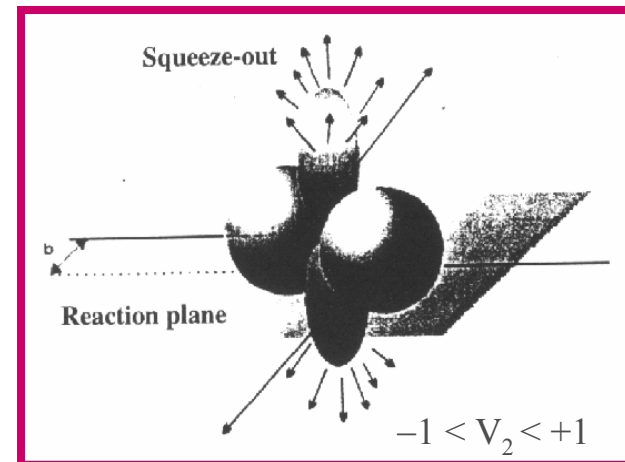
In-plane



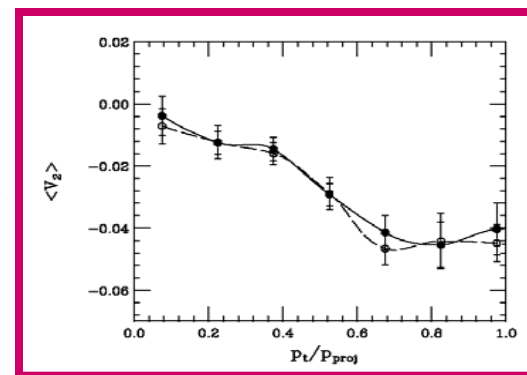
$$V_1(y, p_t) = \langle p_x \rangle / \langle p_t \rangle$$



Out-of-plane



$$V_2(p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



$$V_2 \begin{cases} = -1 & \text{full out} \\ = 0 & \text{spherical} \\ = +1 & \text{full in} \end{cases}$$

$$V_1^{p-n}(y, p_t) = V_1^p(p_t) - V_1^n(p_t)$$

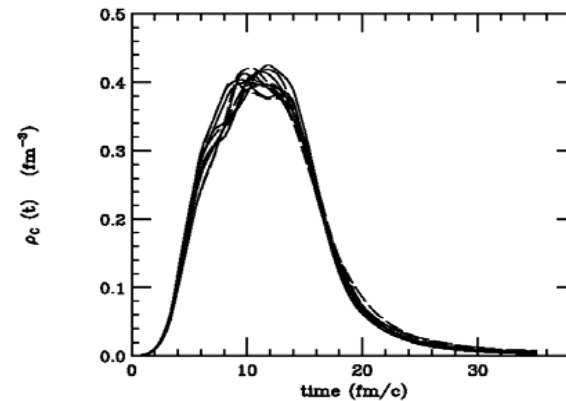
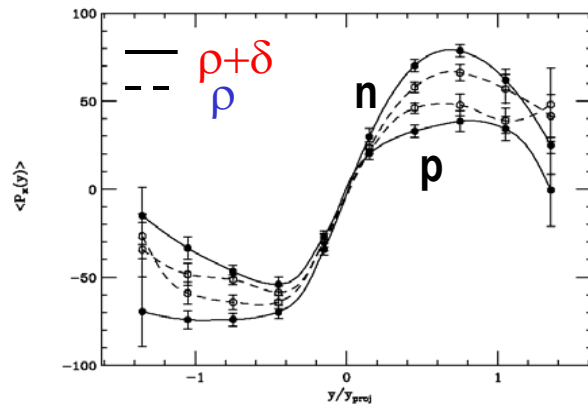
Isospin

Differential flows

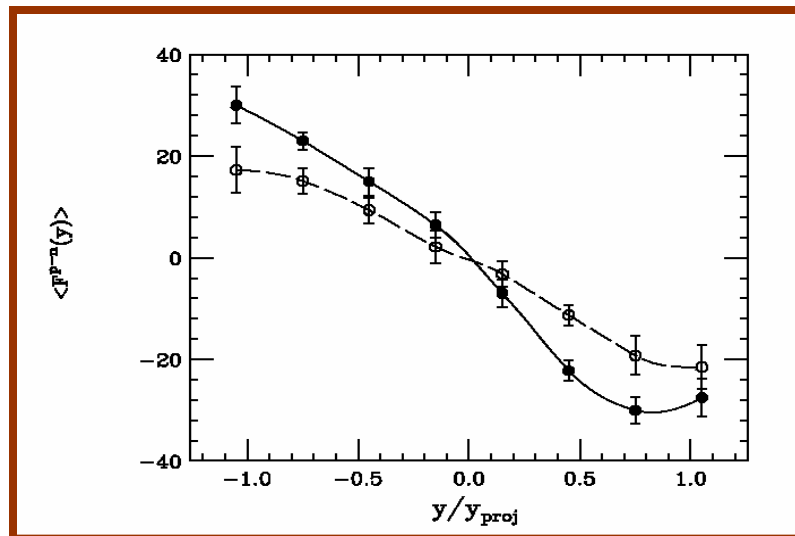
$$V_2^{p-n}(p_t) = V_2^p(p_t) - V_2^n(p_t)$$

Differential Transverse Flow

$^{132}\text{Sn} + ^{132}\text{Sn} @ 1.5 \text{ AGeV } b=6\text{fm}$



- Sensitivity to the isovector part of the mean field
 $\rightarrow E_{\text{sym}}$ around $\rho \approx 2\rho_0$



○ NLH- ρ

• NLH-($\rho + \delta$)

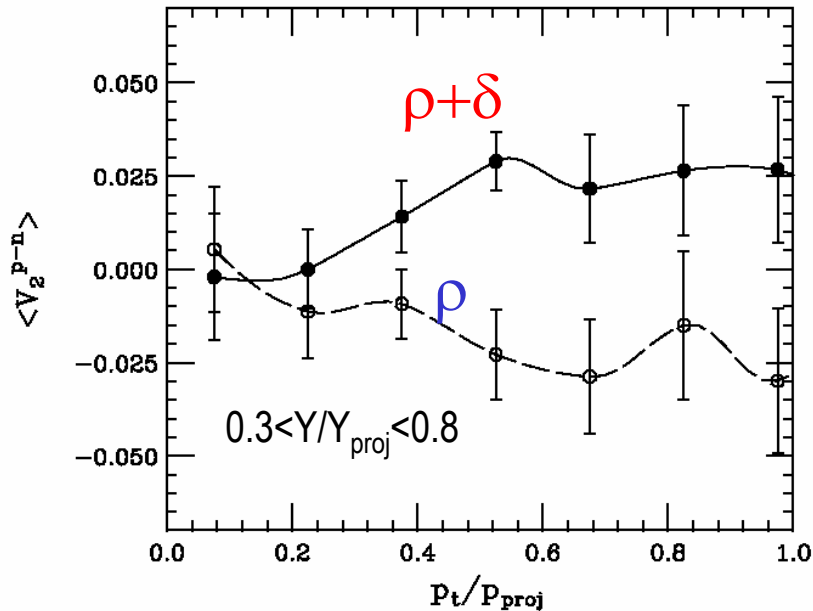
Greater E_{sym}



stiffer $\langle F_{p-n} \rangle$

Elliptic flow

132Sn+132Sn, 1.5A GeV, b=6fm: Test with NL- ρ & NL- $(\rho+\delta)$



✱ Difference at high $p_t \iff$ first stage

← High p_t neutrons are emitted “earlier”

Equilibrium (ρ, δ) dynamically broken
Importance relativistic structure

approximations

$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \vec{\nabla} \rho_3 = \frac{4}{\rho_B} E_{sym}^* \vec{\nabla} \rho_3$$

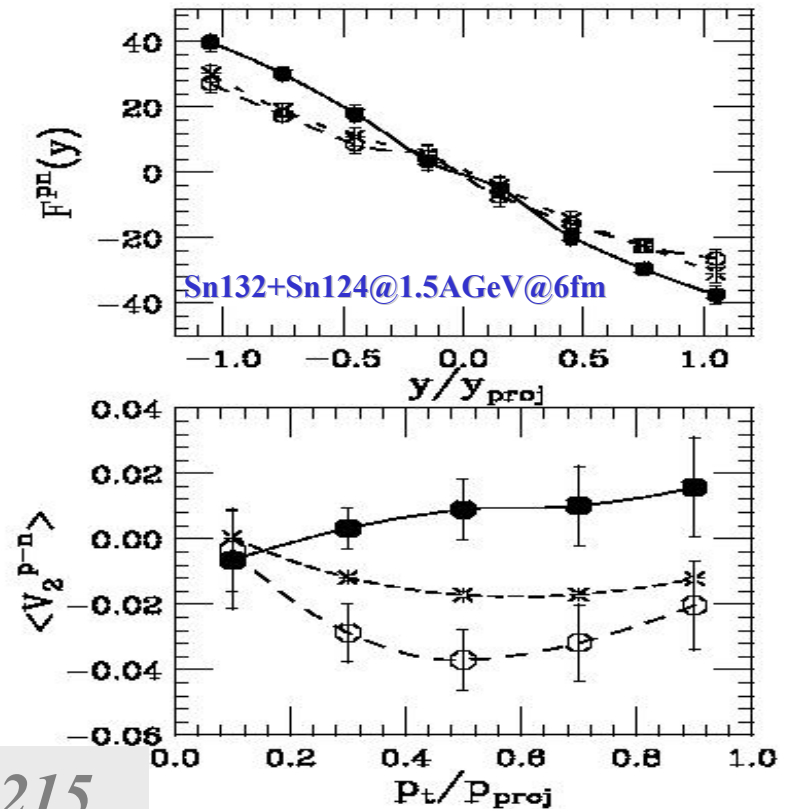
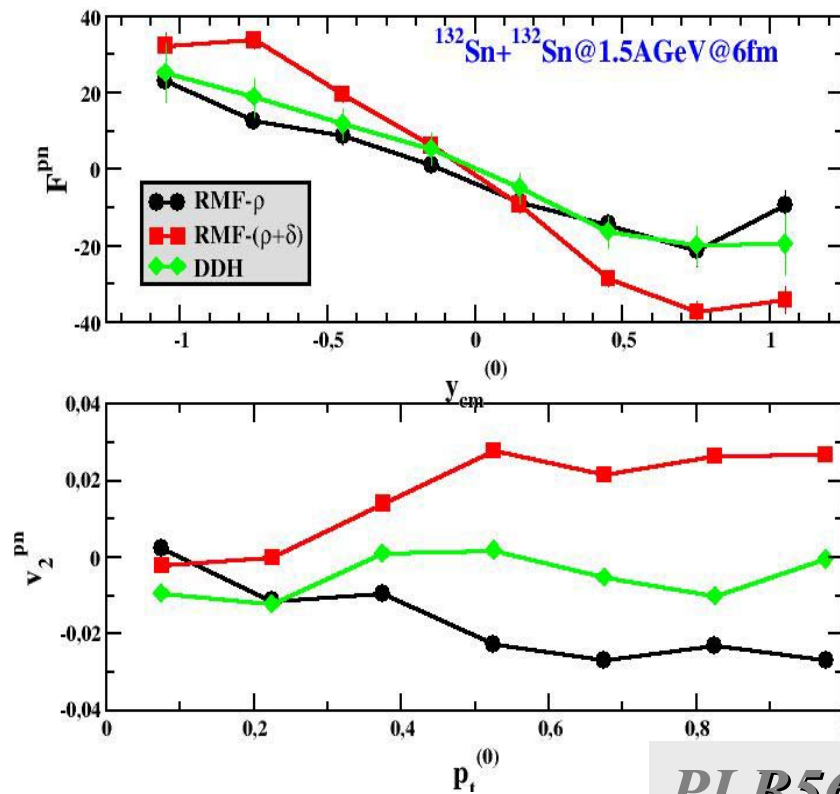


$$2 \left[f_\rho - f_\delta \frac{M^*}{E_F^*} \right] = \frac{4}{\rho_B} E_{sym}^{pot}$$

Dynamical boosting of the
vector contribution

PLB562(2003)

Not just a symmetry energy effect: DDH $\rightarrow$ density dep. f_ρ to reproduce the same symmetry term of $NL\rho\delta$



PLB562(2003)215

Strong isospin dependence of isospin flow

$\rightarrow$ P_t -dependence: Chronometer of collision (high p_t 's reflect earlier high compression)

$\rightarrow$ $NL\rho\delta$: more I-Flow due to Lorentz decomposition of iso-vector channel: $\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \approx 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \nabla \rho$

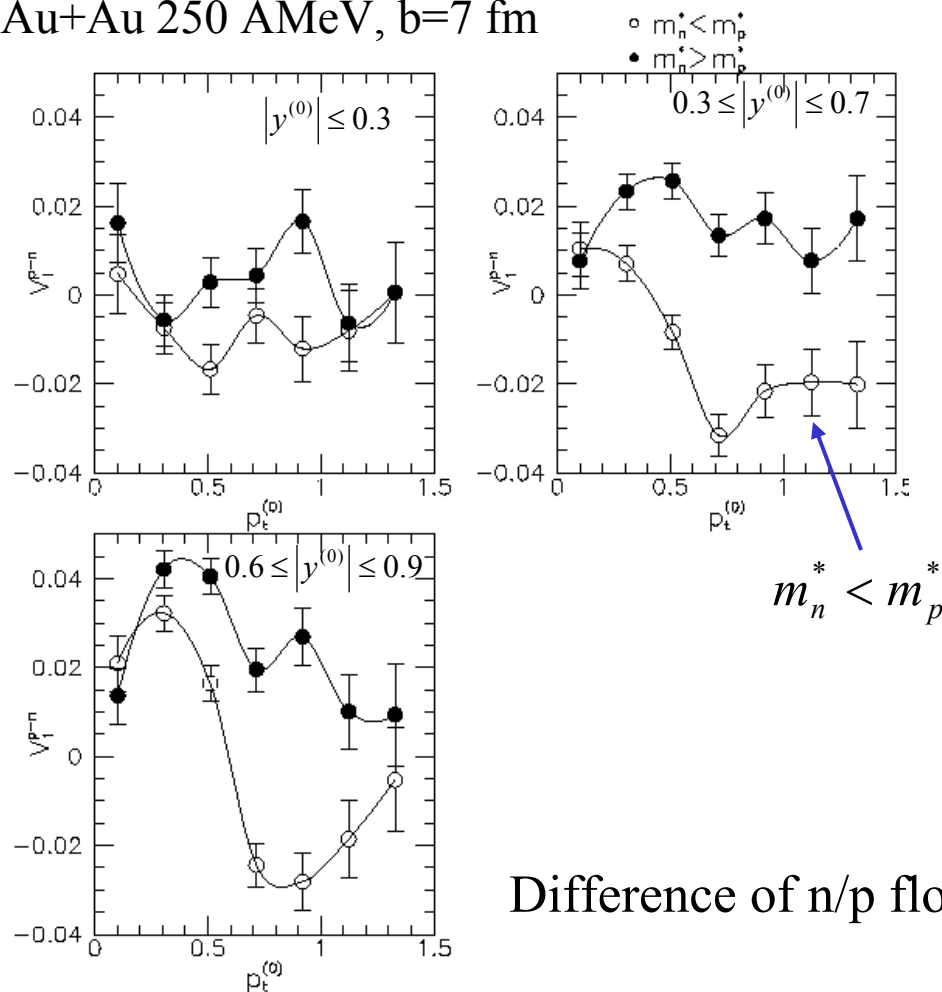
ρ -meson enhanced by γ δ -meson suppressed by scalar density

One needs neutron (light isobars) detection from experiments!

Effective Mass Sensitivity: Differential Transverse Flows

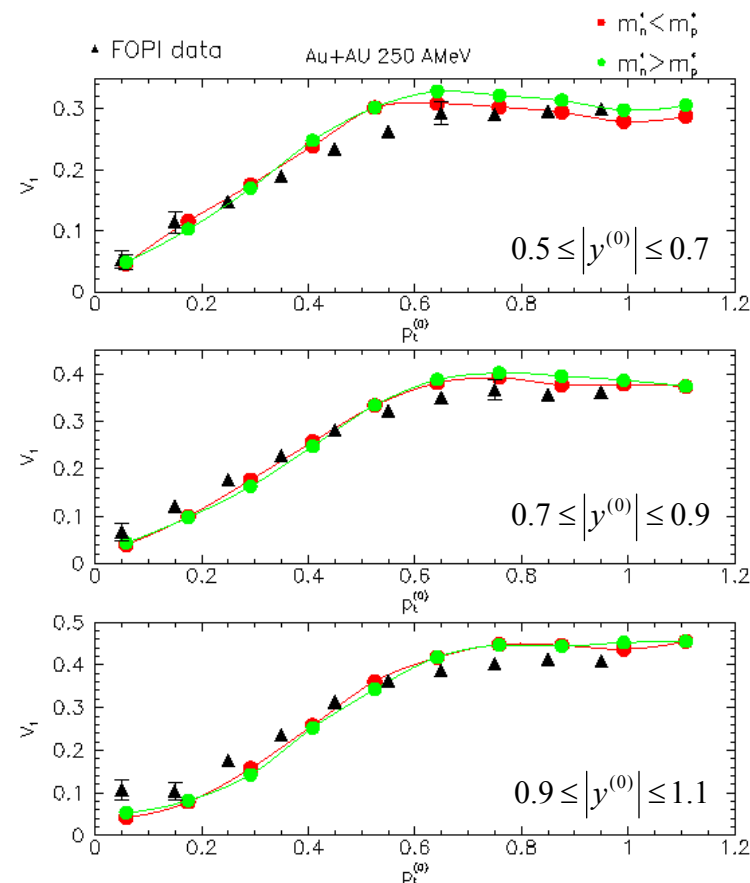
Non-relativistic with both mass-splitting

Au+Au 250 AMeV, $b=7$ fm



Difference of n/p flows

Larger effects at high momenta



Z=1 data

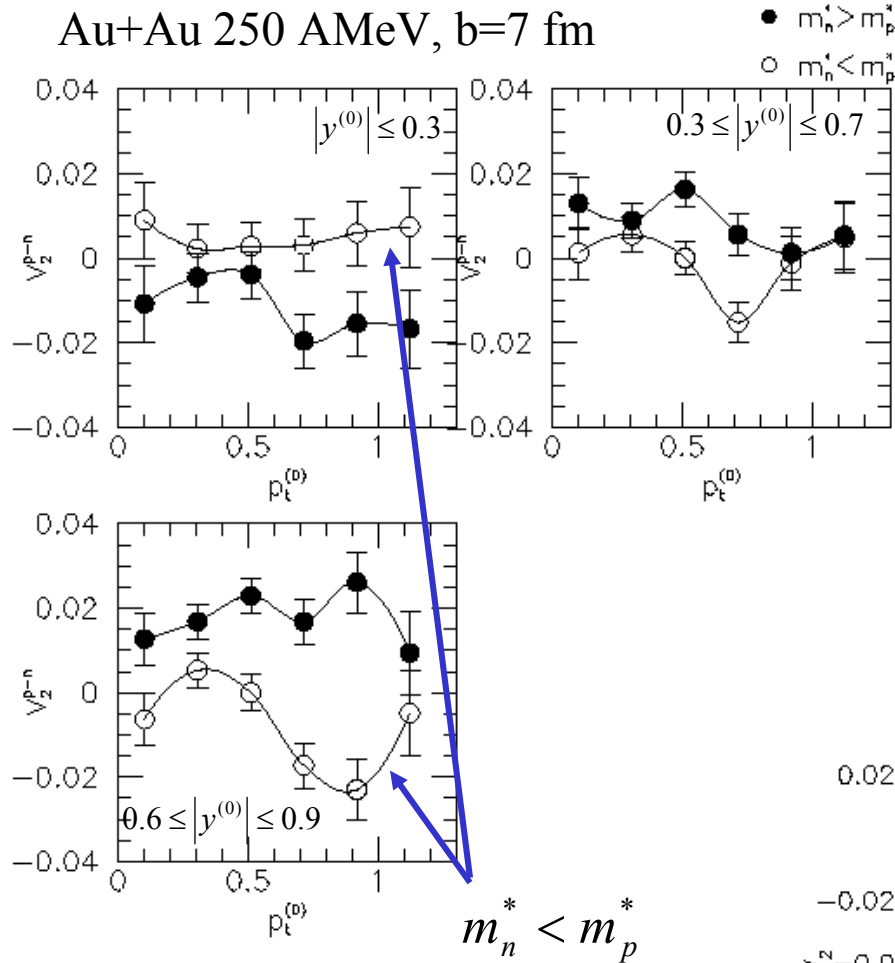
M3 centrality

$6 < b < 7.5$ fm

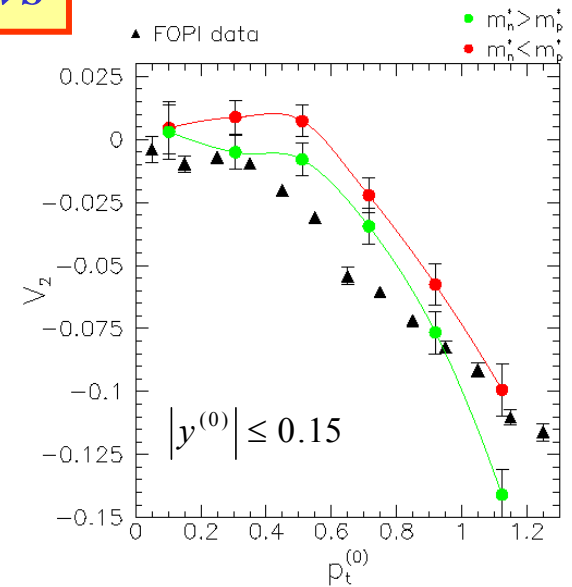
Triton vs. ^3He Flows?

MSU/RIA05, nucl-th/0505013 , AIP Conf.Proc.791 (2005) 70

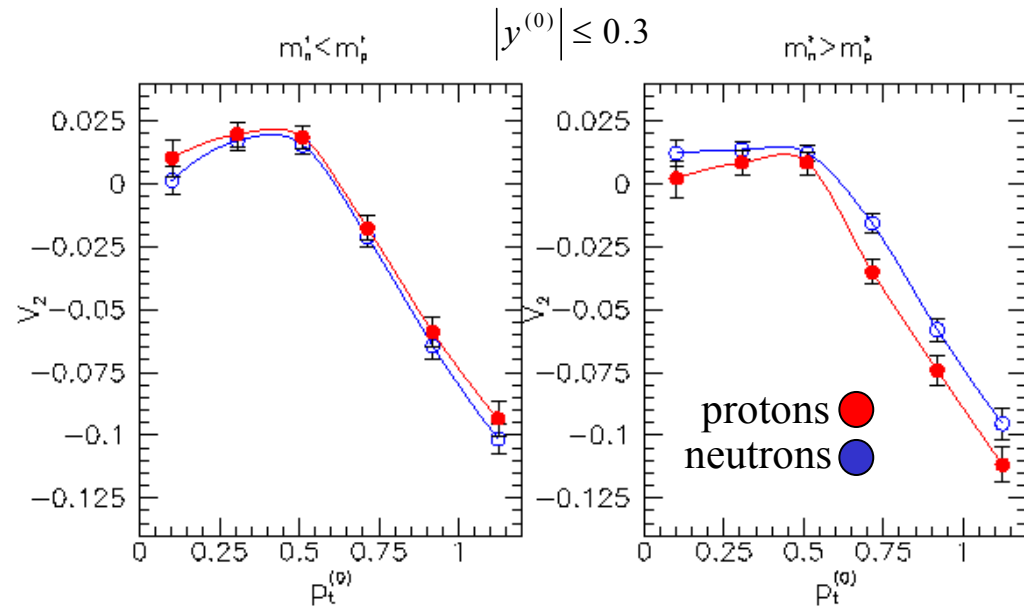
Differential Elliptic Flows



$m^*n < m^*p$: larger neutron squeeze out at mid-rapidity



Z=1 data, M3 centrality, $6 < b < 7.5$ fm



Momentum Dependence and Effective Masses: Fast Nucleon Emission

Non-relativistic with both mass-splitting

Skyrme-like parametrization $\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}(A', A'') + \mathcal{E}(B', B'') + \mathcal{E}(C', C'')$

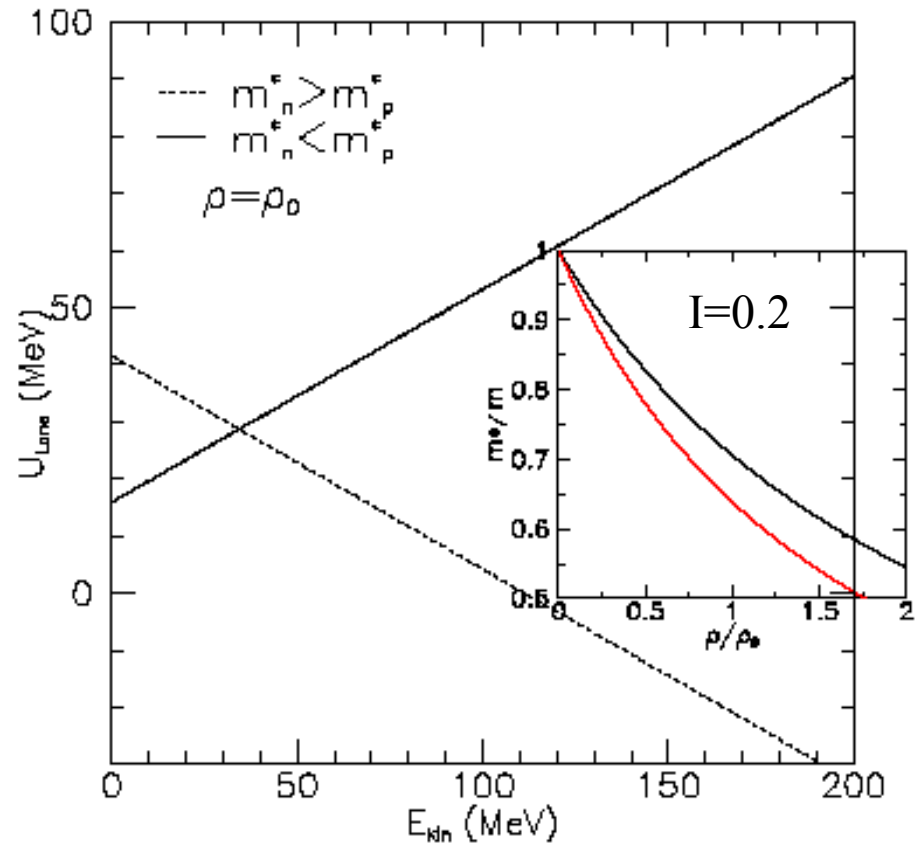
$$\begin{aligned}\mathcal{E}(A', A'') &= (A' + A'' \beta^2) \frac{\rho^2}{\rho_0} \\ \mathcal{E}(B', B'') &= (B' + B'' \beta^2) \left(\frac{\rho}{\rho_0} \right)^\sigma \rho \\ \mathcal{E}(C', C'') &= C'(I_{nn} + I_{pp}) + C'' I_{np}\end{aligned}$$

$$I_{\tau\tau'} = \int d\vec{p} d\vec{p}' f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}') g(\vec{p}, \vec{p}')$$

$$g(\vec{p}, \vec{p}') = 1 - \frac{(\vec{p} - \vec{p}')^2}{\Lambda^2}$$

GBD extension

Correspondence to Skyrme parameters



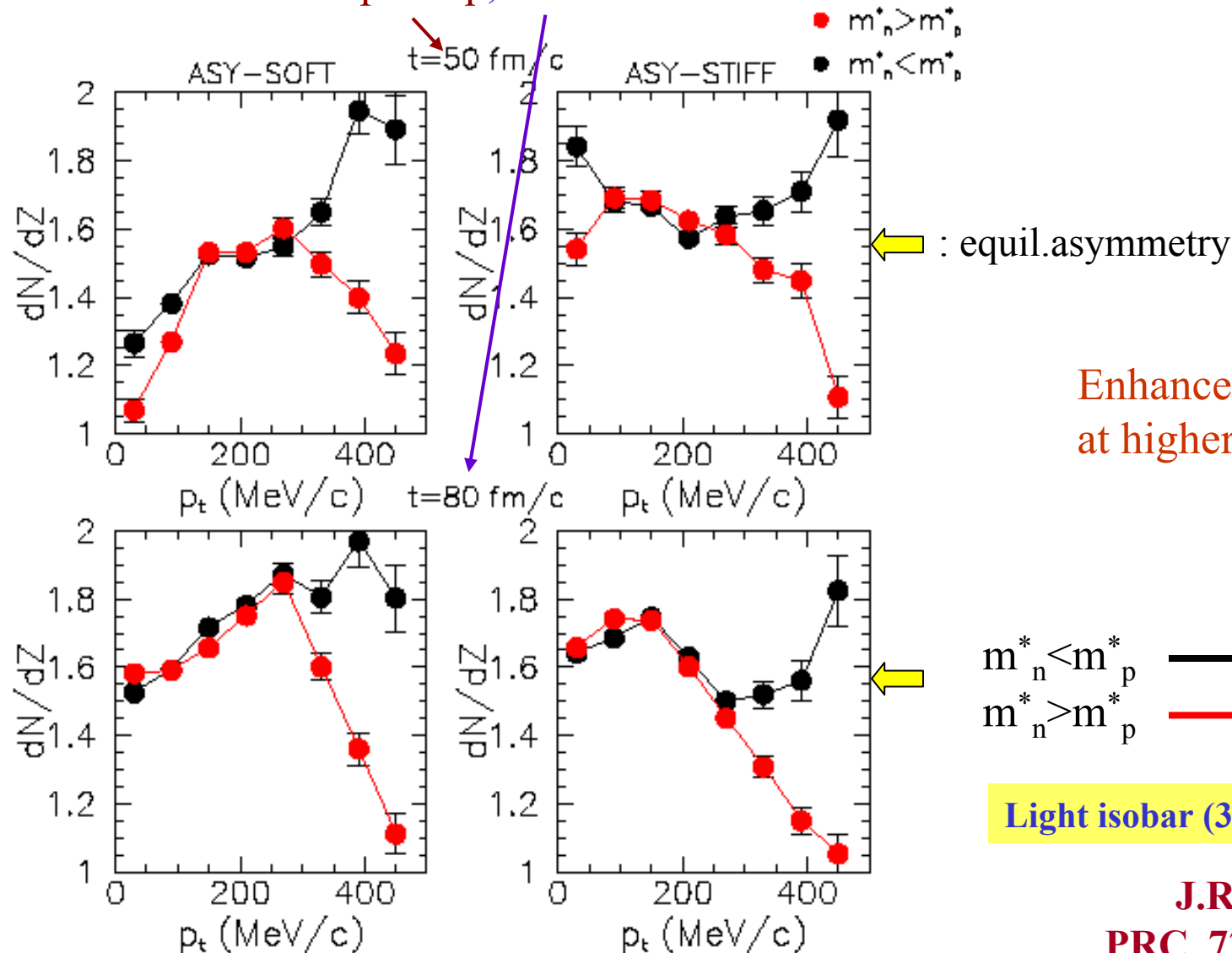
Lane Potential

NPA732(2004)202, nucl-th/0505013, PRC 72 (2005) 064609

Pre-equilibrium emission

Gas asymmetry vs. p_t ($\rho < \rho_0/8$)
different times: end of pre-eq., freeze out

$^{132}\text{Sn} + ^{124}\text{Sn}$, 100 AMeV, $b=2$ fm, $y^{(0)} \leq 0.3$



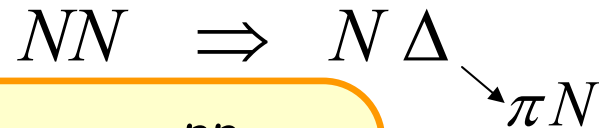
Light isobar ($^3\text{H}/^3\text{He}$) yields?

J.Rizzo et al.,
PRC 72 (2005) 064609

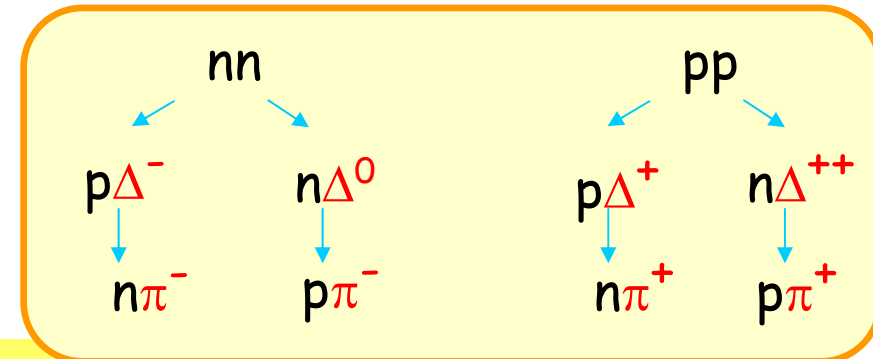
PION PRODUCTION

NPA762(2005) 147

Main mechanism



$$\Rightarrow \frac{\pi^-}{\pi^+}$$



$n \rightarrow p$ "transformation"

1. C.M. energy available: "threshold effect"

$$\varepsilon_{n,p} = E_{n,p}^* + f_{\omega} \rho_B \mp f_{\rho} \rho_{B3} \rightarrow \begin{matrix} s_{nn}(NL) < s_{nn}(NL\rho) < s_{nn}(NL\rho\delta) \\ s_{pp}(NL) > s_{pp}(NL\rho) > s_{pp}(NL\rho\delta) \end{matrix}$$

$\pi(-)$ enhanced
 $\pi(+)$ reduced

2. Fast neutron emission: "mean field effect"

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})} \downarrow \Rightarrow \frac{\pi^-}{\pi^+} \downarrow \Rightarrow \text{decrease: } NL \rightarrow NL\rho \rightarrow NL\rho\delta$$

Compensation
in "open" systems:
HIC

Vector self energy more repulsive for neutrons and more attractive for protons

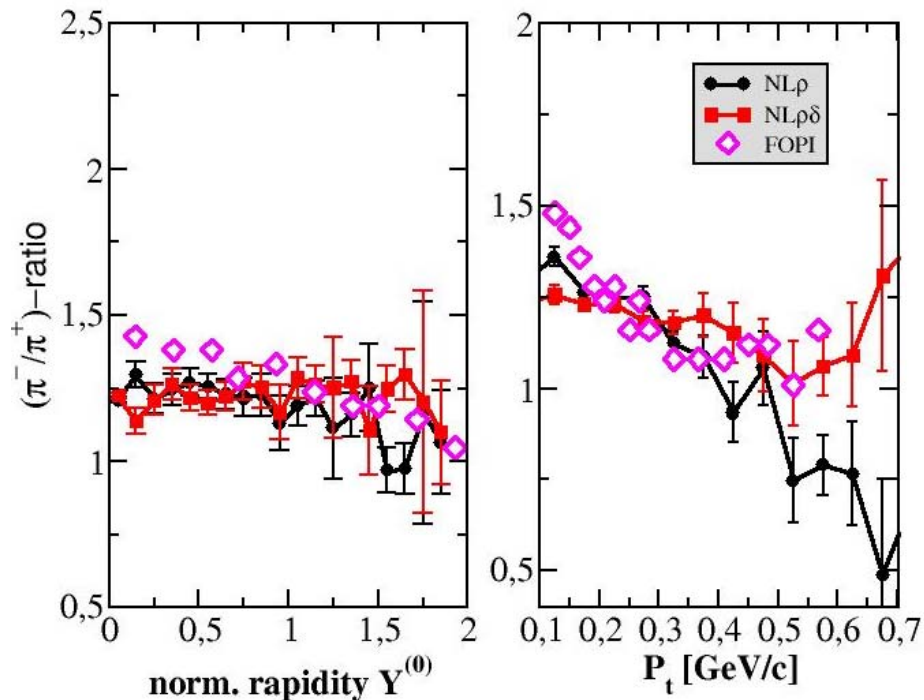
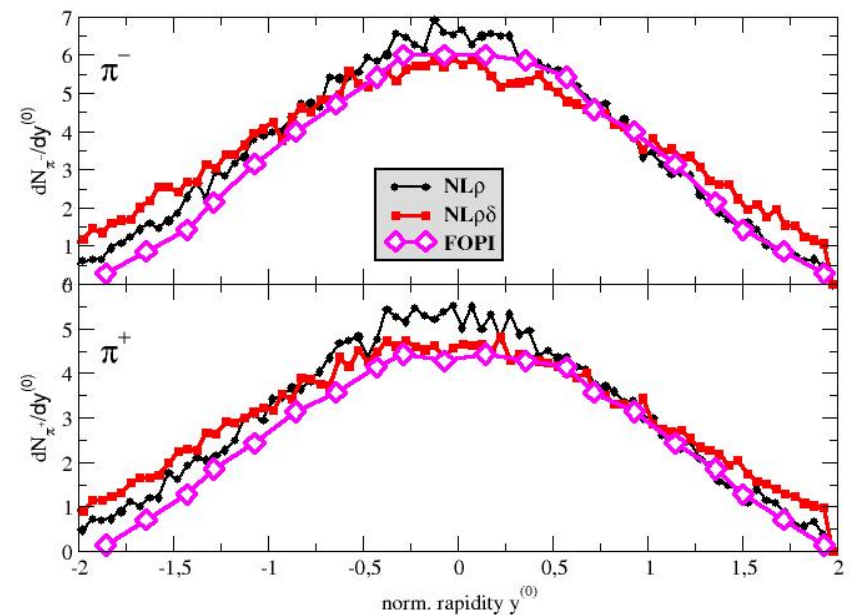
3. Pion absorption

At low energies $\pi(-)$ more absorbed since more energy is available in their production

Pion production at SIS energies: 96Ru+96Ru at 1.53A GeV

Central selection

N/Z=1.18, still some Iso-EOS sensitivity



NL $\rho\delta$ more efficient in “transforming” neutrons into protons at high density, producing π^-

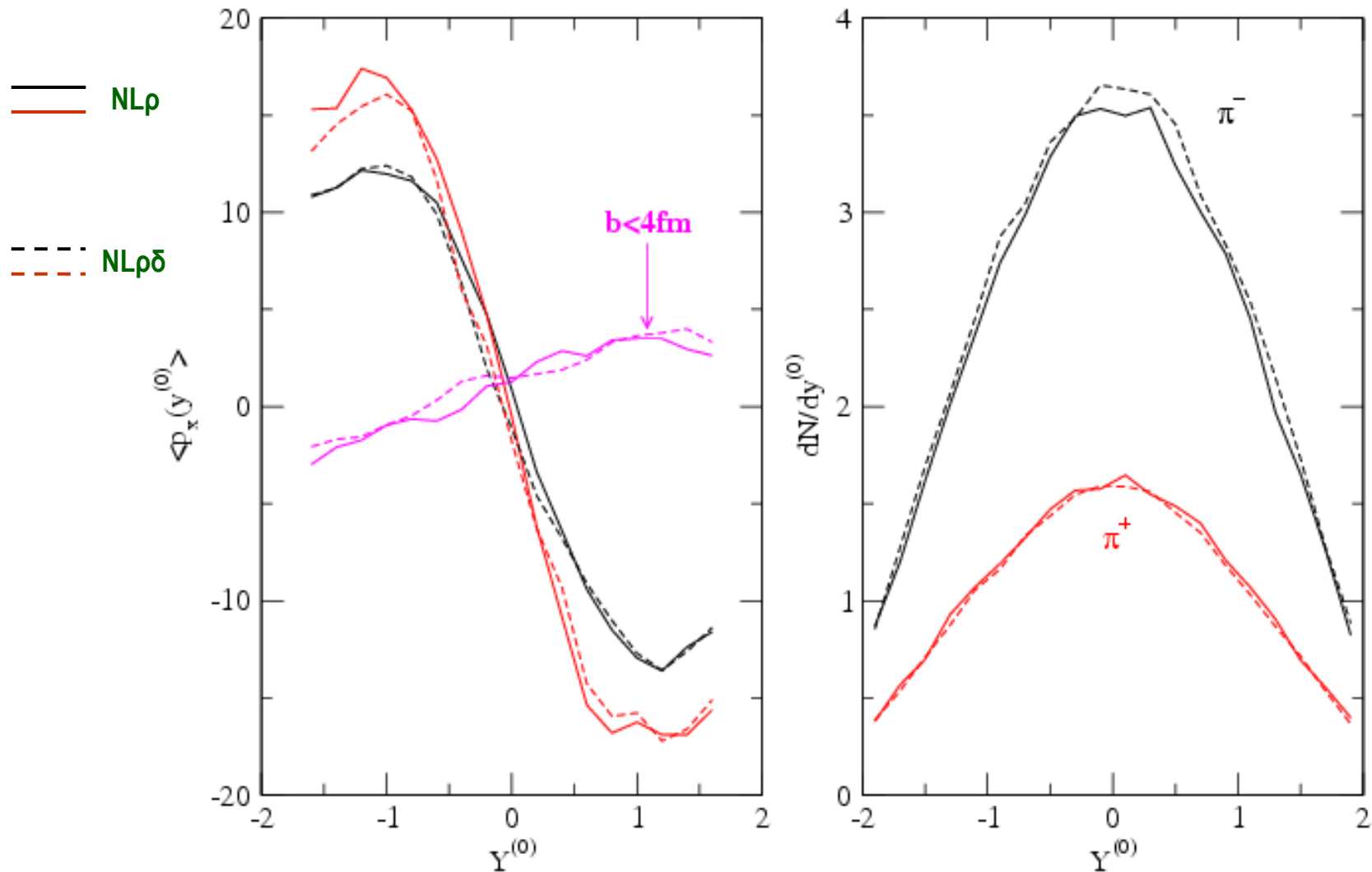
Coulomb effect:
less π^+ present in the high density region

Pion Flows

Semiperipheral: antiflows (shadowing)

More inclusive data→
Not large Iso-EOS effects

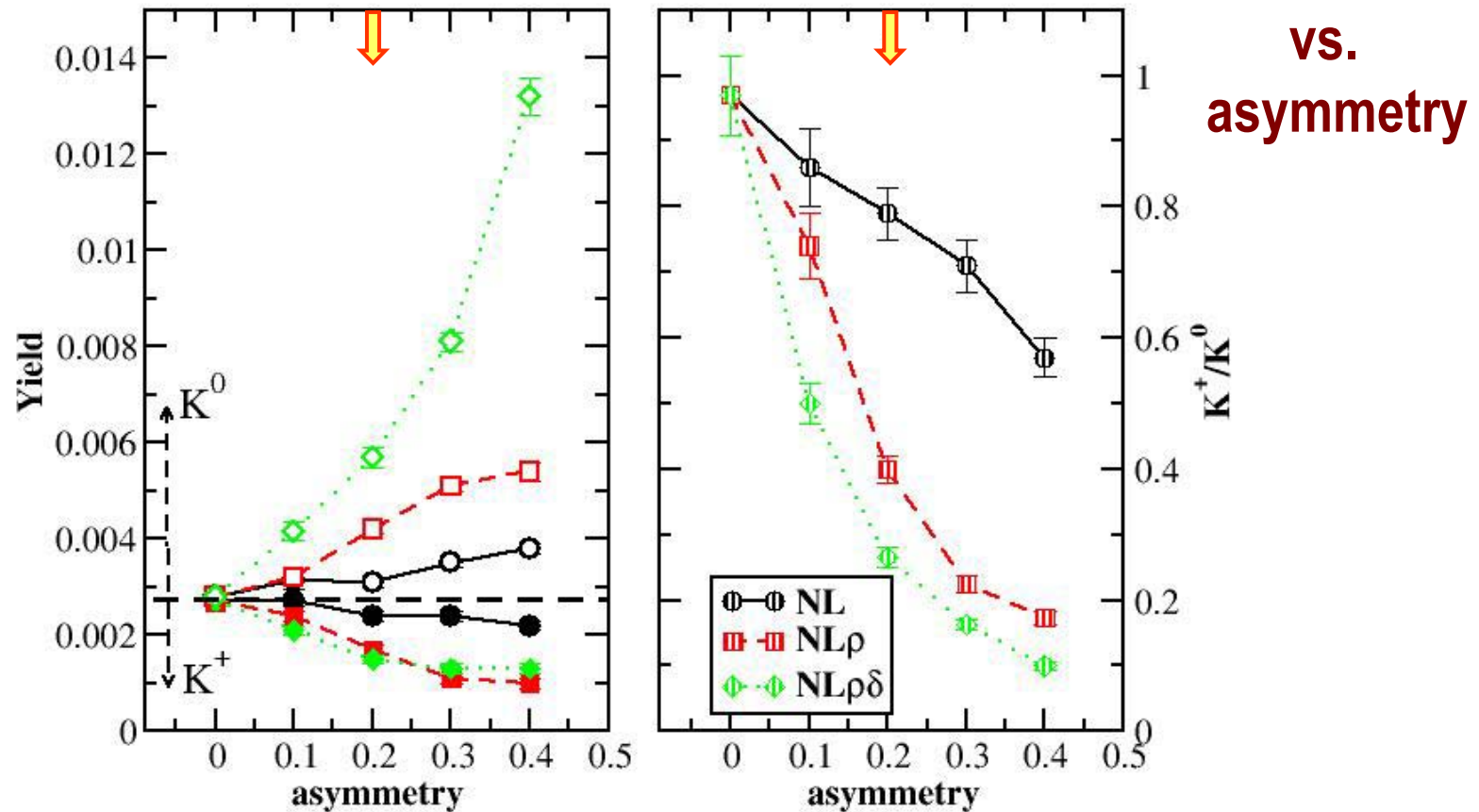
Au+Au@1 AGeV, $b > 4\text{fm}$



Isospin effects on Kaon Production: Nuclear Matter Box Results

NPA762(2005) 147

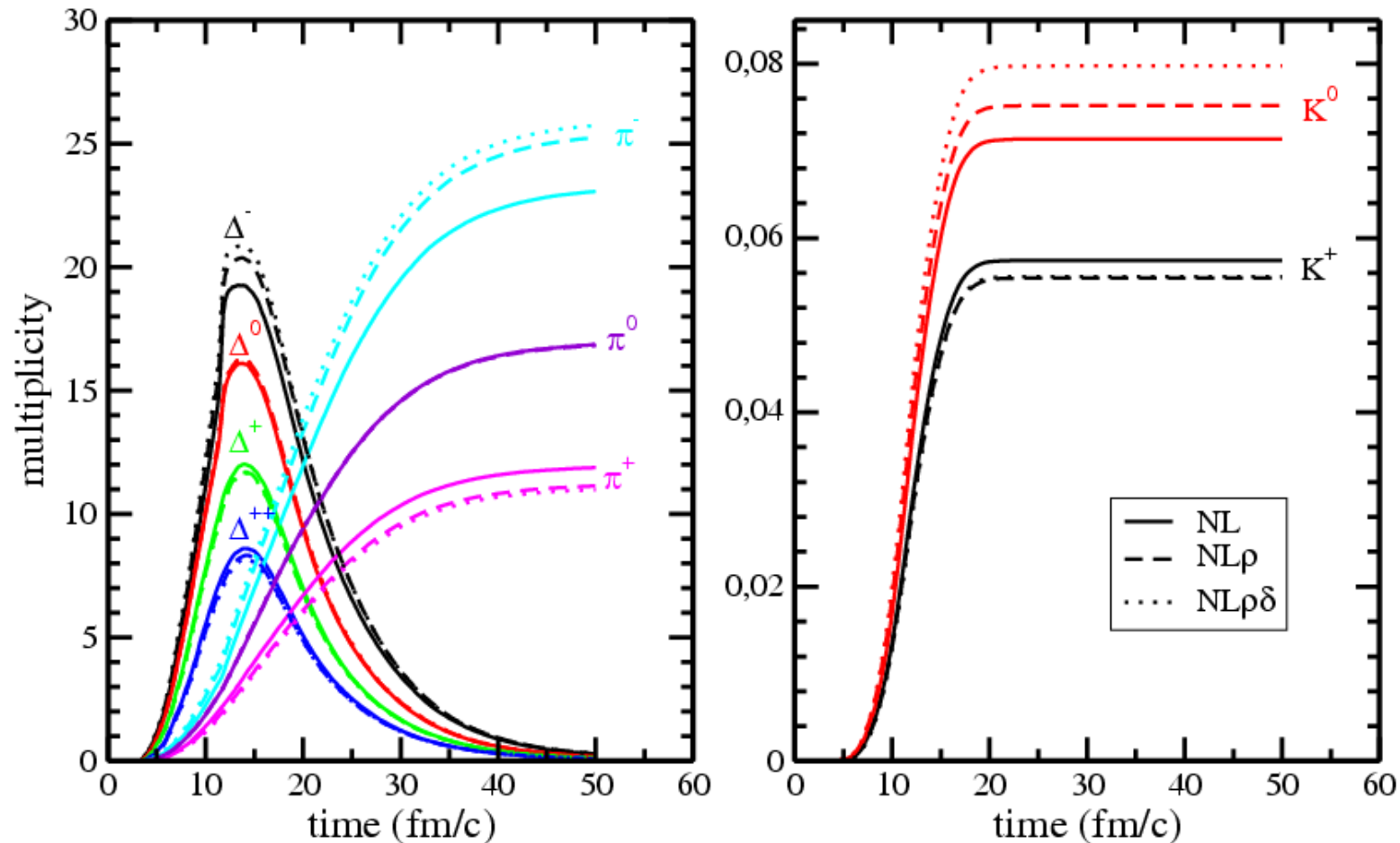
Density and temperature like in Au+Au 1A GeV at max.compression



Large isospin effects: - no neutron escape

- Δ 's in chemical equilibrium $\rightarrow$ less n - p "transformation"

Pion/Kaon production in “open” system: Au+Au 1A GeV, central

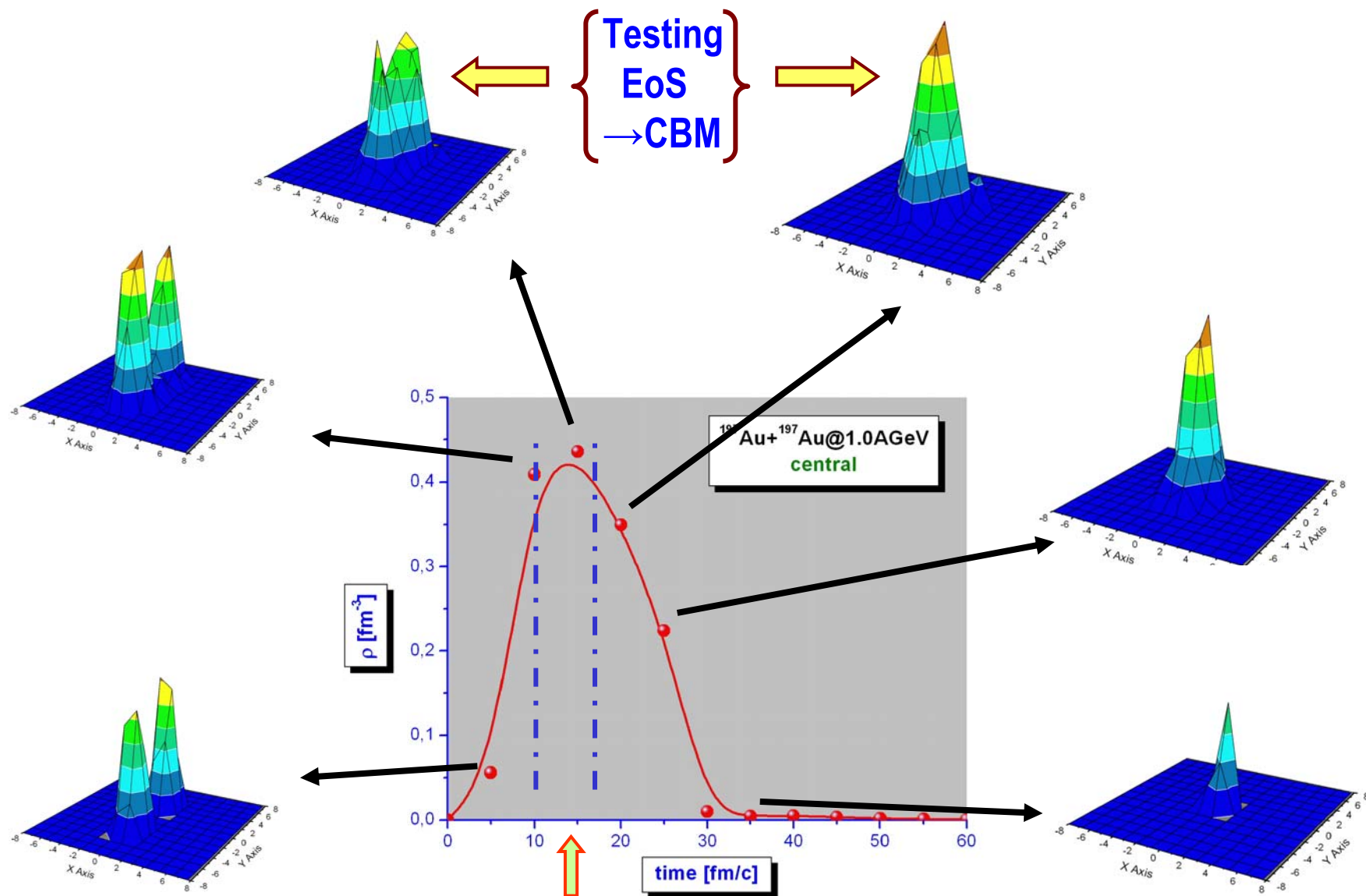


Pions: compensation

Kaons:

- early production: high density phase
- isovector channel effects $\rightarrow$
but mostly coming from second step collisions...
 $\rightarrow$ reduced asymmetry of the source

Au+Au 1AGeV central: Phase Space Evolution in a CM cell



K production

Kaon production in “open” system: Au+Au 1A GeV, central

Main Channels

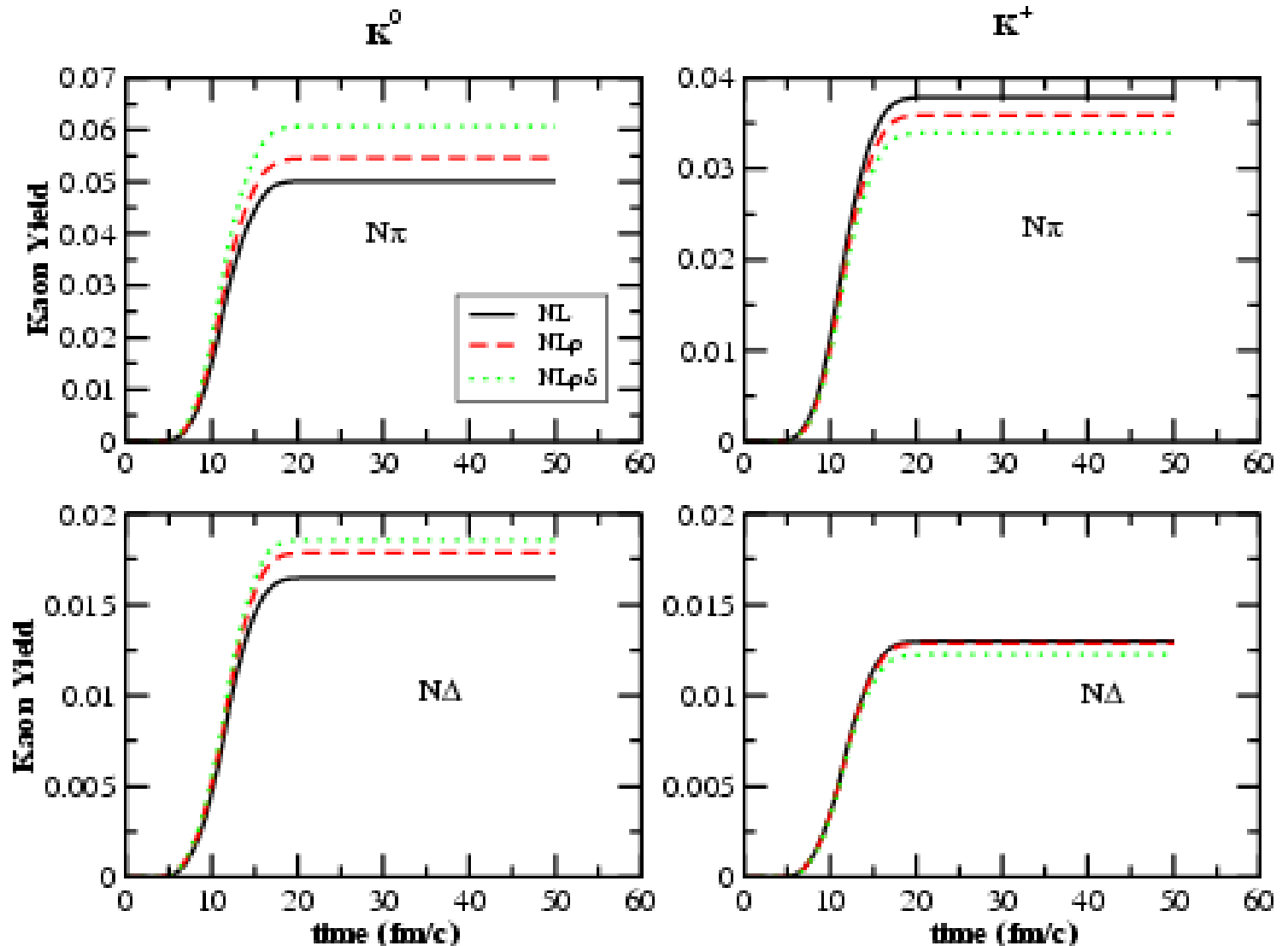
$NN \rightarrow BYK$

$N\Delta \rightarrow BY$

$\Delta\Delta \rightarrow BYK$

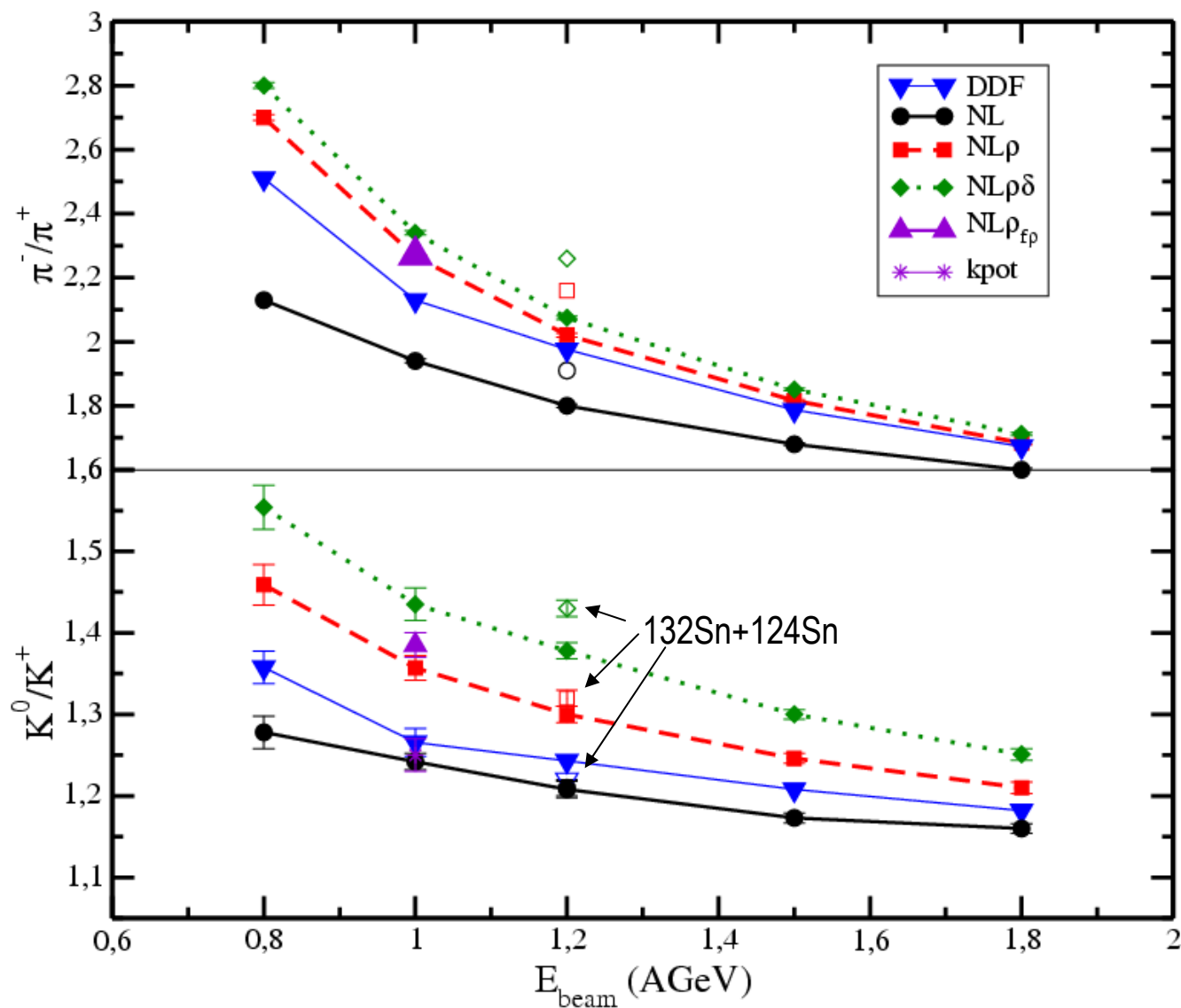
$\pi N \rightarrow YK$

$\pi\Delta \rightarrow YK$



opposite contribution of the δ coupling

Au+Au central: π and K yield ratios vs. beam energy



Kaons:
~15% difference between
DDF and NL $\rho\delta$

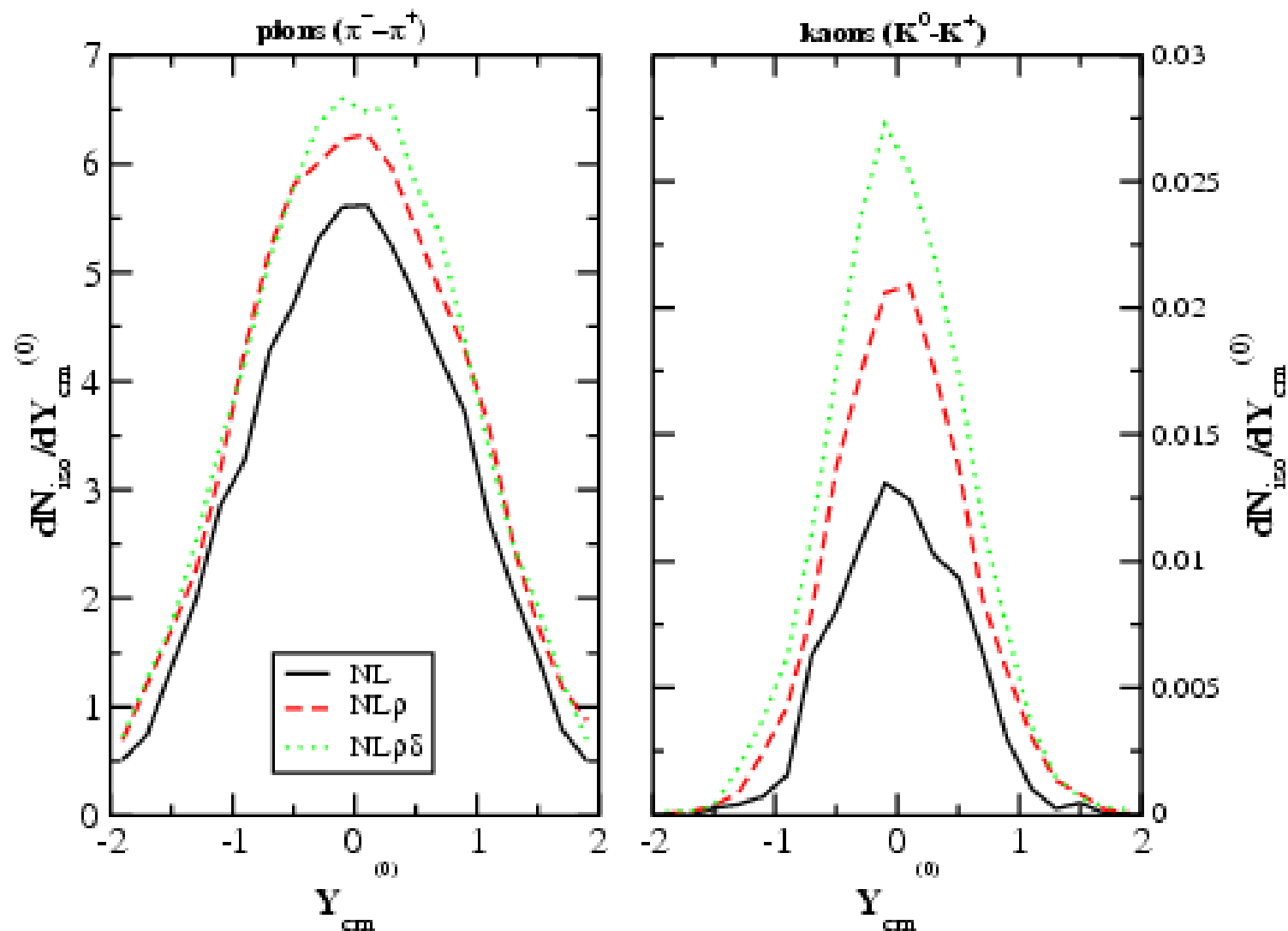
No sensitive to
the K-potential

Pions: less sensitivity ~10%, but larger yields

Inclusive multiplicities

Pion/Kaon production in “open” system: Au+Au 1A GeV, central

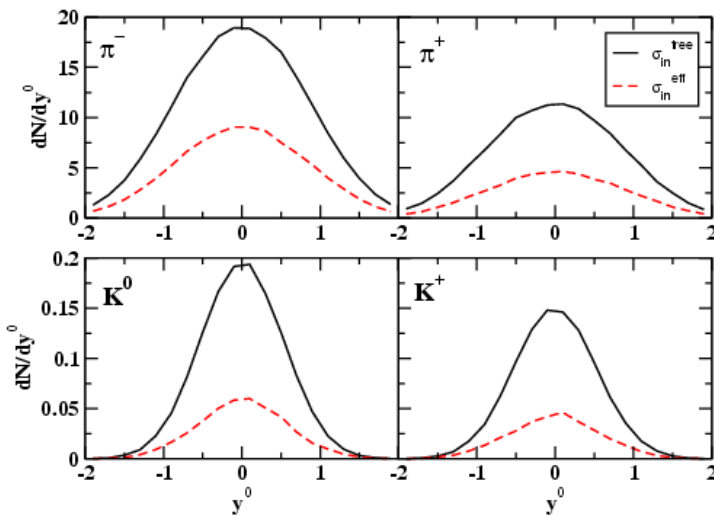
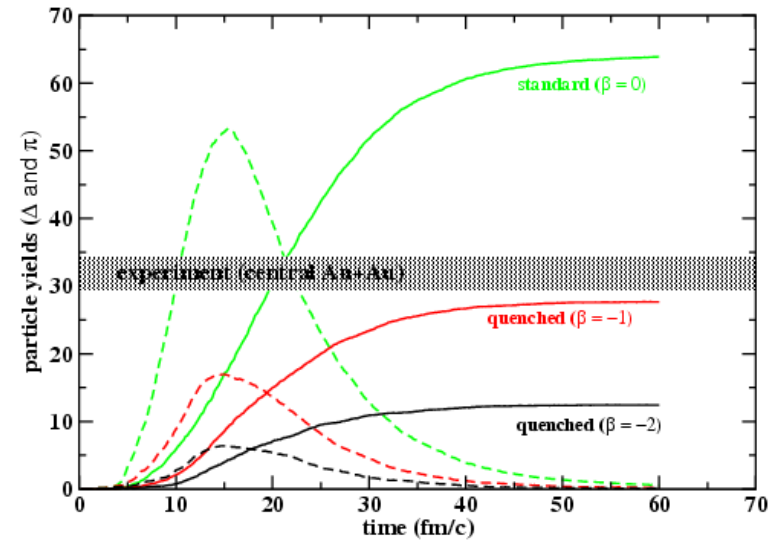
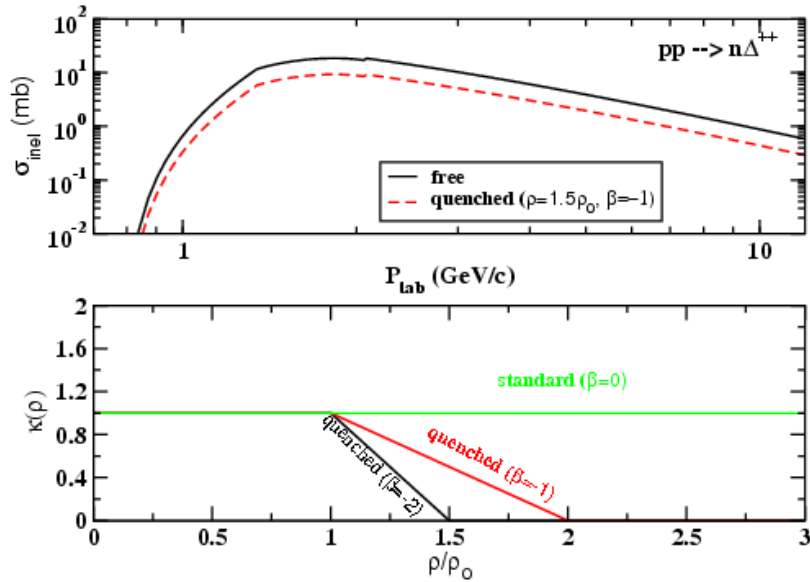
“differential”- rapidity distributions



rather sensitive observable for Kaons

Reduced in-medium inelastic cross sections

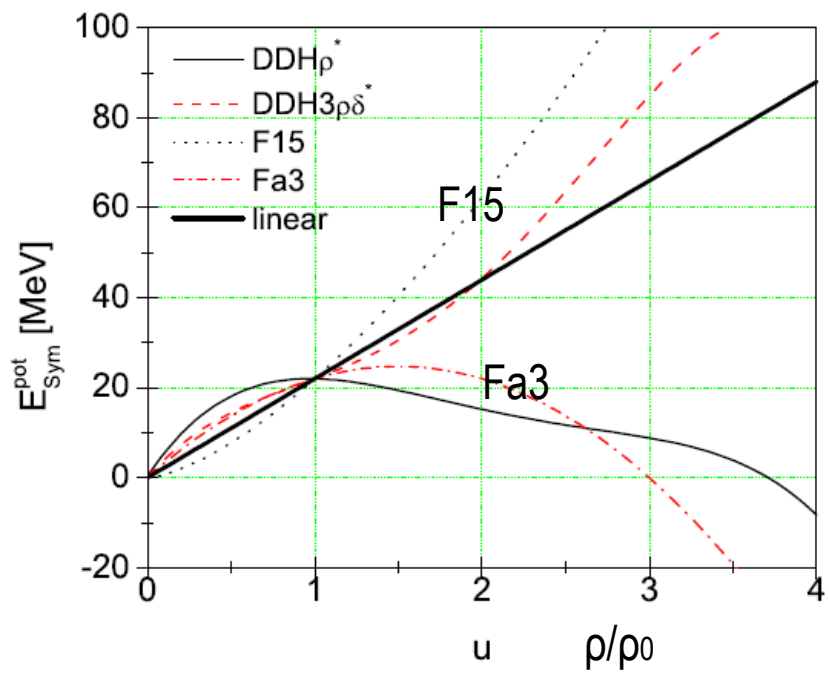
V.Prassa et al. nucl-th/0510035



particle ratio	free σ_{inel}	in-medium $\sigma_{inel}(\beta = -1)$
(π^-/π^+)	$1.65 (\pm 0.22)$	$1.9 (\pm 0.24)$
(K^+/K^0) -ratio	$0.76 (\pm 0.09)$	$0.75 (\pm 0.08)$

K-ratio: robust EOS signal

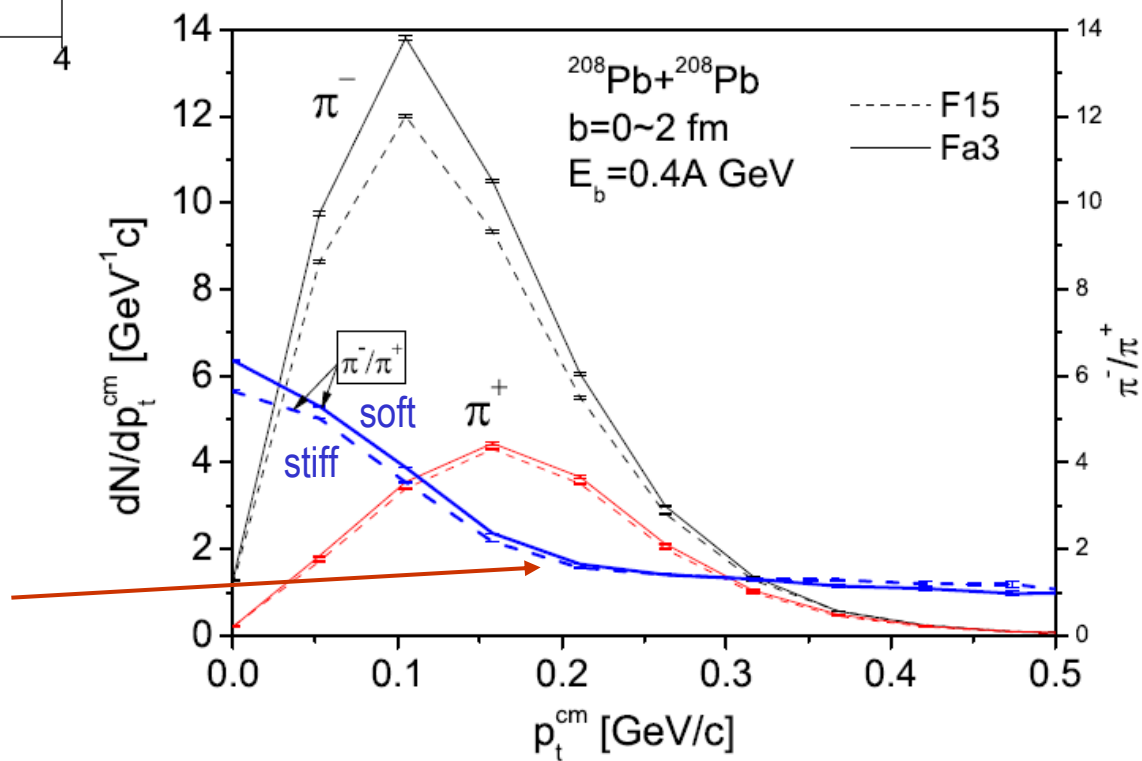
**Pions produced everywhere,
Kaons only at high densities**



Inelastic channels less important
but still crossing at high p_t

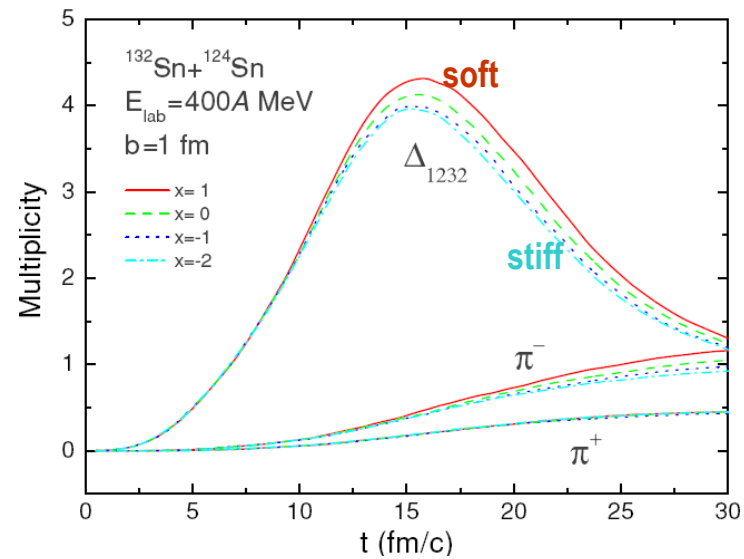
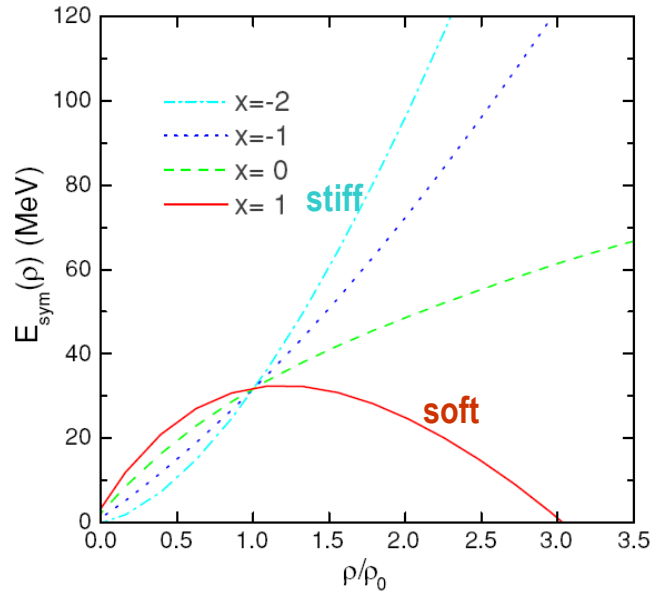
UrQMD : not covariant symmetry term

208Pb+208Pb at 0.4A GeV

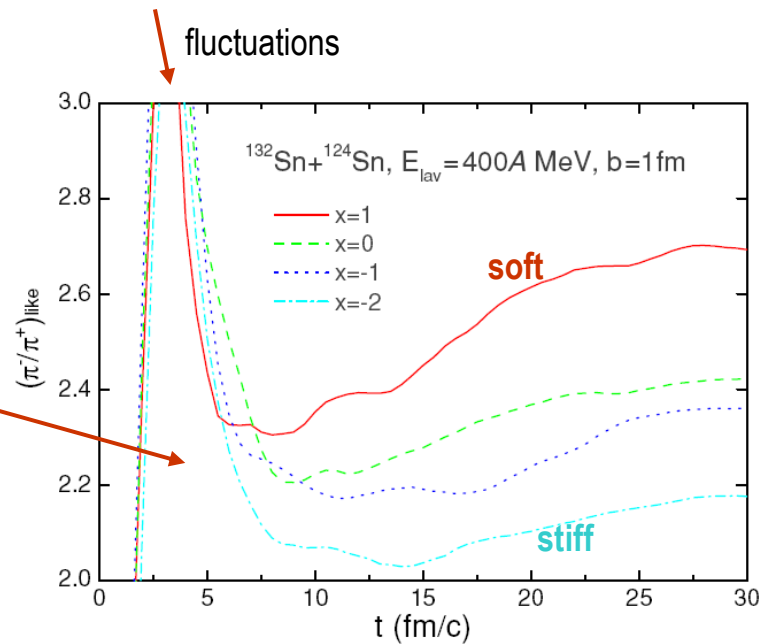


IBUU : not covariant symmetry term

$^{132}\text{Sn}+^{124}\text{Sn}$ at 0.4A GeV



$\pi(-)/\pi(+)$ always decreasing with the iso-stiffness?

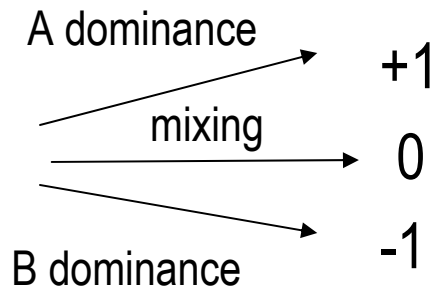


ISOSPIN EFFECTS IN REACTIONS

$$\text{Mass}(A)=\text{Mass}(B) \ ; \ N/Z(A) = N/Z(B)$$

Rami imbalance ratio:

$$R = \frac{2AB - (AA) - (BB)}{(AA) - (BB)}$$



$$\text{isoscailing} \quad \alpha(\beta), \frac{N}{Z}, \frac{t}{^3\text{He}}, \frac{dN}{dY}, \frac{\pi^-}{\pi^+} \dots$$

vs. **Centrality** (fixed y)

vs. **Rapidity** (fixed centrality)

vs. **Transverse momentum** (fixed y , centrality) ?

STOPPING and

ISOSPIN TRACING METHOD (FOPI)

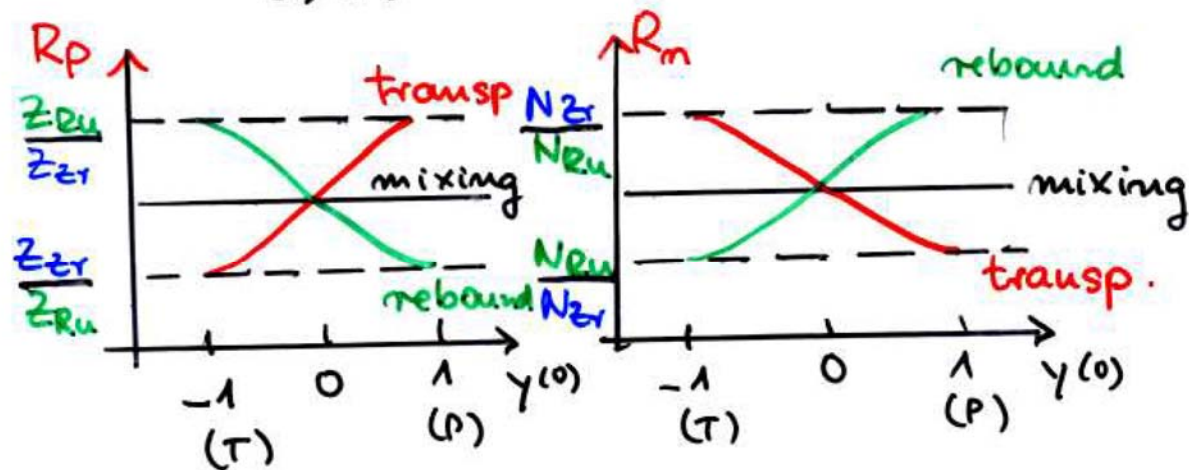
$Ru + Zr$ System

$$\left. \begin{array}{l} {}^{96}_{40}Zr_{56} \quad N/Z = 1.4 \\ {}^{96}_{44}Ru_{52} \quad N/Z = 1.18 \end{array} \right\} \frac{Z_{Zr}}{Z_{Ru}} = 0.91; \frac{N_{Zr}}{N_{Ru}} = 1.08$$

Isospin tracing Ratio

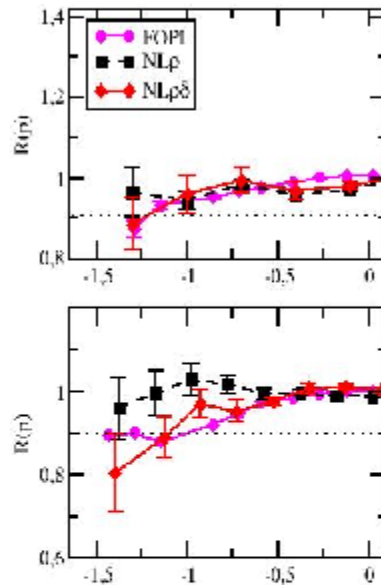
$$R_i = \frac{Y_i^{RuZr}}{Y_i^{ZrRu}}; i = p, n, d, t, {}^3He, \pi^\pm, \dots$$

$\uparrow \quad \uparrow$
 $(P) \quad (T)$

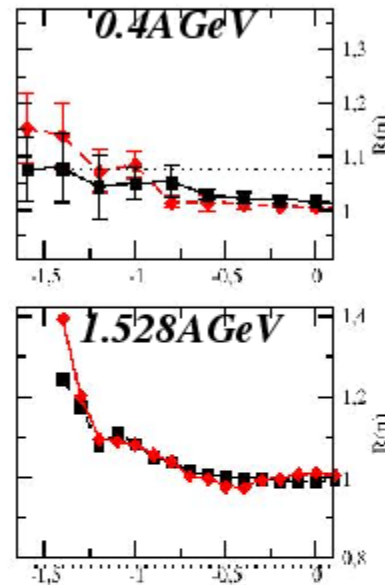


Isospin transparency@SIS

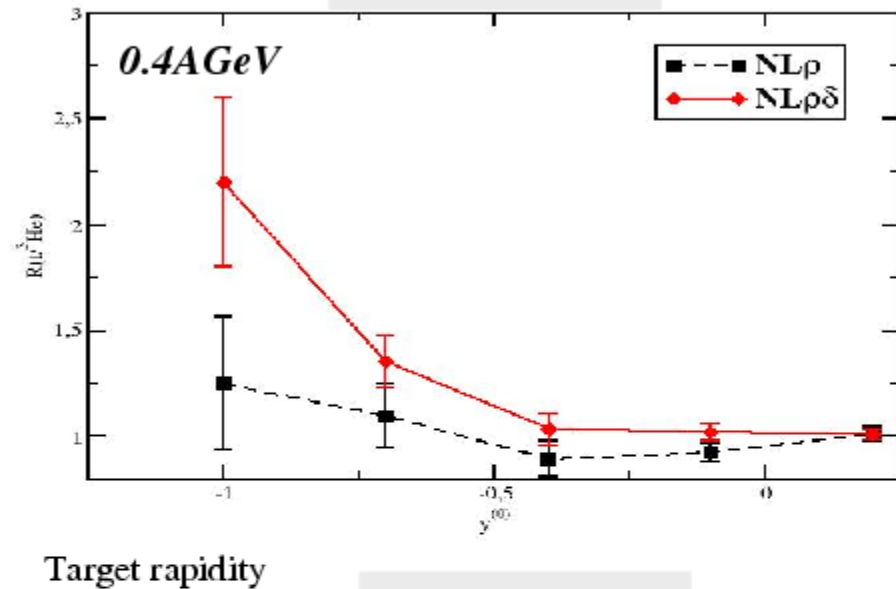
protons



neutrons



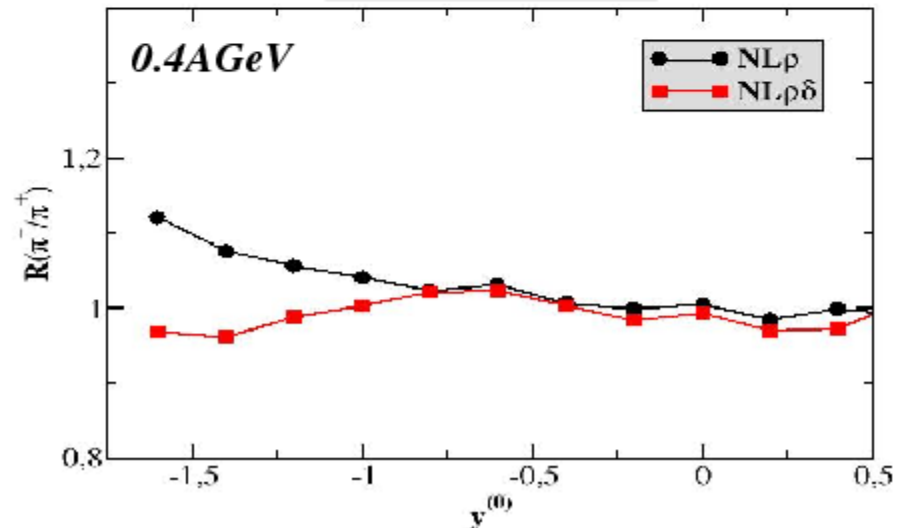
$t / {}^3\text{He}$



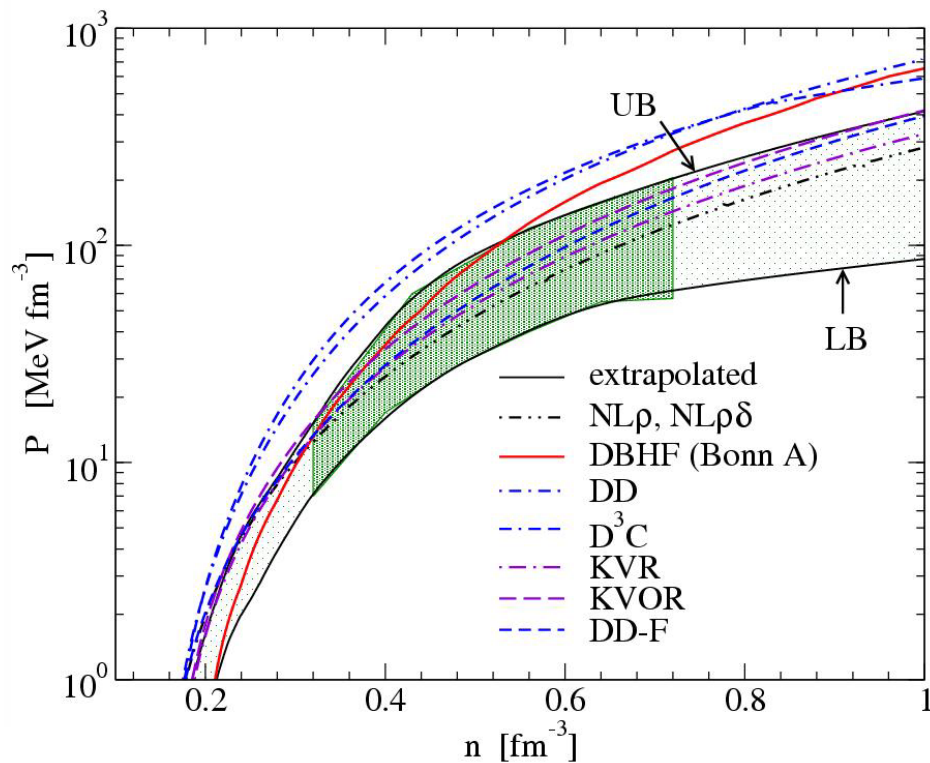
Mixed systems Ru(Zr)+Zr(Ru)
A=96, n-poor(n-rich)+inverse

$$R(n, p, \pi, \text{fragments}, \dots) = \frac{N_y^{\text{Ru+Zr}}}{N_y^{\text{Zr+Ru}}}$$

π^-/π^+



Gaitanos et al. PLB595 (2004)



Testing deconfinement with RIB's?

(T, ρ_B, ρ_{B3}) binodal surface

Hadron-RMF

Quark-bag model

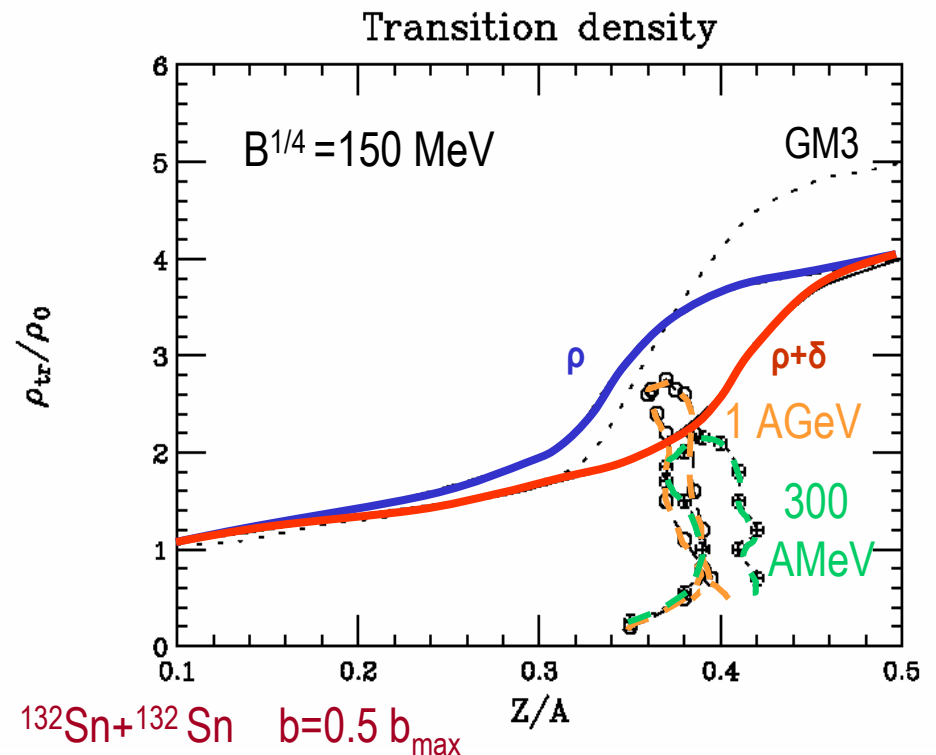
ρ_{trans} → onset of the mixed phase

Signatures?

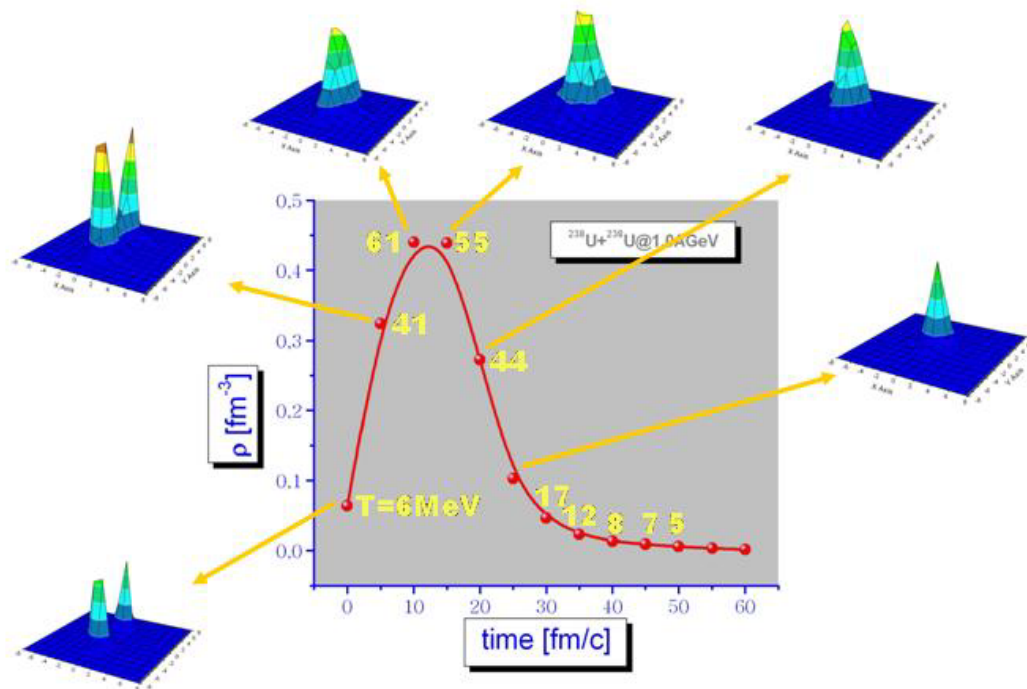
NPA722(2003), nucl-th/0602052 → NPA2006

EOS softening at high baryon density?

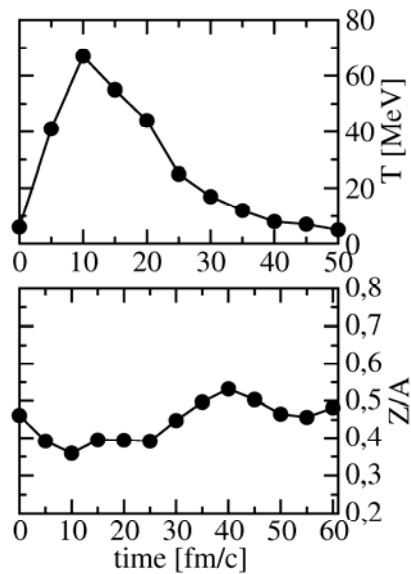
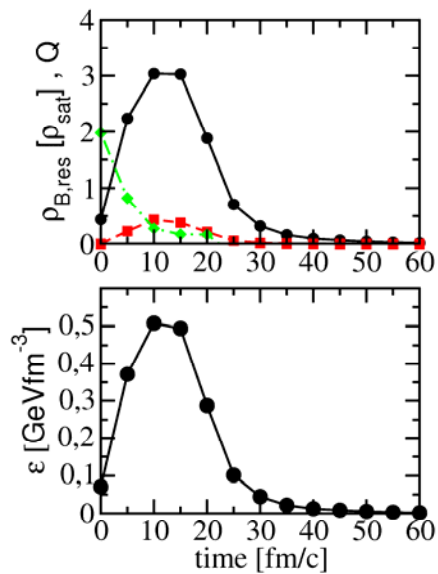
→ asymmetric matter?



System Size Dependence & Equilibration (U+U)



In a C.M. cell



$$^{238}\text{U} + ^{238}\text{U}, 1\text{AGeV}, \quad b = 7\text{ fm}$$

Exotic matter over 10 fm/c ?

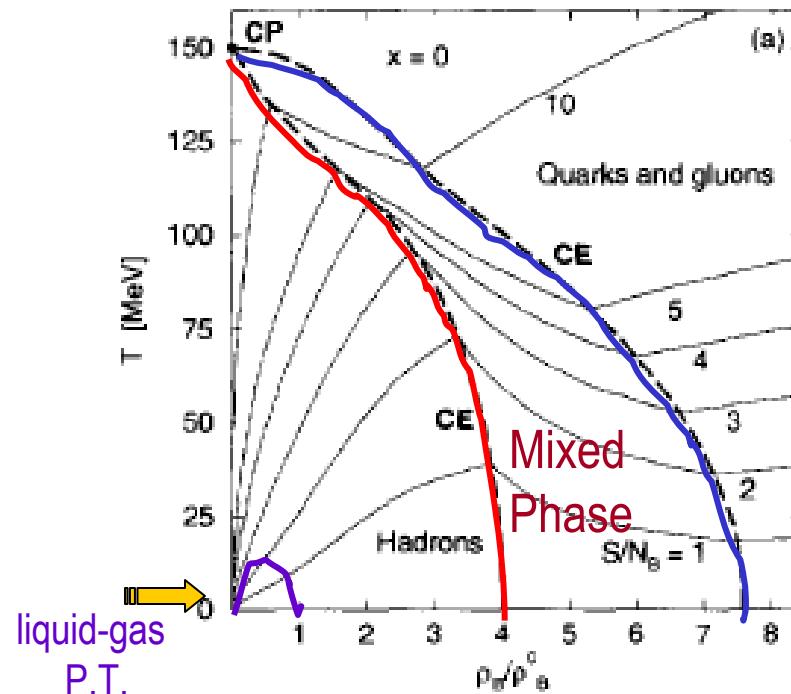
Transition to deconfined phase at high baryon density

H.Mueller NPA618(1997)

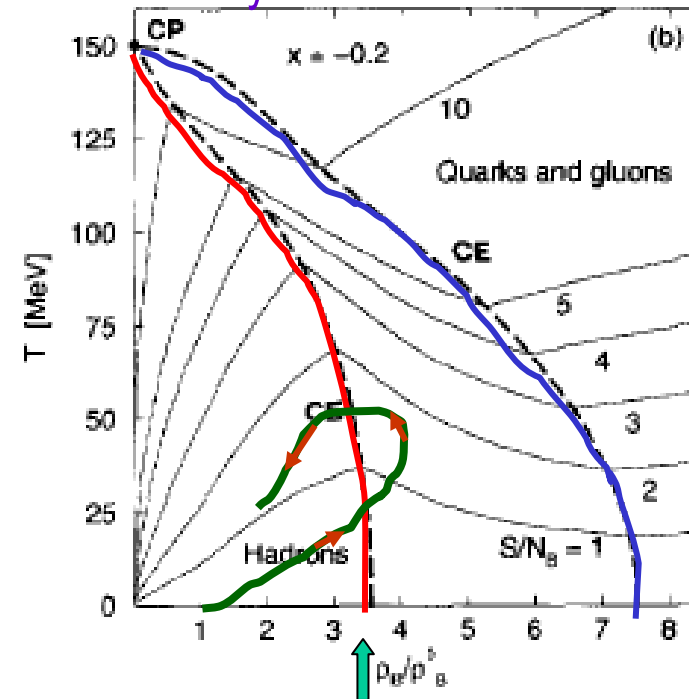
Hadron EOS : QHD

Quark EOS: MIT-Bag Model

Symmetric



Asymmetric: $I=0.4$



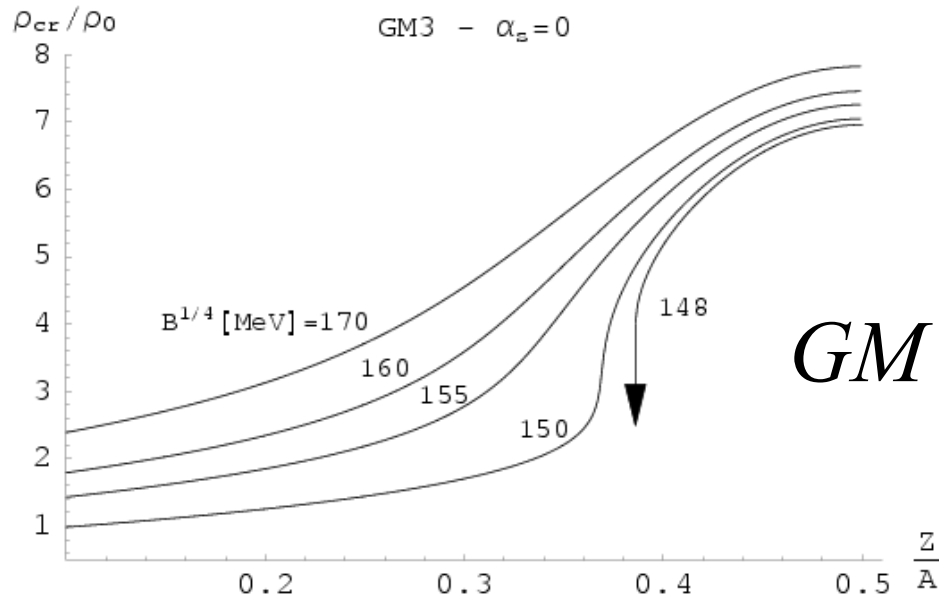
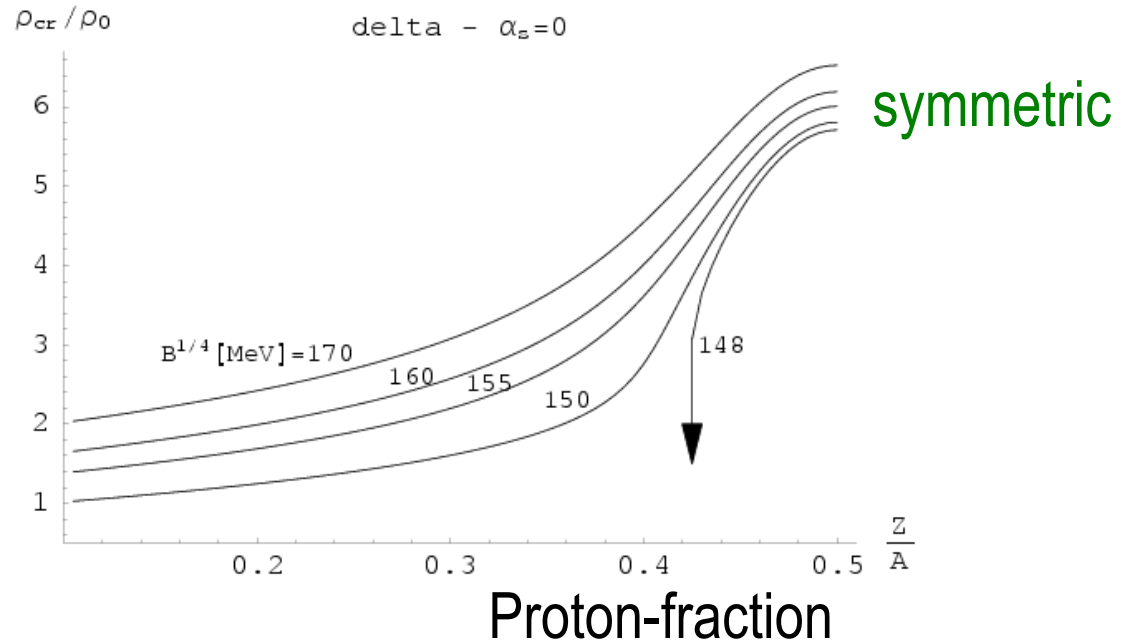
Reduced transition density

1. Earlier transition at high isospin density
2. Worse model choice? Hadron: rho-meson only, Quark: $B/4=190\text{MeV}$ large Bag-Pressure

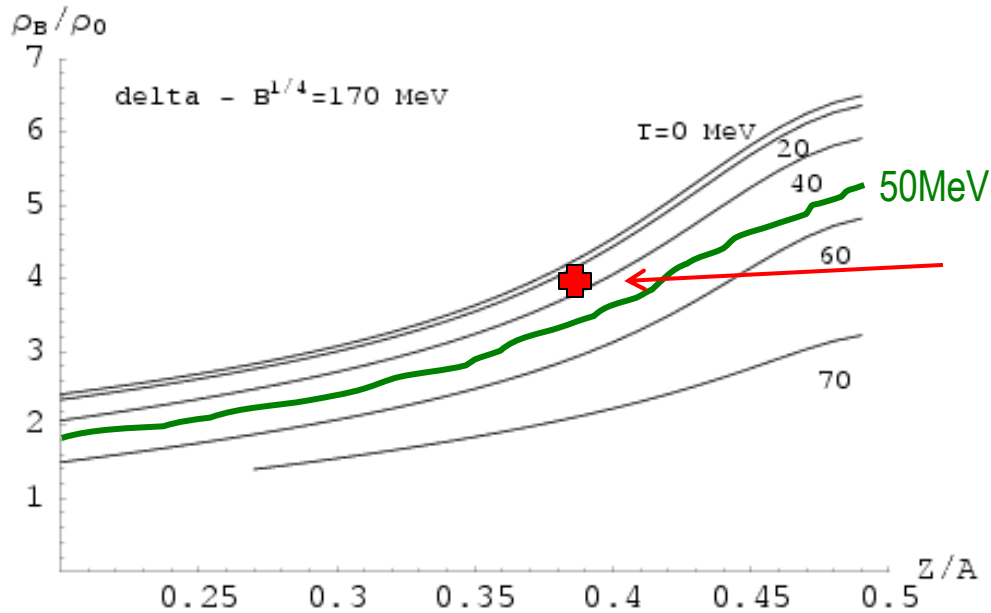
Lower Boundary of the Binodal Surface vs. NM Asymmetry

Hadron : $NL\rho\delta$

vs. Bag-constant choice

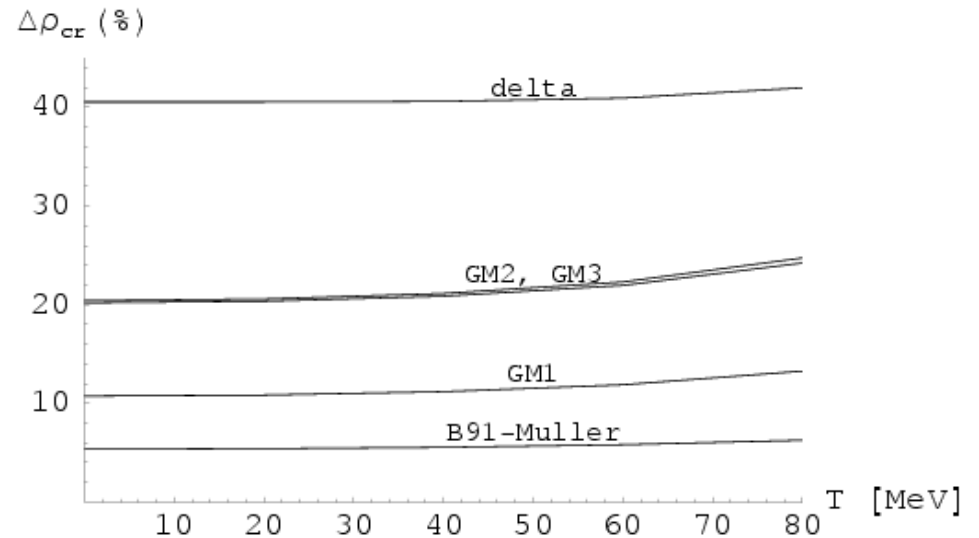


Temperature variation of the crossing density

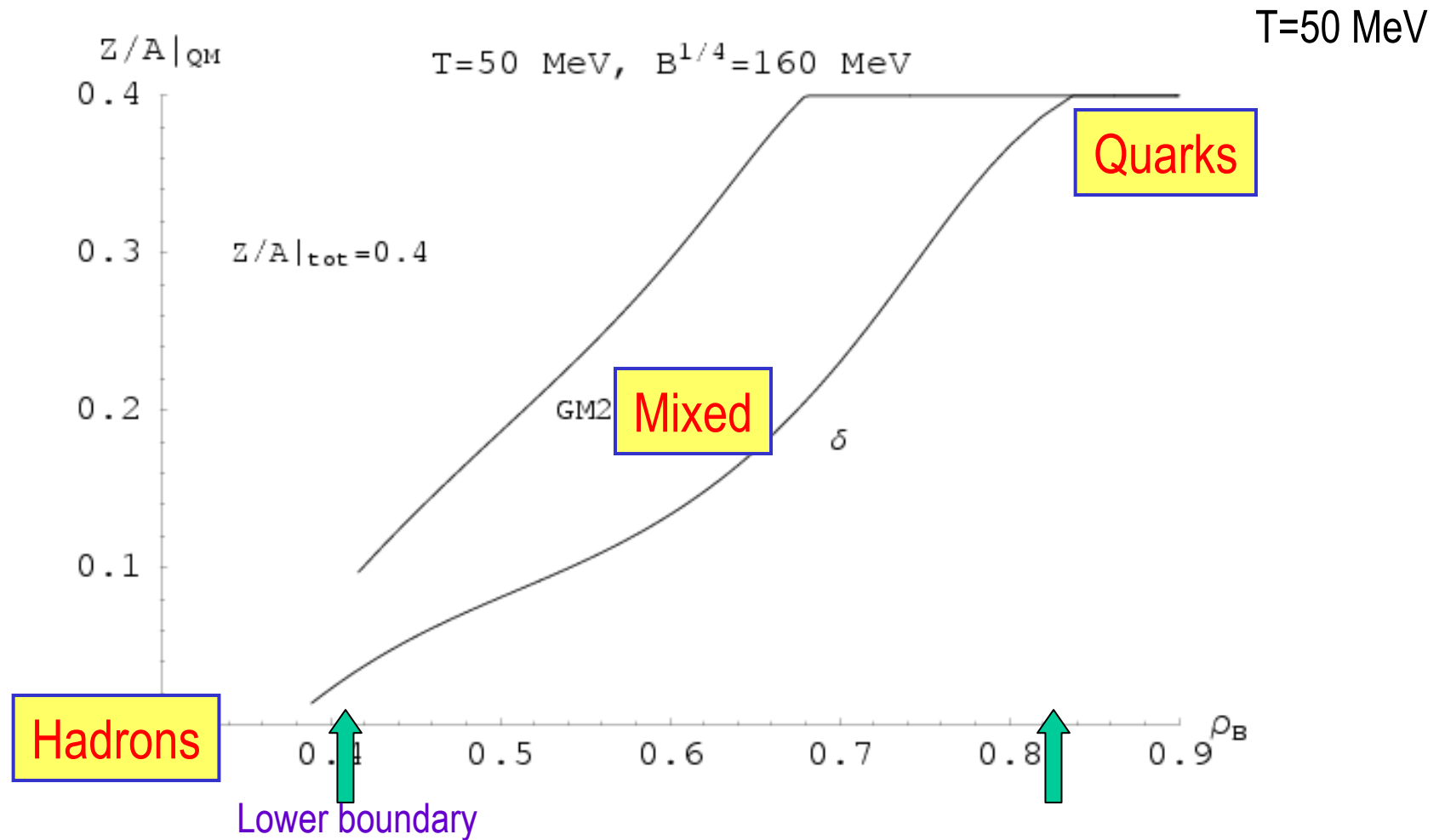


U+U, 1AGeV, semicentral

Reduction of the crossing density vs. T :
delta-meson very efficient!



Isospin content of the Quark Clusters in the Mixed Phase



Signatures? Neutron migration to the quark clusters (instead of a fast emission)

ISOSPIN IN RELATIVISTIC HIC: EOS - SENSITIVE OBSERVABLES

1. *Fragments in Central Collisions: N/Z*
2. *n – p collective flows* $\longrightarrow$ *light isobar flows*
3. *Kaon Yields , (π^-/π^+) ? , flows?*
4. *Iso – stopping*
5. *Deconfinement precursors*

Genuine relativistic effects: - boosting of vector potentials
- baryon and scalar densities
(vector vs. scalar field competition)
- Dirac masses

People: M.Colonna, M.Di Toro, **G.Ferini**, Ch.Fuchs,
Th.Gaitanos, **V.Greco**, Liu Bo, **V.Prassa**,
E.Santini, **S.Yildirim** and H.H.Wolter
+ **A.Drago**, **A.Lavagno**

Relativistic Transport Dynamics

Effective Lagrangian → Transport Equations → Event simulation

Hadronic: High Baryon and Isospin Densities → New Physics?

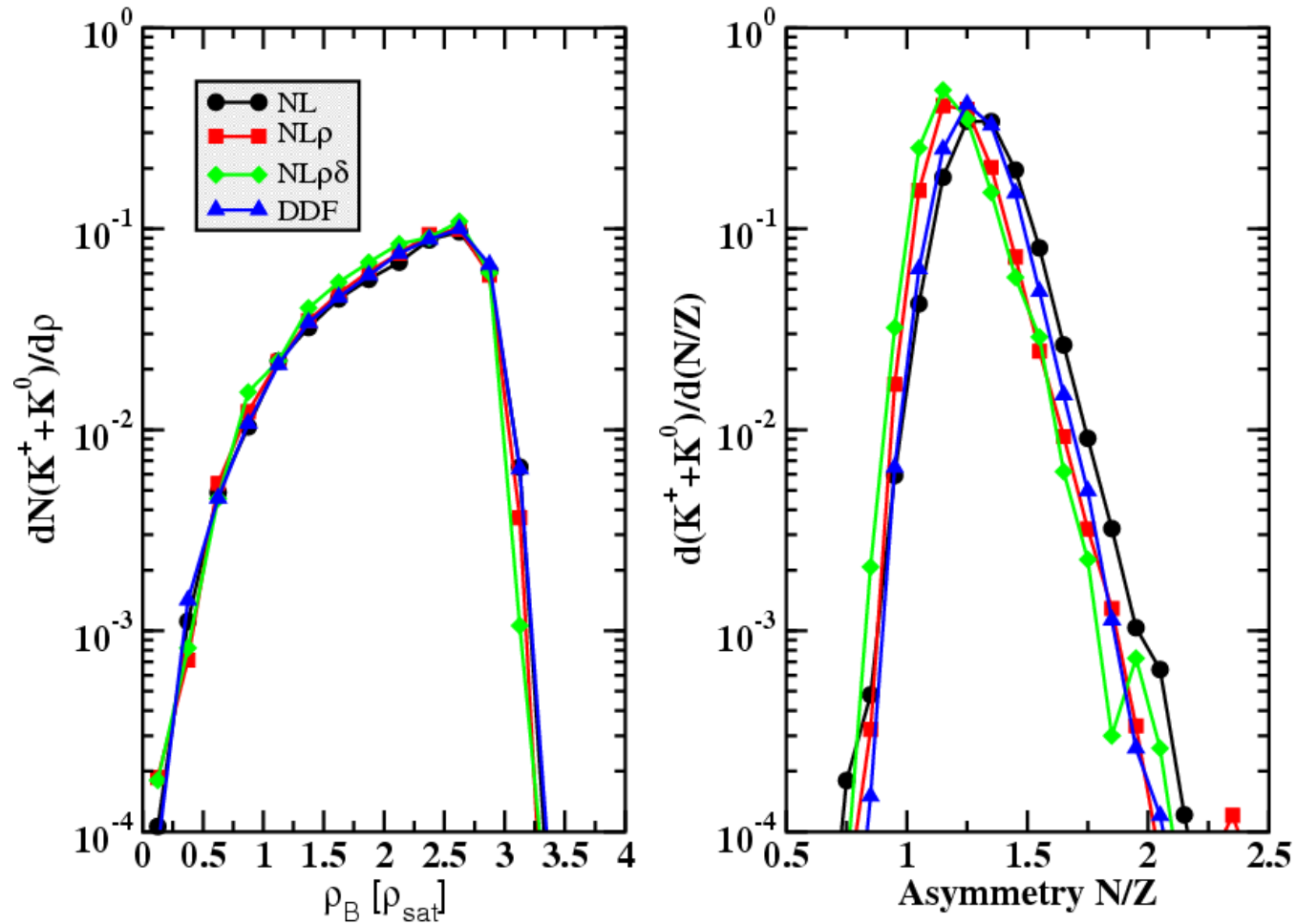
- Pion, proton multiplicities (saturation?)
- Meson, baryon spectra vs. transverse momentum
- Elliptic Flows (EOS softening?)
- Isospin structure of particles at high p_t

Partonic: beyond “Cascade”

- Hadronization (coalescence) dynamics
- Hydro limit
- Collective flows
- Spinodal mechanism for hadronization

Au+Au 1A GeV: density and asymmetry of the Kaon source

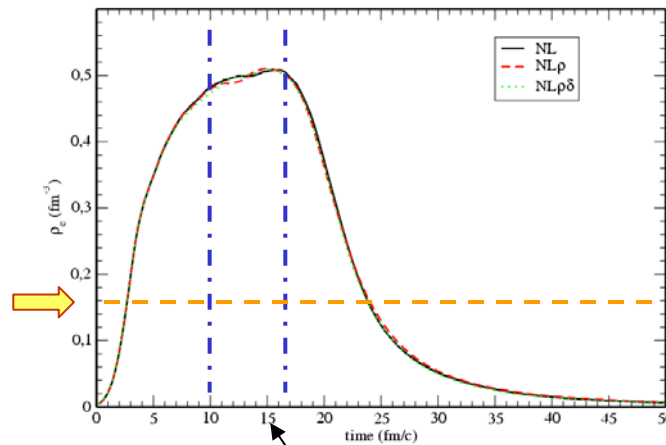
Au+Au@1.0A GeV, $b=0\text{fm}$



NL $\rightarrow$ DDF $\rightarrow$ NL ρ $\rightarrow$ NL $\rho\delta$: more neutron escape and more $n \rightarrow p$ transformation
(less asymmetry in the source)

Au+Au 1A GeV: density and isospin of the Kaon source

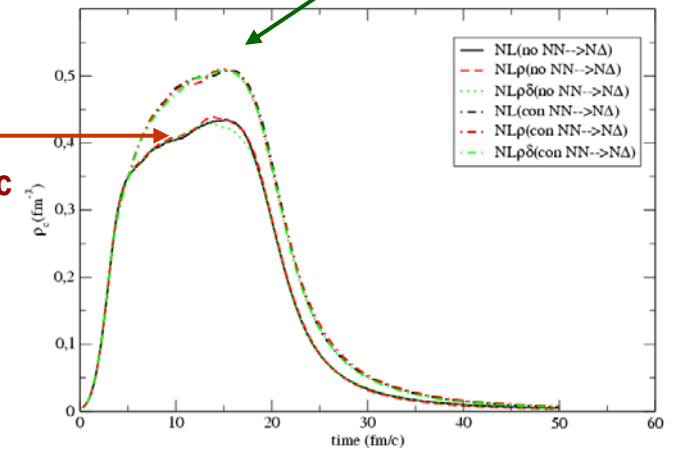
“central”
density



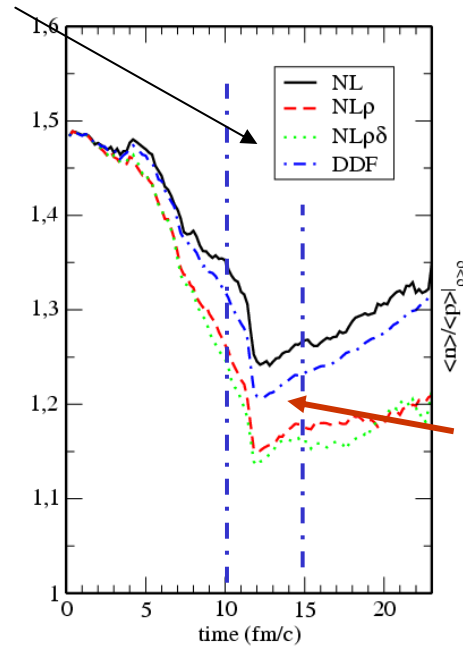
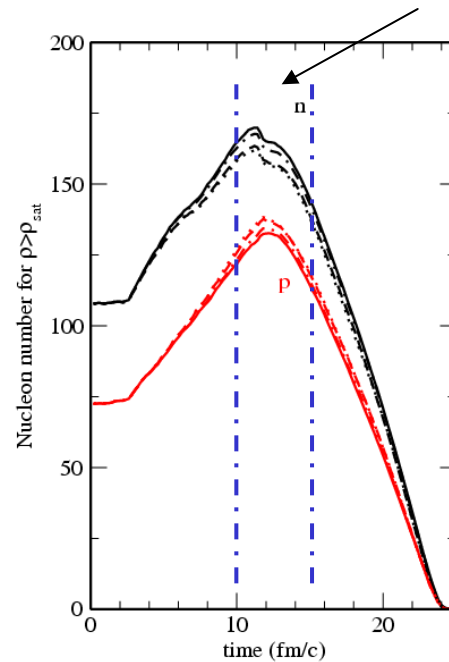
Time interval of Kaon production

only
elastic

Higher baryon density
with inelastic channels



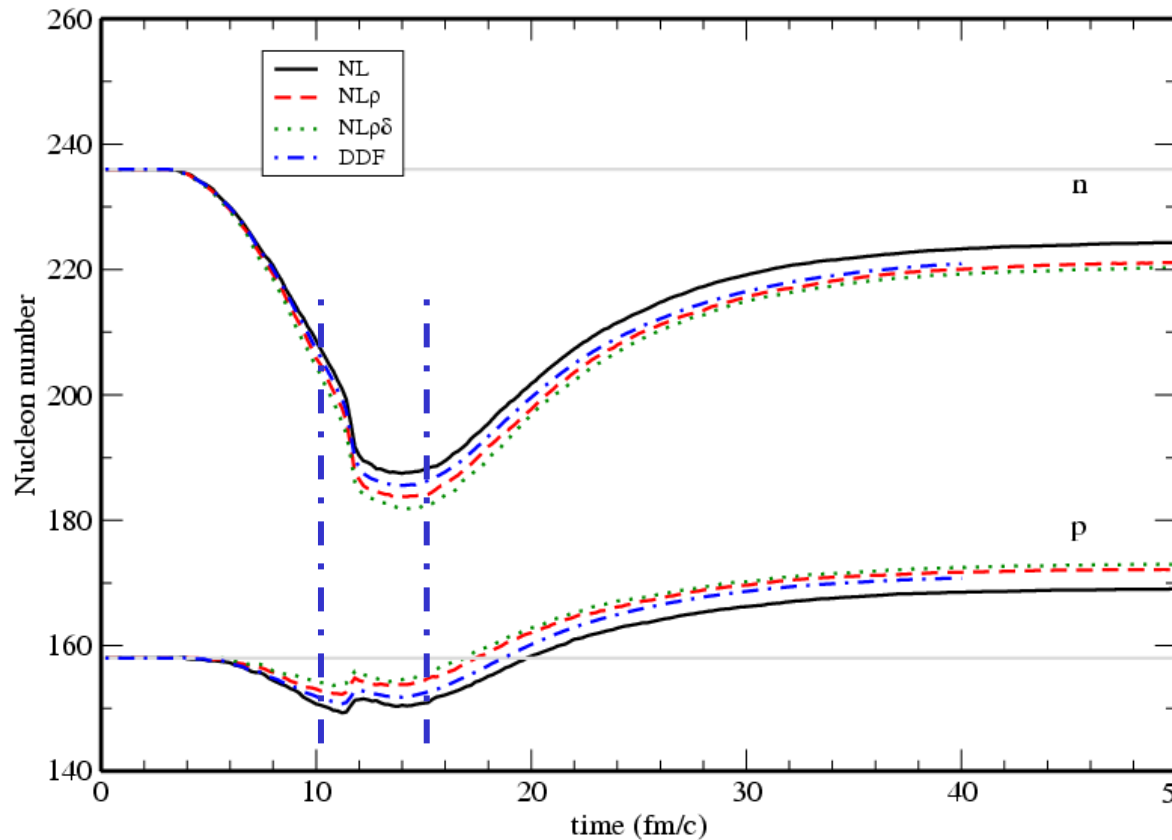
n, p at
High density



n/p at
High density

Drop:
Competition of fast neutron emission
and
Inelastic channels:
n→*p* transformation

Au+Au 1A GeV: time evolution of the total number of nucleons



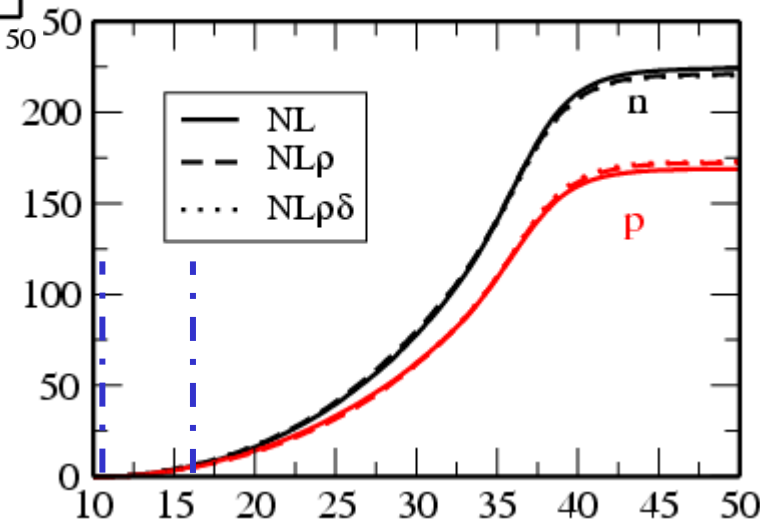
*Large $n \rightarrow p$ transformation
at early times:
Less asymmetry in the Kaon source*

f_ρ increasing sequence
 $NL < DDF < NL\rho < NL\rho\delta$

$\rho < 0.02 \text{ fm}^{-3}$

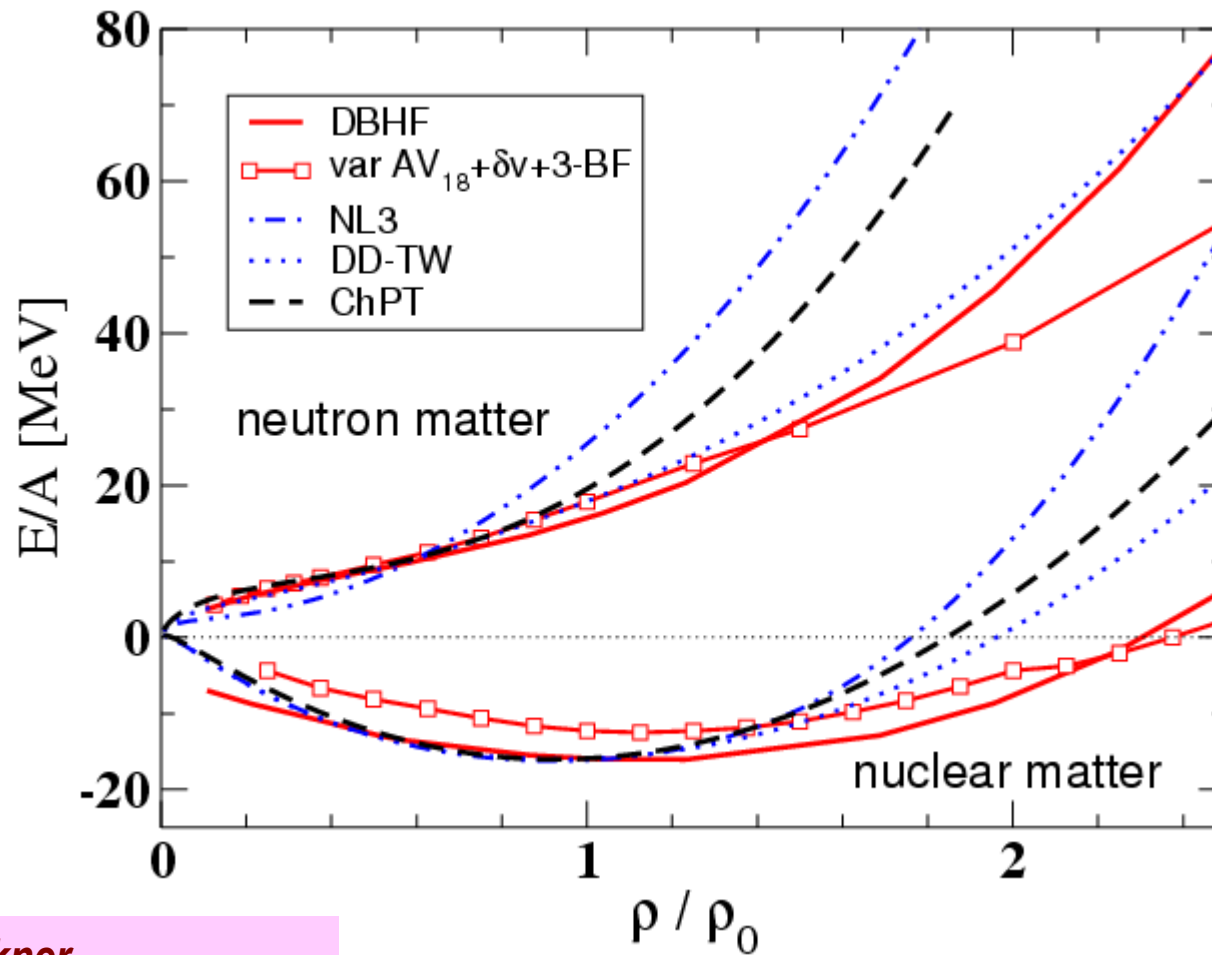
Check:
 π^-/π^+ , n/p , $K(0)/K(+)$ vs
emission time (P_t)

Free nucleons



→ Different behavior at lower energies, reduced inelastic competition

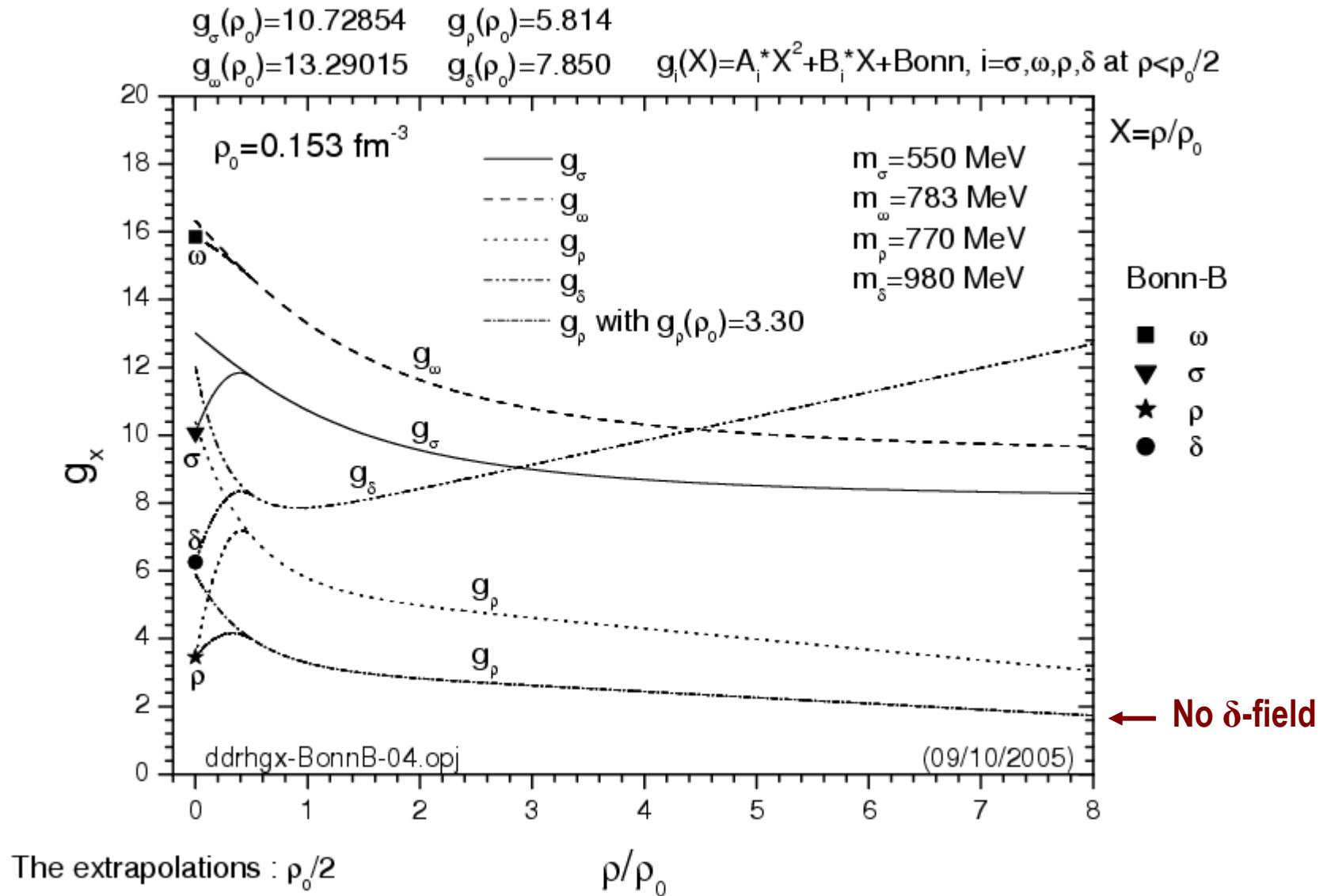
EOS of Symmetric and Neutron Matter



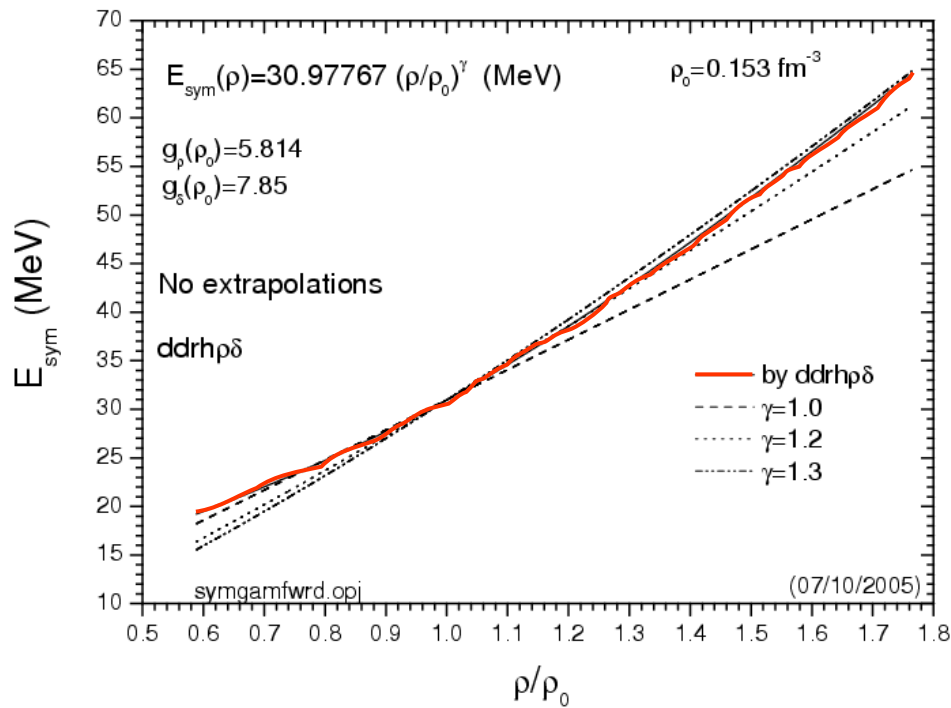
Dirac-Brueckner
 Variational+3-body(non-rel.)
 RMF(NL3)
 Density-Dependent couplings
 Chiral Perturbative

Ch.Fuchs, WCI Final Report 2006

DBHF-density dependent effective couplings

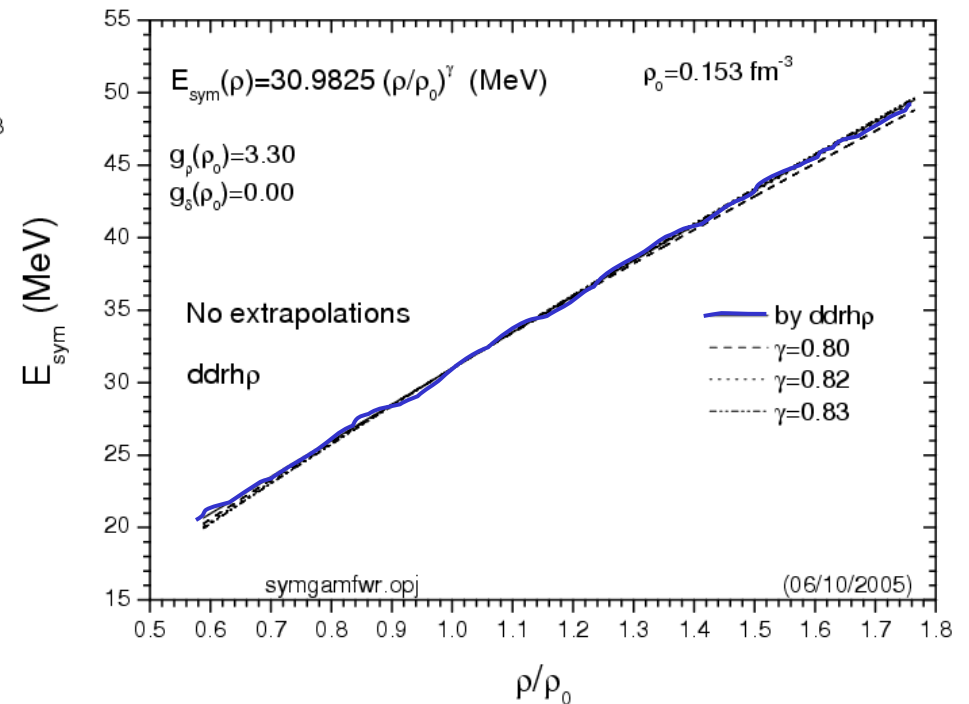


Symmetry energy around saturation (1)

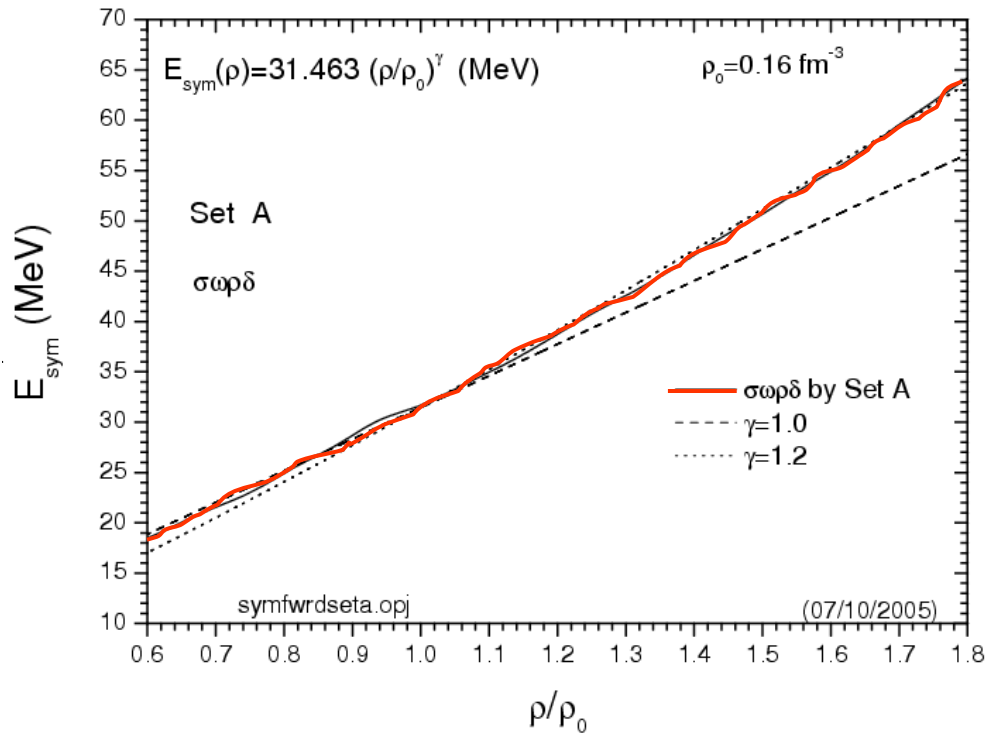


Liu Bo calcs.

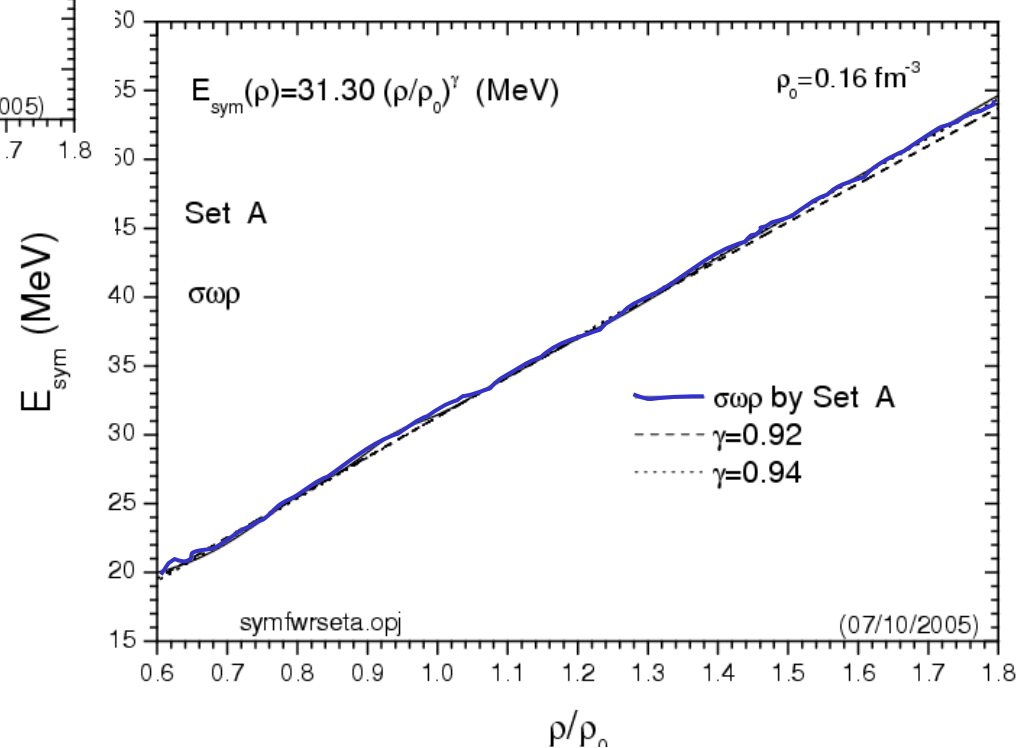
RMF calculation with Density dependent couplings



Symmetry energy around saturation (2)

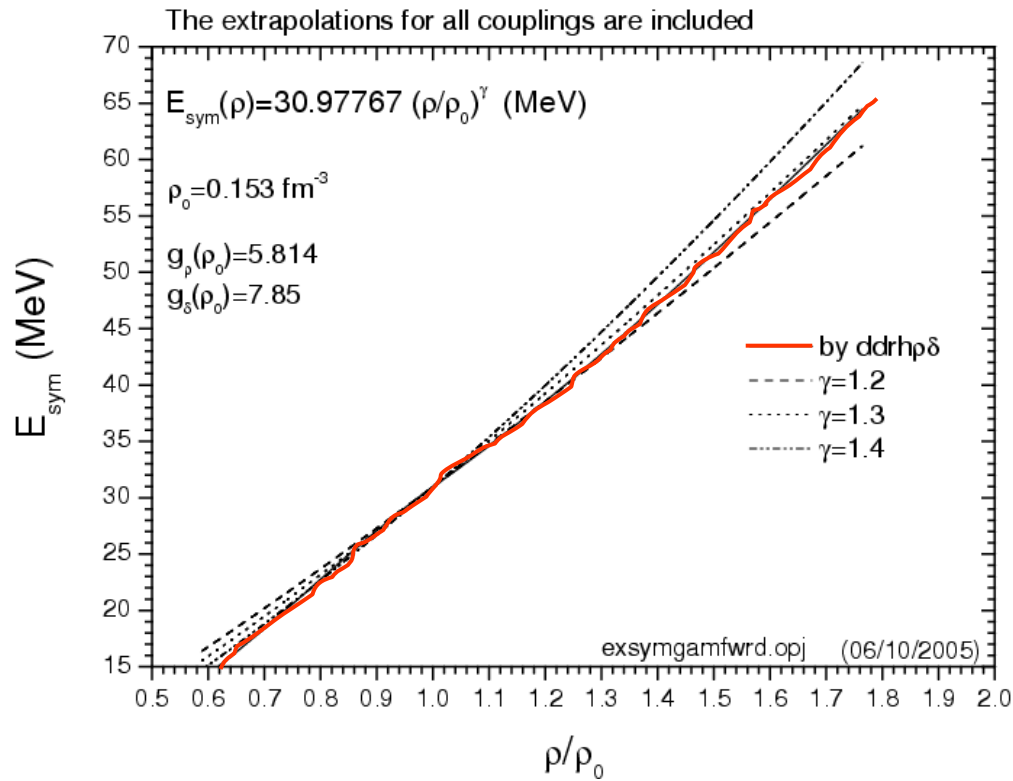


**RMF calculation with
constant couplings**

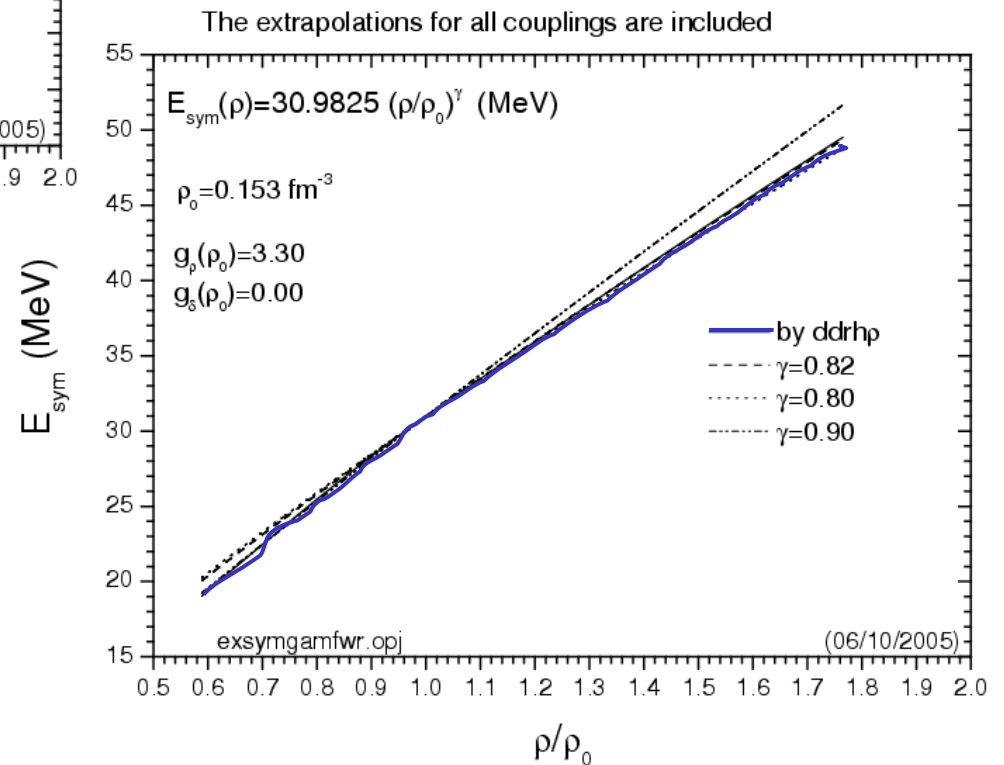


Liu Bo calcs.

Symmetry energy around saturation (3)



Density dependent couplings with free value extrapolations



Liu Bo calcs.